NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

The Role of Mathematics in Engineering Practice and in the Formation of Engineers

Volume 1 of 2

Eileen Goold

B. E., M.Eng.

Thesis Submitted for the Award of Doctor of Philosophy

National University of Ireland Maynooth

Department of Design Innovation

June 2012

Head of Department: Dr. Frank Devitt

Supervisor: Dr. Frank Devitt

TABLE OF CONTENTS

Page number

LIST OF FIGURES

Page number

LIST OF TABLES

Page number

SUMMARY

This research investigated the role of mathematics in engineering practice and whether there is a relationship between students' experiences with school mathematics and their choice of engineering as a career. The study was inspired by the observation that there is a lacuna in the scholarly literature concerning the nature of mathematics' role, if any, as a significant cause of the declining number of students entering professional engineering courses. Additionally there is currently no broad picture of the mathematical expertise required or used by practising engineers.

The population of interest in this study comprises professional engineers practising in Ireland. A sequential explanatory mixed methods design, where the subsequent collection and analysis of interview data builds on the survey findings, is employed. Engineers' use of mathematics is considered in three parts: *curriculum mathematics*, *mathematical thinking*, and *engaging* with mathematics. *Curriculum mathematics* usage is measured by a derivation of de Lange's mathematics assessment pyramid and with reference to three dimensions: mathematics domain, usage type, and academic level. *Thinking* usage relates to mathematical modes of thinking. *Engaging* usage is the motivation to take a mathematical approach. Engineers' experiences of school mathematics, factors that contributed to their engagement with school mathematics and the impact of their feelings about mathematics on their choice of engineering careers are investigated.

The findings show that (i) engineers' feelings about mathematics are a major influence on their choice of engineering as a career; (ii) teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics; (iii) while almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, mathematical *thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*; (iv) professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and role and their mathematical *thinking* usage is independent of discipline and role; (v) engineers show high affective engagement with

mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation; and (vi) the focus on "objective" solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice.

ACKNOWLEDGEMENTS

This existence of this thesis is the result of the combined efforts of a number of people who advised me and supported me during the past four and a half years.

In particular I wish to sincerely thank my supervisor, Dr. Frank Devitt: for the opportunity to conduct this research and for learning so much in doing so; for steering me through the design, development and completion of the study; and for guidance, patience and encouragement throughout the study. I will miss our enlightening conversations and I look forward to future collaboration. Thanks Frank.

I am grateful for the assistance given by Damien Owens, Registrar, Engineers Ireland and James Reilly, Statistician, Institute of Technology Tallaght.

I wish to thank all the study participants for their time and contributions to the study.

I also wish to acknowledge the encouragement and support from my family, friends and colleagues in both NUI Maynooth and the Institute of Technology Tallaght.

Finally, I wish to thank my daughter, Amy, for her patience, albeit at the promise of a dog, while I pursued my dream.

DEDICATION

I would like to dedicate this thesis to the memory of Suzi.

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

It is reported that "engineering has never mattered more" (National Academy of Engineering 2005; National Academy of Sciences et al. 2010; Robinson 2010; Sheppard et al. 2009; Tapping America's Potential Coalition 2008). However while engineering expertise is key to sustaining a modern economy and to the advancement of civilisation, the interest of young people to pursue careers as engineers has diminished, in western Europe and the USA in particular (Elliott 2009; Forfás 2008; King 2008; McKinsey 2011; Organisation for Economic Co-Operation and Development 2010). In Ireland the declining interest in engineering careers is evident in the dramatic reduction of CAO[1] points required for entry into level 8[2] engineering programmes in Ireland over the past twenty years. Less than 8% of all entrants to level 8 degree programmes in Irish universities choose engineering and technology programmes compared to 24% who chose Humanities and Arts subjects, 23% who chose Social Science, Business and Law subjects and 16% who chose Science subjects (Higher Education Authority 2011).

In Ireland there are two state administered exams: the Junior Certificate at mid secondary school (age 15) and the Leaving Certificate at completion of secondary school (age 18). Students sitting these exams can choose either the ordinary level mathematics curriculum or the more advanced higher level curriculum. Participation in higher level mathematics in Ireland is low, with only 45% of Junior Certificate mathematics students and 16% of Leaving Certificate mathematics students taking the higher level papers (State Examinations Commission 2011a).

It had been determined in Ireland, that mathematics achievement is a strong predictor of third level persistence generally (Mooney et al. 2010). However Ireland's

---

[1] CAO: Central Applications Office, Ireland's central administration for management of the competitive points system for entry to third level education.

[2] Level 8: Honours Bachelor Degree.

PISA[3] performance in mathematics is below the OECD average score and is showing a declining trend over recent years (Perkins et al. 2010). A national survey of Junior Certificate students found that almost 60% found mathematics difficult and less than 50% found the subject interesting (National Council for Curriculum and Assessment 2007). A major revision of the school mathematics curriculum is currently taking place in Ireland, under the direction of the National Council for Curriculum and Assessment (NCCA). The new initiative called "Project Maths" involves the introduction of revised syllabi for both Junior and Leaving Certificate mathematics. According to the NCCA, Project Maths "involves changes to what students learn in mathematics, how they learn it and how they will be assessed" (National Council for Curriculum and Assessment 2010b).

It is widely thought that mathematics is the "the key academic hurdle" in the supply of engineering graduates (Croft and Grove 2006; King 2008). Students wishing to pursue an engineering degree course are required to be proficient in mathematics. In Ireland the entry requirement to level 8 accredited engineering courses is a grade of C3 (≥55%, <60%) or higher in higher level Leaving Certificate mathematics. Lynch and Walsh (2010) have shown this minimum mathematics requirement contributes to students' hesitancy in pursuing an engineering degree course and there is a link between students' experience of second level subjects and their perception of future careers (Lynch and Walsh 2010). Many students have "no idea" what role mathematics will play in their future careers (Wood et al. 2011). Most students view engineering education as further engagement in school science and mathematics (Brickhouse et al. 2000). "Some see mathematics as the gateway to engineering, paving the way for sound design; others see mathematics as a gatekeeper, denying entry to otherwise talented would-be engineers" (Winkelman 2009). Many third level engineering students struggle with the mathematics in their courses (James and High 2008) and "it is now generally accepted that students entering the tertiary level suffer a lack of mathematical skills and no longer find mathematics to be an enjoyable

[3] PISA: Programme for International Student Assessment, worldwide evaluation in OECD member countries of 15-year-old school pupils' scholastic performance.

subject … this decline in mathematical skills leads students to avoid overly analytical subjects in later years of degree programmes" (Irish Academy of Engineering 2004).

For decades mathematics has been regarded as the fundamental knowledge underpinning engineering practice. Besides this, it is arguable that traditional engineering careers cannot interest modern young people to the same extent as twenty years ago. In the same period, technology usage and associated practices in the broader society have changed significantly and young people's ranges of interests, skills and activities have altered dramatically. The average modern teenager lives in a world of mobile phones, iPods and iPads where communications, information and entertainment are now available anytime, anywhere and at low cost. In addition there is a belief among some practising engineers that the mathematics they learned is not applicable to their work (Cardella 2007; Pearson 1991; Underwood 1997). There is a view that mathematical and engineering worlds are very different and it is reported that there is a significant difference between what a mathematician calls "doing mathematics" and what an engineer calls "doing mathematics" (Bissell and Dillon 2000). There is also a view, with advancements in technology, knowledge diffusion and almost instant information availability, that teaching "engineers to think analytically will be more important than helping them memorise algebra theorems" (Katehi 2005). There is a further view that the human and "societal aspect" of engineering practice is becoming increasingly important "with constraints on engineering solutions becoming less and less technical and more and more societal, regulatory and human" (Grimson 2002).

Research suggests that while professionals in numerate fields draw upon their mathematics school learning, they do so in a distinctly different manner from the way in which they experienced mathematics in school. However, in the case of engineering practice, research concerning the type of mathematics used by engineers in their work is sparse (Alpers 2010c; Cardella 2007; Gainsburg 2006; Trevelyan 2009). While there are a number of studies that investigate engineers' use of mathematical thinking, most of these are conducted in academic workplaces. Difficulties associated with investigating "real" engineers' mathematics usage are that access to engineers is difficult and with many different branches and job profiles within engineering, there is no unique identity as "'the' engineer". Furthermore studies of engineers' use of mathematics have tended to take a qualitative approach that involve a small number of engineering functions and engineers and thus the findings may not represent engineers generally (Alpers 2010b).

1.2 MOTIVATION

This research was inspired by the observation that there is a lacuna in the literature concerning the nature of mathematics' role, if any, as a significant cause of the declining number of students entering professional engineering courses. On the one hand, students' difficulty with higher level school mathematics is often blamed for the declining number of entrants to engineering degree courses (Croft and Grove 2006; King 2008; Prieto et al. 2009). Coupled with this, there is a view that engineering is not mathematics, and the close linkage between the two that exists in the public perception negatively influences the perception of engineering (Winkelman 2009). On the other hand research concerning the mathematical expertise that is in fact used in engineering practice is sparse (Alpers 2010b; Cardella 2007; Trevelyan 2009). The many different branches of engineering (e.g. civil, electronic and mechanical) and the many interpretations of mathematical activity (e.g. school mathematics, mathematical thinking and understanding) present obstacles to investigating the role of mathematics in general engineering practice (Alpers 2010b).

There is currently no broad picture of the mathematical expertise required or used by practising engineers. A goal of this project is to address this lacuna and provide a research-based insight into the role of mathematics in engineering practice.

The decline in engineering and technology degree enrolments is a major threat to global economic growth (Borrus and Stowsky 1997 ; Boskin and Lau 1992; Boskin and Lau 1996; Grübler 1998; Solow 1957). Interventions such as attempts to improve school mathematics grades, introduction of engineering science subjects in schools, students' participation in engineering projects and activities and students' exposure to engineering role models have not regenerated students' interest in engineering careers (Heywood 2005). It Ireland, the NCCA has observed that many students have a disaffection with mathematics (National Council for Curriculum and Assessment 2007). Further, many students in Ireland with demonstrated high ability in mathematics choose non-numerate careers (Higher Education Authority 2011; State Examinations Commission 2011a). Career choice theory suggests that interest, values, self-efficacy, emotional experiences and socialiser's attitudes are the major career choice influencers (Ginzberg et al. 1951). There is a corresponding view that enriching students' mathematics experiences holds the key to increasing enrolments in engineering education (Maltese and Tai 2011; Prieto et al. 2009). Hence, the second goal of this project is to provide a research-based insight into the relationship between students' experiences with school mathematics and whether they chose engineering as a career.

1.3 RESEARCH QUESTIONS

There are two main research questions in this study.

1. What is the role of mathematics in engineering practice?
    a) How can mathematics usage in engineering practice be measured?
    b) How do engineers use mathematics in their work?
    c) What motivates engineers to engage, or not, with mathematics?

2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?
    a) To what degree do students' feelings about mathematics influence engineering career choice?
    b) What factors in mathematics education influence students' affective engagement with mathematics?

1.4 SIGNIFICANCE

The main aim of this study is to generate new knowledge in relation to engineers' mathematics usage in their work and to determine if mathematics experiences influence school-leaving students' decisions to choose engineering careers. It is anticipated that the findings from this study will contribute to knowledge on the worldwide problem of students' declining interest in engineering careers. It is anticipated that new knowledge on the value of mathematics in engineering practice will inform prospective engineering students and, particularly, engineering educators and the engineering profession. Given that mathematics is of central importance to modern society and is crucially important, too, for the employment opportunities and achievements of individual citizens, the findings of this study will have implications for school mathematics education, engineering education, engineering practice and society generally.

1.5 ORGANISATION OF THESIS

This thesis describes a mixed methods approach to investigating the role of mathematics in engineering practice and the relationship, if any, between students' experiences with school mathematics and whether they chose engineering as a career. The thesis comprises two volumes.

Volume 1

Chapters 1 to 8 and the associated references are included in Volume 1. The remaining chapters in Volume 1 are organised as follows:

Chapter 2: Literature Review

This chapter contains a review of literature about mathematics education, career choice, engineering education and engineering practice. The purpose of this chapter is to establish the current available knowledge about the role of mathematics in engineering practice and its role in engineering career choice. Included in this chapter are: an exploration of what mathematics is; the different general learning theories relating to mathematics learning and teaching; career choice factors and the selection of engineering careers; a review of mathematics in engineering education; a discussion about engineering practice; and a summary of research concerning engineers' use of mathematics.

Chapter 3: Research Design

This chapter describes the study design. The purpose of this chapter is to describe the research methodology employed and the study design for measuring engineers' mathematics usage and for determining whether or not engineers' feelings about mathematics influenced their choice of career. Included in this chapter are: a background theory based framework for the research design; a description of the methodology employed to measure engineers' mathematics usage which is based on de Lange's mathematics assessment pyramid and Project Maths; a description of the methodology employed to measure engineers' feelings about mathematics which is based on motivation theory; the rationale for choosing a sequential explanatory strategy mixed methods (survey followed by interviews) research design; data collection methodologies; identification of the study population; quality considerations; and ethical considerations.

Chapter 4: Survey Methodology and Data Analysis

This chapter presents the methodology used for the collection and analysis of quantitative data from practising engineers in relation to the research questions. The purpose of this chapter is to show how the quantitative first phase of the study was conducted. Included in this chapter are: identification of the survey population; design of the survey questionnaire; survey administration and data collection; and a description of the methodologies used to analyse the survey data.

Chapter 5: Survey Findings

This chapter presents the results of the survey data analysis. The purpose of the chapter is to present the survey findings. Included in this chapter are: five main survey findings; generalisation of the survey findings; and a discussion of the survey findings.

Chapter 6: Interview Methodology & Data Analysis

This chapter presents the methodology used for the collection and analysis of qualitative data from a sample of practising engineers in relation to the research questions and the survey findings. The purpose of this chapter is to show how the qualitative second phase of the study was conducted. Included in this chapter are: a description of the methodology used to select interview participants; the interview design; the process of conducting the interviews; and the interview data analysis.

Chapter 7: Interview Findings

This chapter presents the results of interview analysis involving a sample of practising professional engineers in relation to the research questions and the survey findings. The purpose of the chapter is to present the interview findings. Included in this chapter is a discussion of the ten themes that emerged from the interview data.

Chapter 8: Concluding Discussion

This chapter discusses the overall findings. The purpose of this chapter is to present the overall findings and conclusions. Included in this chapter are: a summary of the interview findings in the context of both the survey findings and the two main research questions; a discussion of both the survey and interview findings; contributions to research knowledge; implications of this new knowledge; limitations of the methodology employed; and suggestions for further work.

Volume 2

The appendices are included in Volume 2. These are:

Appendix 1: Survey Questionnaire

A copy of the survey questionnaire distributed to practising professional engineers and used to collect quantitative data is included in Appendix 1 in Volume 2 of this thesis.

Appendix 2: Survey Support Document

A copy of a separate "Survey INFO" document that accompanied the survey questionnaire is included in Appendix 2 in Volume 2 of this thesis. This survey support document was designed to assist survey participants when completing the questionnaire and it describes and illustrates each of the five mathematics usage types that are measured in the survey analysis. The document also contains instructions for completing and returning the survey questionnaire.

Appendix 3: Survey Distribution Emails

Copies of survey distribution emails and notices are included in Appendix 3 in Volume 2 of this thesis. Engineers Ireland, the professional body representing engineers in Ireland, distributed the survey questionnaire and the survey support document by direct email, to its 5,755 chartered members. Engineers Ireland also included a direct link to the survey questionnaire on its weekly newsletters on 9th and 16th March, 2011 which were emailed to its entire 21,700 members.

Appendix 4: Survey Data Analysis

Survey analysis is included in Appendix 4 in Volume 2 of this thesis.

Appendix 5: Interview Participants' Emails

A copy of the email sent to a sample of practising Chartered Engineers requesting their participation in the interview study is included in Appendix 5 in Volume 2 of this thesis.

Appendix 6: Interview Protocol

A copy of the interview protocol compiled to assist the semi-structured interview process is included in Appendix 6 in Volume 2 of this thesis. An interview protocol is a list of questions and predetermined inquiry areas that the interviewer wants to explore during each interview and it helps to make interviewing multiple participants more systematic.

Appendix 7: Interview Participants' Stories

The interview participants' stories are included in Appendix 7 in Volume 2 of this thesis. These are engineers' individual stories about their background, their mathematics education experiences, their career decisions and their work in engineering practice.

Appendix 8: Interview Data Codes

The interview data codes are included in Appendix 8 in Volume 2 of this thesis. In the first cycle of coding, 107 descriptive codes, representing sections of the transcript data that were likely to be helpful in addressing the research questions, were identified. Following subsequent coding cycles, ten overarching themes, characterising key concepts of the analysis, emerged from the data.

Appendix 9: Interview Data Analysis

Interview data analysis is included in Appendix 9 in Volume 2 of this thesis. This includes: a profile of interviewees; a profile of engineers' mathematics teachers; engineers' motivation to engage with school mathematics; feelings about engineering mathematics; feelings about mathematics in engineering practice; feelings about mathematics outside of engineering; engineers' paths to engineering education; engineers' job descriptions; engineers' views about engineering practice; engineers' *curriculum mathematics* [4] usage; engineers' *curriculum mathematics* usage by discipline and role; engineers' views about and usage of mathematics in engineering practice; the need for a mathematical approach in engineering practice; and the value of mathematics education in engineering practice.

[4] Curriculum mathematics: Term devised in this study to represent engineers' mathematics education at school and university.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a review of research literature relevant to the two main research questions:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

The literature review is organised under six themes:

Page number

2.2 MATHEMATICS

2.2.1 What is Mathematics?

In order to investigate the role of mathematics in engineering practice and in the formation of engineers, there is a need to explore what mathematics is. As expected, there are many different perspectives of what mathematics is. Most people consider mathematics to comprise arithmetic, algebra, geometry, trigonometry, statistics and probability, a subset of logical thinking and/ or a mechanism for reasoning. In 1962, some 75 well-known U.S. mathematicians produced a paper wherein they stated "to know mathematics means to be able to do mathematics: to use mathematical language with some fluency, to do problems, to criticize arguments, to find proofs, and, what may be the most important activity, to recognise a mathematical concept in, or to extract it from, a given concrete situation" (Ahlfors et al. 1962).

Defining mathematics is conditional since each person and even each time period, tends to emphasise different aspects of the subject. Many people have attempted to define or describe mathematics and words such as logical ideas, interconnected ideas, relationships, patterns, communications and numbers appear regularly in such descriptions. Orton and Wain (1994) define mathematics as "an organised body of knowledge, an abstract system of ideas, a useful tool, a key to understanding the world, a way of thinking, a deductive system, an intellectual challenge, a language, the purest possible logic, an aesthetic experience, a creation of the human mind" (Orton and Wain 1994). Greer and Mukhopadhyay (2003) say that mathematics is characterised as "the purest form of reasoning, embodying the highest standards of proof; and as a training in dispassionate, objective, rational thinking" (Greer and Mukhopadhyay 2003). Paul Ernest from the University of Exeter in the United Kingdom presents two perspectives of mathematics, one is the "absolutist" perspective where maths is viewed "as an objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic." His other view is the "fallibilist" philosophy of mathematics where mathematics is viewed as "human, corrigible, historical and changing … the outcome of social processes … open to revision" (Ernest 2004b).

Mathematics is often associated with certainty and with being able to get the right answer. For example, Lampert (1990) suggests that "doing" mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question and a mathematical "truth" is determined when the answer is ratified by the teacher" (Lampert 1990). However there appears to be a distinction between mathematics as a study subject and mathematics that is useful. Thomas Romberg (1992) is of the view that rather than "passing on a fixed body of mathematical knowledge by telling students what they must remember and do … society today needs individuals who can continue to learn, adapt to changing circumstances, and produce new knowledge". He says this mathematical literacy "involves moving beyond a knowledge of concepts and procedures produced by others to gathering and interpreting information about open-ended problems, making conjectures, and building arguments to support or reject hypotheses" (Romberg 1992). Burton's (2004) view of mathematics as the "product of people and societies" contrasts with the commonly held view of mathematics "as objective knowledge, codified and transmitted inertly and separated from the people who learn and do mathematics" (Burton 2004). According to Chambers (2008), pure mathematicians are of the view that mathematics is: "objective facts"; "a study of reason and logic"; "a system of rigour, purity and beauty"; "free from societal influences"; "self-contained"; and "interconnected structures". The purist view of mathematics is that "applications are inferior to the set of structures that make up pure mathematics" and "mathematics is a higher-level intellectual exercise, an art form and an example of the creativity of the human mind". With a focus on economic success, applications became the most important part of mathematics in the 1980s when learning how to do mathematics was perceived to be more important than understanding the underlying principles. Since then mathematics is often characterised as "a tool for solving problems, the underpinning of scientific and technological study and providing ways to model real situations" (Chambers 2008). According to Evans (2000) doing mathematics includes processing, interpreting and communicating numerical, quantitative, spatial, statistical mathematical information in ways that are appropriate for a variety of contexts (Evans 2000).

Ernest (2010) believes that there is much more to mathematics than numbers and what is taught in school and that there are many reasons for and capabilities desired in teaching and learning mathematics. He lists three types of necessary mathematics, these are: functional numeracy (for successful functioning in society and minimum requirement for general employment at end of schooling); practical work-related knowledge (solve industry and work-centred practical problems, not necessary for all) and advanced specialist knowledge (specialist high school or university mathematics needed by a minority). He adds that there is also mathematics that has “personal, cultural and social relevance”. This includes deploying mathematical knowledge and powers in both posing and solving mathematical problems, being confident in one’s personal knowledge of mathematics and being able to identify and critique the mathematics embedded in social, commercial and political systems. Ernest’s last capability is an appreciation of mathematics as an element of culture including its role in history, culture and society in general. Ernest lists some of the “big ideas of mathematics” such as: “pattern; symmetry; structure; proof; paradox; recursion; randomness; chaos and infinity” (Ernest 2010).

Given the ubiquitous use of information technology in the workplace, Hoyles, Wolf, Molyneux-Hodgson and Kent (2002) found that mathematical skills in the workplace are changing and “mathematical literacy” is displacing numeracy in the workplace. They say that mathematical literacy reflects the skills needed in businesses and the communication of mathematically expressed decisions and judgements within businesses (Hoyles et al. 2002). De Lange (2001) defines mathematical literacy as “an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen” (De Lange 2001). Hoyles, Noss, Kent and Bakker (2010) introduce the term “techno-mathematical literacies” whereby individuals “need to be able to understand and use mathematics as a language that will increasingly pervade the workplace through IT-based control and administration systems as much as conventional literacy has pervaded working life for the last century" (Hoyles et al. 2010). In the context of engineering where mathematics is regarded as the fundamental undergirding engineering practice, Radzi, Abu, and Mohamad (2009) are of the view that mathematics "should not merely serve as a subject that provides only the basic knowledge needed in engineering" but as importantly, "to inculcate essential and effective critical thinking skills". Mathematics oriented thinking skills include "the ability to interpret information presented in a mathematical manner and to use mathematics accurately to communicate information and solve problems" (Radzi et al. 2009). Another perspective of mathematics in engineering is that the engineer's burden of truth is lighter than that of the mathematician where truth is nothing less than absolute, generalised proof. According to Chatterjee (2005) "the unique charm of mathematics in engineering lies in the many levels and forms in which it is evoked, revoked, used, abused, developed, implemented, interpreted and ultimately put back in the box of tools, before the final engineering decision, made within the allotted resources of time, space and money, is given to the end user" (Chatterjee 2005).

Given the importance of mathematics outside the classroom, mathematics within the classroom is evolving from "objective knowledge" to being mathematically prepared for an increasingly technological world. Mathematics curricula and instruction are being transformed. For example, in the context of the new "Project Maths"[5] mathematics curriculum in Ireland, the National Council for Curriculum and Assessment (NCCA) state that "mathematics is a wide-ranging subject with many aspects. On one hand, it is about pattern; the mathematics of which can be used to explain and control natural happenings and situations; it is about logical analysis; and it provides the basic language and techniques for handling many aspects of everyday and scientific life. On the other hand, it deals with abstractions, logical arguments, deduction, calculation and fundamental ideas of truth and beauty, and so it is an intellectual discipline and a source of aesthetic satisfaction" (National Council for Curriculum and Assessment 2010a). Rather than assess mathematical knowledge, the

[5] Project Maths: Major revision of the second level school mathematics curriculum in Ireland.

OECD Programme for International Student Assessment (PISA) assesses students' mathematical literacy. Students' mathematics literacy is assessed in relation to: content (space and shape, change and relationships, quantity, uncertainty); competencies (reproduction, connections, reflection) and situations (personal, educational/ occupational, public, scientific). PISA uses six proficiency levels to represent groups of tasks of ascending difficulty ranging from level 1 where students can answer questions involving familiar contexts where all relevant information is present and the questions clearly defined, up to level 6 where students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations (Organisation for Economic Co-Operation and Development 2009). Another international assessment of students' mathematics is the Trends in International Mathematics and Science Study, (TIMSS)[6]. In the TIMSS assessment, mathematics is classified into "content domains" and "cognitive domains". The 2011 framework has four content domains: number (30%); algebra (30%); geometry (20%); and data and chance (20%) and three cognitive domains: knowing (35%); applying (40%); and reasoning (25%) (International Association for the Evaluation of Educational Achievement 2011).

Rather than present school mathematics in the traditional sense of lists of topics, Niss (2003) identifies eight competencies in mathematics, these are: thinking mathematically (mastering mathematical modes of thought); posing and solving mathematical problems; modelling mathematically (analysing and building models); reasoning mathematically (proof and proving); representing mathematical entities (objects and situations); handling mathematical symbols and formalisms; communicating in, with, and about mathematics and making use of aids and tools (information technology included). He is of the view that each mathematical competency has three dimensions: the degree of coverage (the extent to which the person masters the characteristic aspects of the competence); the radius of action (the spectrum of contexts and situations in which the person can activate that competence); and the technical level (how conceptually and technically advanced the

[6] TIMMS: Trends in International Mathematics and Science Study, an international assessment of the mathematics and science knowledge of fourth grade and eighth grade students around the world.

entities and tools are with which the person can activate the competence) (Niss 2003).

2.2.2 Mathematical Thinking

Mathematical thinking is a form of mathematics that that is considered necessary in many workplaces. According to Breen and O'Shea (2010), mathematical thinking involves "conjecturing, reasoning and proving, abstraction, generalisation and specialisation" (Breen and O'Shea 2010). Schoenfeld (1992) is of the view that a mathematics "curriculum based on mastering a corpus of mathematical facts and procedures is severely impoverished" and especially lacking in mathematical thinking. He says that mathematics is multidimensional and he considers metacognition, beliefs and mathematical practices as critical aspects of thinking mathematically (Schoenfeld 1992). According to Schoenfeld, "learning to think mathematically means (a) developing a mathematical point of view – valuing the processes of mathemisation and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure – mathematical sense-making". Schoenfeld's five aspects of mathematical thinking are: the knowledge base; problem solving strategies; effective use of resources; mathematical beliefs and affects; and engagement in mathematical practices. The knowledge base includes: "informal and intuitive knowledge about the domain; facts and definitions, and the like; algorithmic procedures; routine procedures; relevant competencies; and knowledge about the rules of discourse in the domain". Schoenfeld notes the limited capacity of short term memory and the complexity of accessing information from long term memory (Schoenfeld 1992). According to Ernest (2011) there are two forms of mathematics knowledge, these are explicit (theorems, definitions) and tacit (personal know how). Ernest's view is that knowledge is usually learned in a social context. He says that the transfer of learning between contexts often does not take place and that it is the social context that elicits the skills and knowledge from long term memory (Ernest 2011).

Problem solving in mathematics is the process of "doing" mathematics and differs from learning how to do "textbook" problems which are a reinforcement of knowledge (Ernest 2011). Problem solving strategies are methods or procedures that guide the choice of skills or knowledge to use at each stage in problem solving and they offer no guarantee of success. George Pólya (1945) developed systems of heuristics and he suggested ways of teaching problem solving strategies to students (Pólya 1945). Typical stages to problem solving are: understanding the problem: devising a plan; applying strategies; and reviewing the solution. Ernest (2011) lists the following thought strategies and processes: "imaging; representing; symbolising; explaining; describing; discussing; hypothesising; generalising; taking special cases; classifying; interpreting; rule-making; and proving" as part of the problem solving process. While problem solving includes cognitive activities such as using and applying mathematical knowledge, there is also a metacognitive aspect (Ernest 2011). Metacognition refers to monitoring, self-regulation and resource allocation during cognitive activity and problem solving. Metacognitive activities include "planning, controlling and monitoring progress, decision making, choosing strategies, checking answers and outcomes and so on" (Ernest 2011). Schoenfeld (1992) showed that students' problem solving performance is enhanced when engaging in self-monitoring and controlling activities. While there is little work on the effectiveness of teaching problem solving strategies to students, Schoenfeld's work demonstrates that teacher interventions can raise the level of metacognitive activity and effectiveness in problem solving among students (Schoenfeld 1992).

Schoenfeld (1992) is also of the view that an individual's beliefs and affects toward mathematics impact how and when they use mathematics and engage in mathematical thinking. The affective domain[7] includes a person's internal feelings, such as liking of mathematics, confidence in one's mathematical ability, anxiety towards mathematics and importance of mathematics. The affective domain in the context of mathematics learning is discussed in Chapter 3. Experiences in school mathematics form the basis for the conception, appreciation and images of

[7] The affective domain: The manner in which people deal with things emotionally, including for example feelings, values, attitudes and beliefs.

mathematics constructed by learners. Researchers have found a significant correlation between teachers' attitudes and student achievement in mathematics (Schoenfeld 1992).

Schoenfeld's fifth aspect of mathematical thinking is engagement in mathematical practices. While experience gained from engagement in mathematical performances leads to increased knowledge and confidence, Schoenfeld notes that there is a considerable difference between school mathematics and the way experts engage in mathematical practices. He suggest that mathematics classrooms should engage in practice type mathematics that includes: classroom discussions; defending claims mathematically, coming to grips with uncertainty; engaging in a science of patterns, extracting mathematical tools from the solutions of complex problems; reflecting on thought process; having a mathematical point of view and mathematical sense-making (Schoenfeld 1992).

2.2.3 Is Mathematics a Special Subject?

There is some evidence to suggest that mathematics is a special subject compared to other school subjects. According to Smith (2004) it is widely recognised that "mathematics occupies a rather special position". He refers to mathematics as "a major intellectual discipline," providing "the underpinning language for the rest of science and engineering and, increasingly, for other disciplines in the social and medical sciences," underpinning major sectors of modern business and industry and providing "the individual citizen with empowering skills for the conduct of private and social life and with key skills required at virtually all levels of employment" (Smith 2004). Smith identifies what is widely known as the 'mathematics problem' where mathematics education "fails to meet the mathematical requirements of learners, fails to meet the needs and expectations of higher education and employers and fails to motivate and encourage sufficient numbers of young people to continue with the study of mathematics post-16". He maintains that there is a tendency for schools to see choosing high level mathematics as a higher risk in terms of outcome than in many other disciplines (Smith 2004). A study of student participation in upper secondary mathematics education in 24 countries found evidence of students behaving strategically by not choosing mathematics, particularly advanced mathematics, because it is perceived as being more difficult than other subjects or one in which it is harder to achieve higher grades (Hodgen et al. 2010).

Compared to most other subjects, mathematics is a "hierarchical subject" where later learning depends critically on earlier learning and students perfect their technique at each lower level before they progress to the next level (Chambers 2008; Ridgway 2002). Compared to other subjects, mathematics concepts are more abstract, and learning the subject involves manipulation of symbols with little or no tangible meaning (Brown and Porter 1995; Nardi and Steward 2003). Students' attainment in mathematics and their attitudes about mathematics are strongly inter-related (Betz and Hackett 1983; Brown et al. 2008; Carmichael and Taylor 2005; Hackett and Betz 1989; Hannula 2002; Hannula et al. 2004; Nardi and Steward 2003). Many students see mathematics as being uniquely difficult. For example, a longitudinal study of students' experiences of the curriculum in the first three years of their post-primary schooling in Ireland found that, compared to other Junior Certificate subjects, students perceive mathematics to be the most difficult and the least interesting subject (National Council for Curriculum and Assessment 2007). Studies show that even relatively successful students perceive that they have failed at the subject and they do not feel that they are good enough to cope with mathematics at more advanced levels and there are also reports about the perception of "elitism" in mathematics where only a 'clever core' of students are capable of learning mathematics (Brown et al. 2008; Hodgen et al. 2009; Matthews and Pepper 2007; Nardi and Steward 2003). Paul Ernest (2009) reinforces this view, he states that the perception of mathematics "in which an elite cadre of mathematicians determine the unique and indubitably correct answers to mathematical problems and questions using arcane technical methods known only to them" puts "mathematics and mathematicians out of reach of common-sense and reason, and into a domain of experts and subject to their authority. Thus mathematics becomes an elitist subject of asserted authority, beyond the challenge of the common citizen" (Ernest 2009). While Ernest (2009) argues that "higher mathematical knowledge and competence is not needed by the majority of the populace to ensure the economic success of modern industrialised society" one special value of higher level Leaving Certificate mathematics in Ireland is that students are awarded a greater number of CAO points compared to other school subjects.

There is a real disaffection in students towards mathematics and, by extension, other numerate studies. Skemp (1987) says "not only do we fail to teach children mathematics, but we teach many of them to dislike it" and he admits that "for too many children, the word "mathematics" has become a conditioned anxiety stimulus" (Skemp 1987). Nardi and Stewart (2003) found that the characteristics of classroom mathematics include: tedium; isolation; rote learning, elitism; and depersonalisation (Nardi and Steward 2003). It is reported that there is a sense of fear and failure regarding mathematics among a majority of children (National Council of Educational Research and Training 2006). In a study of second-level mathematics classroom practices in Ireland Lyons, Lynch, Close, Sheerin and Boland (2003) found that all students, regardless of the level of mathematics studied or the type of school attended, had "a fear of being seen to be 'wrong'" and many suffered "mathematics anxiety" when teachers taught at a very fast pace, when teachers were critical of students who made errors or sought help and when teachers pressurised students to achieve without giving positive support (Lyons et al. 2003). Jo Boaler (2006) notes the narrowness by which mathematics success is judged where "executing procedures quickly and correctly" is valued above all other practices in mathematics learning and consequently "some students rise to the top of classes, gaining good grades and teacher praise, while others sink to the bottom with most students knowing where they are in the hierarchy created" (Boaler 2006). Richard Skemp (1987) asks "why should anyone want to learn mathematics?" His response is "motivation … towards satisfaction of some need" and in the classroom short-term motivations are "the desire to please the teacher and the fear of displeasing her or him" (Skemp 1987). Paul Ernest also asks "what is the purpose of teaching and learning maths?" He believes that the aims of teaching mathematics "can be a hotly contested area." An absolutist-like view of "giving students mainly unrelated routine mathematical tasks which involve the application of learnt procedures, and by stressing that every task has a unique, fixed and objectively right answer, coupled with disapproval and criticism of any failure to achieve this answer" lead to "mathephobia" or a feeling that "mathematics is cold, hard, uncaring, impersonal, rule-driven, fixed and stereotypically masculine" (Ernest 2004b).

Mathematics is a minority subject whereby only minorities of students take the subject at higher level compared to other subjects. For example, in Ireland only 16% of all Leaving Certificate mathematics students (and 14% of all Leaving Certificate students) take the higher level option compared to 64% for English, 32% for Irish, 66% for History; 78% for Geography; 75% Biology and 76% for Art. The number of higher Leaving Certificate mathematics students is approximately the same as the number of students taking higher level Art (State Examinations Commission 2011a). An international comparison of upper secondary mathematics education found that fewer than 20% of pupils in the United Kingdom take mathematics in any form during the "upper secondary" years. The study found that in the eight countries where all students (95-100%) study mathematics the subject is compulsory for all upper secondary students; these countries are the Czech Republic, Estonia, Finland, Japan, Korea, Russia, Sweden and Taiwan. On the other hand, mathematics is entirely optional in the four United Kingdom countries, Australia, Ireland and New Zealand once a minimum level is reached. The study also found that participation in advanced mathematics in upper secondary school is low (0-15%) in Germany, Ireland, Russia, Spain, England, Wales and Northern Ireland; is medium (16-30%) in Australia, Estonia, Finland, France, Sweden, USA (Massachusetts) and Scotland; and is high (31-100%) in Japan, Korea, New Zealand, Singapore and Taiwan. The study also observed that in countries where participation is higher in advanced mathematics, it generally follows that participation in any mathematics is also higher - at least in countries where upper secondary general education is not targeted to a relatively small elite (Hodgen et al. 2010).

2.3 LEARNING MATHEMATICS

A research question in this study is to query if there is a relationship between students' experiences with school mathematics and their choice of engineering as a career. This requires consideration of the theories of mathematics learning and mathematics teaching.

2.3.1 Mathematics Learning Theory

Research literature shows that there are a variety of different general learning theories that are applied to mathematics learning. Mathematics is often described as a hierarchical subject, where later learning depends on understanding of earlier concepts (Chambers 2008). Skemp (1987) asserts that "the amount which a bright child can memorise is remarkable, and the appearance of learning mathematics may be maintained until a level is reached at which only true conceptual learning is adequate to the situation. At this stage the learner tries to master the new tasks by the only means known – memorising the rule for each kind of problem. That task being now impossible, even the outward appearance of progress ceases, and, with accompanying distress, another pupil falls by the wayside" (Skemp 1987). Skemp (1987) also asserts that "mathematics is the most abstract, and so the most powerful of all theoretical systems" where "more abstract means more removed from experience of the outside world". Skemp believes that "mathematics cannot be learnt directly from the everyday environment, but only indirectly from other mathematicians". He says that mathematics learning is "very dependent on good teaching" and that "to know mathematics is one thing and to be able to teach it – to communicate it to those at a lower conceptual level – is quite another; and I believe it is the latter which is most lacking at the moment" (Skemp 1987).

Most mathematics learning theories refer to Jean Piaget whose work established constructivism as a leading theory in mathematics learning (Chambers 2008; Ernest 2011; Jaworski 2002; Zimmerman and Schunk 2003). Constructivism is founded on Piaget's belief that learning is an active process whereby new knowledge is accommodated into previously understood knowledge. Piaget (1896-1980) identified four stages of learning through which people progress from birth to adulthood, these are: sensor-motor (up to 2 years); preoperational (2 to 7 years); concrete operational (7 to 11 years) and formal operational (11 years and older). Teaching involves using methods that are appropriate to a child's stage of development and children move through these levels in the defined order; they cannot skip a stage. Constructivism is based on the theory that thinking is an internalised activity and that new knowledge is constructed based on experiences. When a child encounters a learning experience, mental structures or schemas are constructed to represent perceptions of what they experience. New experiences result in new schemas or the reinforcement or modification of existing schemas. Assimilation is the process where new knowledge is fitted into existing schemas and accommodation is the process of adapting schemas to fit new perceptions (Chambers 2008; Ernest 2011; Jaworski 2002; Zimmerman and Schunk 2003).

Deriving from Piaget's work, Lev Vygotsky developed a theory of social constructivism based on the idea that social interaction with others provides the foundation for individuals coming to understand ideas for themselves (Vygotsky 1978). Social constructivism adds the dimension of language and communication to Piaget's idea of learning through constructing new understanding. In Vygotsky's theory of learning, he links the content that is learned with the social context in which it is learned. He suggests that thought and thinking depend on language that is acquired in discussion and conversation with others. According to Vygotsky learning is fundamentally a social process whereby knowledge exists in a social context and is initially shared with others instead of being represented solely in the mind of an individual. He says that the stimulus for learning comes from outside the individual and the individual's construction of knowledge is secondary to the social context. Building on this theory, Vygotsky developed the idea of the student's zone of proximal development which he defines as "the distance between the actual development level as determined by independent problem-solving and the level of potential development as determined by problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky 1978). According to this view, there is a difference between what learners could achieve by themselves and what they could do with assistance from a skilled person. Vygotsky highlights the key role of teachers in mathematics learning whereby skills are developed through the interaction and guidance of teachers, who provide scaffolding on which students construct their learning. Scaffolding is a means whereby a more skilled person imparts knowledge to a less skilled person through language and communication. Vygotsky findings suggest that learning environments should involve interaction with experts and that discussion between teacher and students and amongst students themselves enhance students' mathematical thinking and communication (Vygotsky 1978).

2.3.2 Effective Mathematics Teaching

Vygotsky's theory of social constructivism suggests that understanding and social interaction are key components of effective mathematics learning. Accordingly teacher interaction with the learner is essential for effective mathematics teaching. Learning mathematics is an active process where learners engage in tasks and make sense of concepts rather than just passively receive facts and skills. It is up to teachers to structure tasks that present an appropriate challenge for learners to engage in. Mason and Johnston-Wilder (2004) hold that mathematics learners require: "relevant experiences from which to extract, abstract and generalise principles, methods and ways of working with mathematics; stimuli appropriate to the concepts to be worked on; and a supportive and compatible social environment in which to work" (Mason and Johnston-Wilder 2004).

Mathematics has a number of dimensions, including: developing knowledge and skills; applying mathematics in a range of contexts; relating mathematical ideas to each other; and expressing mathematics. It is the teacher's task to facilitate this learning. For example, Pietsch (2009) says that "mathematics teachers need to be comfortable with a wide range of mathematical abstractions, techniques, concepts, ideas and generalisations". They also "need to feel comfortable working with individuals, with people who are fundamentally unpredictable, beyond complete understanding, each person representing a unique exemplar of multiple overlapping abstractions" (Pietsch 2009). One reason advanced to explain the decline in mathematical competencies of students in Ireland is the number of untrained and under-qualified teachers of mathematics. It is estimated that only 20% of Leaving Certificate mathematics syllabus is taught by those with degrees in the subject. One concern about unqualified teachers is that they fear having to teach mathematics and consequently “the problem-solving power and logical basis of mathematical manipulations is often lost and replaced by attempts by students to learn by rote and memorise numerous sets of complex rules”. Another concern about higher level Leaving Certificate mathematics is that the course is considered too long and offered too much choice resulting in both teachers and students omitting significant parts of the course (Irish Academy of Engineering 2004).

According to Vygotsky’s theory of social constructivism, the method by which students construct their own meaning based on accommodating new ideas into their already understood set of knowledge, understanding is critical in mathematics learning. Teaching for conceptual understanding requires a radically different approach compared to teaching for skill development. It is claimed that many teachers overstress methods, routine tasks and skills at the expense of long term learning strategies and that consequently students are poor at transferring their skills (Pietsch 2009). For example, in Ireland mathematics teachers generally rank lower-order abilities (e.g. remembering formulae and procedures) more highly and higher-order abilities (e.g. providing reasons to support conclusions, thinking creatively and using mathematics in the real world) less highly: than do teachers in many other countries (Lyons et al. 2003). Schoenfeld recommends a shift from memorising towards conjecturing and mathematical reasoning (Schoenfeld 1992). Vygotsky’s theory regarding students’ zone of proximal development suggests that mathematics teachers should present students with the right level of challenge and teachers should assist students perform tasks just beyond their current level of understanding.

The key to Vygotsky’s theory of social constructivism is the idea that learning is constructed in a social context and that classroom discussion, rather than the teachers’ transmission of knowledge, is an essential part of mathematics learning. Developing specific mathematical forms of discourse that can be internalised by individual students is an important part of effective mathematics teaching (Pietsch

2009). In Ireland there is little evidence of group work, individualised work, whole class discussion or reflection in mathematics classrooms (Lyons et al. 2003). Classroom discussion, dialogue and collaboration are critical components of social constructivist theory of mathematics learning. Dialogical classrooms, while challenging teachers, allow students to ask questions and consider different perspectives, create rich learning environments. Collaborative learning, where a group of students work together dealing with different perspectives and a common goal, encourages interaction between students. The peer tutoring element of collaborative learning benefits both students who are tutoring as they are encouraged to clarify their own thinking and those who are being tutored as they can address their areas of misunderstandings. Collaborative learning opportunities encourage students to verbalise their ideas and challenge other students (Pietsch 2009).

There are numerous mathematics classroom teaching practice views and the majority of these recommend a shift away from isolated facts and memorisation of procedures and a move towards conceptual understanding and problem solving (Chambers 2008; Jaworski 2002; Pietsch 2009; Schoenfeld 1994; Watson and Mason 2008). The National Council of Teachers of Mathematics (NCTM) in the U.S. is probably the most active initiative aimed at reforming school mathematics teaching. The NCTM released standards for school mathematics in 1989; these were subsequently updated and re-released in 2000 and they are called "Principles and Standards for School Mathematics". The NCTM's Principles and Standards for School Mathematics highlight students' need to learn mathematics with understanding by actively building new knowledge from existing knowledge and experience. The council also highlights the need to focus on "important mathematics" that will prepare students for continued study and for solving problems in a variety of school, home and work settings (National Council of Teachers of Mathematics 2000). The NCTM present six principles and ten standards to guide teachers who seek to improve mathematics education in their classrooms and schools. The six principles for school mathematics address overarching themes of: Equity ("excellence in mathematics education requires equity-high expectations and strong support for all students"); Curriculum (“a curriculum is more than a collection of activities: it must be coherent, focused on important mathematics and well-articulated across the grades”); Teaching (“effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well”); Learning (“students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge”); Assessment (“assessment should support the learning of important mathematics and furnish useful information to both teachers and students”; and Technology (“technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning”) (National Council of Teachers of Mathematics 2000). In the teaching principle the NCTM confirms that “students’ understanding of mathematics, their ability to use it to solve problems and their confidence in and disposition toward mathematics are all shaped by the teaching they encounter in school”. For teachers to be effective, they “must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks … make curricular judgments, respond to students’ questions, and look ahead to where concepts are leading and plan accordingly … need to know the ideas with which students often have difficulty and ways to help bridge common misunderstandings”. Because “students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know”. Teachers need to establish and nurture an environment conducive to learning mathematics that “encourages students to think, question, solve problems and discuss their ideas, strategies and solutions”. Teachers who engage in effective teaching motivate students to engage in mathematical thinking and reasoning and provide learning opportunities that challenge students at all levels of understanding”. The NCTM note that learning mathematics without understanding is a big problem and a major challenge in mathematics education (National Council of Teachers of Mathematics 2000). Conceptual understanding is an important component of mathematics proficiency and mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways (Schoenfeld 1988). The NTCM present that classroom interactions, problem solving, reasoning and argumentation enhance mathematics learning with understanding (National Council of Teachers of Mathematics 2000). The NCTM's ten standards describe what mathematics instruction should enable students to know and do. These ten standards are divided into two groups titled Content and Process. The five Content Standards (Number and Operations, Algebra, Geometry, Measurement, Data Analysis and Probability) explicitly describe the curriculum or the content students should learn in their mathematics classes. The five Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) are interwoven throughout the curriculum to provide a context for learning and teaching mathematical knowledge. The NTCM present that by learning problem solving in mathematics, students develop new mathematical understandings and they acquire ways of thinking, habits of persistence and curiosity and confidence in unfamiliar situations. When engaged in problem solving students develop metacognition and they frequently monitor their progress and adjust their strategies accordingly. Reasoning and proof include: developing ideas; exploring phenomena; justifying results (arguments consisting of logically rigorous deductions or conclusions); and using mathematical conjectures (informed guessing). The NCTM confirms that communication is an essential part of mathematics and mathematics education in that it is a way of sharing ideas and clarifying understanding. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing and they also develop new levels of understanding mathematics. The NCTM believes that communicating mathematics is neglected in mathematics education. It holds mathematics is an integrated field of study and that mathematical connections to contexts outside of mathematics should be part of students' mathematics learning experiences. By emphasising mathematical connections, students build a disposition to use connections in solving mathematical problems rather than see mathematics as a set of disconnected, isolated concepts and skills. Another contribution from the NTCM is that the ways in which mathematical ideas are presented are fundamental to how people can understand and use those ideas. Diagrams, graphs and symbolic expressions are not ends in themselves but rather are supports to students' understanding of mathematical concepts, communicating mathematics, recognising connections and applying mathematics to realistic problem situations. When students gain access to mathematical representations they have a set of tools that significantly expand their capacity to think mathematically. Technological tools offer students opportunities to use new forms of representations and they allow students to explore complex models of situations. The NCTM maintains that students' use of representations to model physical, social and mathematical phenomena should grow through their school years (National Council of Teachers of Mathematics 2000).

While constructivism provides the theoretical basis for mathematics education generally, there is a more recent view in research literature whereby "all of the goals of mathematics education do not need to be achieved through the processes of personal construction and not all the mathematics students learn needs to be invented by students" (English 2007). English (2007) holds that studies of the nature and role of mathematics used in the workplace and other everyday settings should contribute to how students are taught mathematics. Her view is that the increasing sophistication and availability of new technologies is changing the nature of the mathematics needed in the workplace. Students' future lives involve a world governed by complex systems and the body of research on complex systems and complexity theories should have an impact on mathematics education. Complexity is the study of systems of interconnected components whose behaviour cannot be explained solely by the properties of their parts but from behaviour that arises from their interconnectedness. In order for students to be able to deal with such complex systems beyond school, they need to learn the following abilities: "constructing, describing, explaining, manipulating and predicting complex systems; working on multi-phased and multi-component projects in which planning, monitoring and communicating are critical for success; and adapting to ever-evolving conceptual tools and resources". English holds that these abilities can be developed through mathematical modelling. She defines models as "system of elements, operations, relationships and rules that can be used to describe, explain or predict the behaviour of some other familiar system". The inclusion of real-world problems that involve data handling, statistical reasoning and mathematical modelling and applications in school mathematics curricula would equip students for a rapidly advancing and exciting technological world (English 2007).

Another major factor in mathematics learning concerns the affective domain which is explored in more detail in Chapter 3. The affective domain is that area of causes internal to a person that drives their behaviours and includes attitudes, feelings, beliefs, confidence and values. "There is a common and reasonable belief that positive attitudes, particularly liking for, and interest in, mathematics, lead to greater effort and in turn to higher achievement … affective learning outcomes – such as enjoyment, enthusiasm, fascination, appreciation – may be taken into account alongside the more cognitive aspects of learning mathematics which are measured in terms of achievement" (Costello 1991). A study of high achievers in mathematics found that for almost two thirds of the students mathematics was their favourite subject. Being good at mathematics and the ability to get 100% marks in tests were the main reason for students' enjoyment of mathematics. Some people enjoyed mathematics for other reasons including: the "beauty" of the subject; the logical nature of the subject; the clear cut answers; the challenge of problem solving; satisfaction from problem solving and the pleasure of figuring something out that was not initially obvious. The students were generally highly motivated and thrived on challenges. The most exciting mathematics came from opportunities to do advanced mathematical work with mathematically talented peers outside of school. The majority of the students were interested in pursuing a mathematics related career (Leder, 2008). However in Ireland, mathematics learning is often associated with a belief that mathematics is boring and difficult (National Council for Curriculum and Assessment 2007).

2.4 ENGINEERING CAREER CHOICE

Engineering career choice is a central theme in this study. There is considerable evidence in published literature to show that in spite of good career prospects, there has been a decline in both the study of mathematics in schools and engineering at university level. This trend is common to the United States, Australia, Europe, the United Kingdom and Ireland (Elliott 2009; Forfás 2008; King 2008; McKinsey 2011;

Organisation for Economic Co-Operation and Development 2010). Mathematics, misunderstandings about what engineers do and their invisibility as a profession and financial reward are some of the reasons offered for the decline.

While the selection of a career made during students' senior school years is among the most critical in a person's lifetime, there are many factors that enter into the selection of a career including: the choices a person makes (e.g. school subject choice); the values a person holds; the successes and failures a person experiences; the social class in which a person has developed; and the interests, strengths, and capacities of the person (Ginzberg et al. 1951). According to Ginzberg, Ginsburg, Axelrad, and Herma (1951), career development may be viewed as an evolutionary process comprising three periods: fantasy; tentative and realistic. In the fantasy period, the impulses and momentary needs of a young child are translated into career choices without any realisation of facts regarding the occupation. During this period, the child begins to role-play these occupations while the family responds with attitudes toward both the behaviours and the occupations and this plays an important role in influencing the child during the fantasy period. The child is typically aged 11 to 17 years in the tentative period and career choices are based on personal criteria: interests; abilities; and values. During this time adolescents begin to evaluate the occupational activities available, the traits of the people in those occupations and the attitudes of others towards those people and occupations. The adolescents consider the things they enjoy or are interested in doing, their abilities and talents, salary, satisfaction specific occupations offer, work schedule and other value-related facets. In the realistic period, which is the early years of adulthood, the individual begins to balance the personal criteria with the opportunities, requirements, and limitations of the occupations presented in society. It is during this period that the individual determines the specific career choice or the area in which they choose to work. The individual's choice is a compromise of interests and abilities, as well as satisfying values and goals as much as possible (Ginzberg et al. 1951).

Roberts (2002) attributes low engineering enrolments to "poor experiences of science and engineering education among students generally, coupled with a negative image of and inadequate information about, careers arising from the study of science and engineering" (Roberts 2002). Social cognitive career theory posits that greater knowledge of occupation specialities and greater match between one's image of a career and one's self-identity are each associated with greater confidence in career choice (Lent et al. 2002). However Knight and Cunningham in their "Draw an Engineer Test" found that many students, especially younger students, associate engineers with fixing car engines, construction work and with being male (Knight and Cunningham 2004). Studies of young people's perceptions of engineers generally show that engineers' work is viewed as fixing, building, making or working with vehicles, engines, buildings and tools and engineers are generally male. Such misconceptions and stereotypes about engineering make it more difficult to attract students to engineering (Capobianco et al. 2011; Oware et al. 2007a; Oware et al. 2007b). Research literature also shows that even many students in engineering education are not familiar with different career choices (Shivy and Sullivan 2003). While there are many reports highlighting the shortage of people qualified in science, technology, engineering and mathematics (Brown et al. 2008; McWilliam et al. 2008; National Academy of Sciences et al. 2010; Smith 2004), Prieto, Holbrook, Bourke, O'Connor and Husher (2009) note that many of these reports focus on symptoms such as shortages of engineers and fewer students taking science and higher level mathematics in secondary school rather than the causes. They say that the multiple meanings and the wide range of contexts in which engineering takes place lead to misconceptions, mystification and misunderstandings about what engineers do and to a decline in university enrolments in engineering education (Prieto et al. 2009). In their review of research literature on students' interest in mathematics, science and engineering leading to enrolment in engineering education, Prieto, Holbrook, Bourke, O'Connor and Husher (2009) found four main influences contributing to poor enrolments in engineering degrees. These are national investment, sources of information, education and perceptions of the profession. They say that students' image of the engineering profession comes from their parents, family relations and school career advisor. They present a consensus that "college graduates who become teachers have somewhat lower academic skills on average than those who do not go into teaching" and that significant percentages of middle school mathematics and science teachers do not have a major or minor in those subjects. Consequently students' mathematics and science learning is compromised. They say that when they draw all the factors together that raising students' interest in mathematics and science and relating these subjects to engineering is of critical importance. They believe that enriching the mathematics and enabling sciences experience for students holds the key to increasing enrolments in engineering education (Prieto et al. 2009). Similarly, McWilliam, Poronnik and Taylor (2008) are of the view that engaging students in mathematics and science is crucial to their interest in such careers and they say that "schools and universities whose curriculum, pedagogy and assessment remain 'outside' will be increasingly irrelevant to the modes of learning and social engagement that young people choose and to the future of their work" (McWilliam et al. 2008).

Becker (2010) looks at the changing role of engineers and technology and he says that young people "simply do not see it as attractive enough compared to other options" and that "society and the business world send a host of psychological and financial signals that contradict their claims to foster science and technology". Becker claims that engineering has changed from the second half of the nineteenth century, when the challenge was to develop "working innovations", to the current challenge which is "to prevail in an intensively competitive market where a wide array of non-technical factors determine success". He says that current technological performance has become invisible and that engineering primarily involves the computer screen. He adds that "direct hands-on technology experience is nearly impossible in the everyday environment; thus, eliminating a strong incentive for pursuing it" and that "the gap between technology nerds and technology users has widened". Becker believes that a bachelor's engineering curriculum is not relevant for the labour market but instead it is a theoretical foundation for a master's degree. Becker notes that in 2010 only 25% of Siemens' managing board members were scientists and engineers, while in 2001 the percentage was 64%. Becker is of the view that young people know what type of education will lead them to the top positions in companies and in society (Becker 2010). Similarly, Duderstadt (2008) asserts that students "sense the eroding status and security of engineering careers and increasingly opt for other more lucrative and secure professions such as business, law and medicine". He also notes that engineers no longer occupy positions in business and government and he says this is because neither the engineering profession nor the educational system supporting it has kept pace with the changing nature of the "knowledge-intensive society and the global marketplace". Duderstadt (2008) asserts the need "to transform engineering practice from an occupation or a career to a true learned profession, where professional identity with the unique character of engineering practice is more prevalent than identification with employment". He suggests that engineers "would increasingly define themselves as professionals rather than employees. Their primary markets would be clients rather than employers. And society would view engineering as a profession rather than an occupation" (Duderstadt 2008).

Given the underrepresentation of women in engineering, much of the available research on engineering career choice relates to women's participation in engineering. In Ireland, approximately 20% of undergraduate entrants to university engineering courses each year are women (Higher Education Authority 2010). Similarly women represent approximately 20% of bachelor's degrees awarded in engineering in the United States of America (National Science Foundation 2010). Morgan, Isaac and Sansone (2001) found that women are significantly less likely to enter physical/ mathematical science careers than men and women are also significantly less likely to enter physical/ mathematical science careers than enter social services or medical careers. This is because students' perception is that careers in physical/ mathematical science areas are less likely to offer interpersonal rewards and more likely to offer extrinsic rewards when compared to social service careers and medicine (Morgan et al. 2001). A 20-year follow-up study of mathematically gifted adolescents also showed that males as a group were heavily invested in the inorganic sciences and engineering and that there was greater female participation in the "medical arts and biological sciences as well as the social sciences, arts and humanities". The findings show that males placed greater weight on securing career success and females' priorities included career, family and friends. The study also found that "those with exceptional mathematical abilities relative to verbal abilities tend to gravitate toward mathematics, engineering and the physical sciences, while those with the inverse pattern are more attracted to the humanities, law and social sciences" (Benbow et al. 2000). Lubinski and Benbow (2006) say that mathematically precocious females, more often than mathematically talented males, are "endowed with talents that enable them to excel with distinction in domains that require highly developed verbal-linguistic skills". Lubinski and Benbow note that these skills give career flexibility which is useful in "navigating today's multidimensional work environments". They say that women are well suited to working in interface areas that form connections with multiple disciplines (Lubinski and Benbow 2006). In a study of graduates who didn't come from the pool of mathematically gifted students, it was found that male scientists have "exceptional quantitative reasoning abilities, relatively stronger quantitative than verbal reasoning ability, salient scientific interests and values, and, finally, persistence in seeking out opportunities to study scientific topics and develop scientific skills" (Lubinski et al. 2001).

Many studies of the disproportionately low numbers of women compared to men in engineering education and in engineering careers concern women's mathematical self-efficacy. According to Albert Bandura's social cognitive theory, individual's beliefs about their competencies in a given domain affect their choices. Self-efficacy perceptions come from past experiences, observing others, encouragement and emotions (Bandura 1986). A study by Betz and Hackett (1981) found that the strongest predictors of the range of career options were interests and self-efficacy. Self-efficacy expectations are one's beliefs concerning one's ability to successfully perform a given task or behaviour. Self-efficacy expectations are "viewed as both learned and modified via four types of information: (a) performance accomplishments; (b) vicarious learning; (c) emotional arousal, for example anxiety in response to a behaviour or set of behaviours; and (d) verbal persuasion, for example encouragement or discouragement" (Bandura 1986). Betz and Hackett found that the occupation perceived as most difficult among males was that of physician while among females was engineer. The occupation that received the most divergent ratings for the sexes was that of engineer, "70% of males but only 30% of females felt they could successfully complete its educational requirements". The significant sex differences in self-efficacy with regard to occupations such as engineer and mathematician were not paralleled by significant sex differences in ability. Betz and Hackett suggested that "women's lower self-efficacy expectations with regard to occupations requiring competence in mathematics may be due to a lack of experiences of success and accomplishments, a lack of opportunities to observe women competent in math, and/ or a lack of encouragement from teachers or parents" (Betz and Hackett 1981). Lent, Brown and Larkin (1986) also found that self-efficacy is predictive of important indexes of career entry behaviour such as college choices and academic performance (Lent et al. 1986). Social cognitive career theory, which grew out of Bandura's social cognitive theory, posits that "academic and career choice goals and actions are seen as being influenced largely by interests, self-efficacy and outcome expectation as well as by the environmental supports and barriers that people have experienced or expect to experience in relation to particular choice alternatives" (Bandura 1986; Lent et al. 1994). Many studies show that women's mathematical self-efficacy is significantly lower than men's perceptions of their capability to succeed in mathematics and this is a major influence on career choice (Correll 2001; Løken et al. 2010; Zeldin and Pajares 2000). Shelley Correll (2010) presents a social psychological model of career choice whereby students must believe they have the skills necessary for a given career in order to persist on a path leading to that career. In her study of high school students, Correll found that because males assess their mathematical competence higher than their otherwise equal female counterparts, they are more likely to pursue activities leading to a career in science, mathematics and engineering. She says that "boys do not pursue mathematical activities at a higher rate than girls because they are better at mathematics. They do so, at least partially, because they think they are better" (Correll 2001). Løken, Sjöberg and Schreiner found that girls who do choose science, technology, engineering and mathematics (STEM) related careers are highly motivated for success and they often have positive childhood experiences with STEM (Løken et al. 2010).

Morgan, Isaac and Sansone (2001) in their study of college students found that while women were less likely to choose physical/ mathematical science careers than men, the perceived "interestingness of a career" was a significant predictor of career choice for both male and female college students even when perceived competence of related school subjects was controlled. They say that “real or anticipated experience of interest when engaged in career-related activities is a critical influence on career choice” (Morgan et al. 2001). Hardré, Sullivan and Crowson (2009) studied how rural high school student’s self-perceptions and environmental perceptions influence their course-related interest, school engagement and post-graduation intentions. They found that teacher support predicated student interest in subject matter. Learning goals, perceived competence and instrumentality (“learner’s tendency to ascribe worth and benefits to knowledge and skills in the domain, which in turn influences attention, engagement and investment”) demonstrated strong influences on interest and the likelihood of pursuing postsecondary education (Hardré et al. 2009). Maltese and Tai, in their study of graduate students’ interest in science, found that interest in science begins before middle school. In that study the majority of females stated that their interest in science was sparked by school-related activities and for males it was mostly “self-initiated activities” (Maltese and Tai 2010). In another study Maltese and Tai found that the majority of students who choose STEM careers make that choice during high school and that choice is related to their interest in mathematics and science (Maltese and Tai 2011). While Matusovich, Streveller & Miller (2009) say that what is lacking in research findings is an understanding about why students choose engineering careers and their case study analysis investigated how students’ motivational values contributed to their choices to enrol and persist in engineering education. They found four values: attainment; cost; interest; and utility. Attainment is one’s self-identity as an engineer. Cost concerns the effort and sacrifices required to become an engineer. Interest is about enjoyment of activities thought to be associated with engineering and utility is the perceived usefulness of an engineering degree. It was found that all four values influence engineering career choice but that students’ choice of engineering is primarily related to “students’ sense of self” or attainment value. While attainment value concerns one’s sense of identity of becoming an engineer, a student’s reason for pursuing (or not pursuing) engineering is related to the student becoming the type of person who is an engineer (Matusovich et al. 2009). Similarly Sjöberg and Schreiner in their study of how young people in different cultures relate to science and technology found that the more emancipated a society and the greater the range of alternatives that a highly differentiated labour market offers young women, the less likely they will be inclined to opt for professions they do not wish to identify with (Sjöberg and Schreiner 2011).

Engineering career choice was much more popular in 1985 than it is today. The results of a study, conducted by Purdue University in the USA in 1985, found that the challenge of engineering work, salary, creativity and a liking for problem solving were of central importance to students' choices to pursue engineering careers at that time (Jagacinski et al. 1985). However since 1985 major changes have occurred within engineering fields. Also since 1985, there has been a huge "social change" with respect to the supply of students to universities whereby students choose non-traditional subjects in favour of science and technology subjects (Heywood 2005). In the past 30 years, the Irish education system has also experienced huge change. For example, when the Irish CAO system (competitive points system for entry to third level education in Ireland based on Leaving Certificate grades) was conceived in 1977, only 5 universities and 69 courses were part of the system, compared to 2008 when 44 higher education institutions (universities and institutes of technology[8]) offered 778 degree courses and 407 diploma and certificate courses (Central Applications Office 2008). Heywood (2005) says that one consequence of the change in both engineering and education is that entry requirements into engineering studies, as measured by grades in public examinations, have reduced. Consequently science and engineering departments in universities have to adapt to the new student intake. The mathematical ability of students entering engineering is a concern for both direct entry to engineering degree programs and for students progressing to engineering via technician courses[9]. Interventions such as attempts to improve school mathematics grades, introduction of engineering science subjects in schools, students' participation in engineering projects activities and students' exposure to engineering role models have not regenerated students' interest in engineering careers

[8] Institutes of Technology form part of third level education in Ireland. They operate a unique system in that they allow students to progress from two year programmes (level 6) and three year programmes (level 7) to primary degree and postgraduate qualifications.

[9] In Ireland students who achieve high grades in technician courses (level 6) can subsequently enrol in year 3 of engineering degree courses (level 8) and thus bypass the minimum requirement of 55% in higher level Leaving Certificate mathematics.

(Heywood, 2005). Heywood believes that interventions in schools can help teachers acquire knowledge that will better prepare and excite students about engineering careers. Heywood asserts that even though we live in a technological society, that "engineering departments possess a vast knowledge that is not readily available to school teachers". He suggests new types of degrees in which students undertaking an engineering program can also obtain teacher certification. Heywood is also of the view that raising the status of design and technology in schools is difficult when students perceive engineering jobs as "unglamorous" (Heywood, 2005).

A longitudinal study of engineering undergraduate students found that students' views of themselves as future engineers include "being good in math and science, being communicators, being good at teamwork and enjoying activities they believe engineers do, doing problem-solving and having/ applying technical knowledge" (Matusovich et al. 2009). Mathematics is perceived to be the "the key academic hurdle" in the supply of engineering graduates (Croft & Grove, 2006; King, 2008). At the same time the idea that engineers need to be good at mathematics is being very effectively communicated (Baranowski and Delorey 2007). For example, Craig Barrett, former Chairman of Intel Corporation came to Ireland in February 2010 to speak about Ireland's economy and how he sees education as one of the key solutions to Ireland's current economic woes. In his ten-point plan for economic recovery, Barrett told the Irish people that their "future relies on a critical mass of maths and science skills". He gave the same message to the American people: "America's economic future lies with its next generation of workers and their ability to develop new technologies and products. This means we must strengthen math and science education in the U.S" (Barrett 2008). Engineers Ireland, the body that accredits engineering degree programmes in the Republic of Ireland, also emphasises the importance of mathematics in engineering. Engineers Ireland specifies that engineering degree students must have a minimum of grade C3 (55%) or better in higher level Leaving Certificate mathematics or an equivalent mathematics grade approved by the body (Engineers Ireland 2012).

Students' difficulty with higher-level school mathematics is considered to be a major contributor to the declining number of entrants to engineering degree courses

(Bowen et al. 2007; King 2008; Prieto et al. 2009). In a review of the literature in engineering education, James and High (2008) maintain that mathematics is "believed to be one of the confounding variables tripping students in their learning" of engineering. However they were unable to answer the following question: "is there a correlation between people choosing engineering as their field of study and those who enjoy applications of mathematics?" (James and High 2008). Similarly Ifiok Otung, from the University of Glamorgan, questions the "wisdom of scaring away potentially successful engineers with a mathematical content that is rarely used during the career of 98% of practitioners" (Otung 2002). According to Smith (2004) many of the problems identified across science and engineering manifest themselves most acutely in the area of mathematics. He expresses a deep concern about many young people's perception of mathematics as being "boring and irrelevant" and "too difficult, compared with other subjects" (Smith 2004). Winkelman (2009) maintains that "mathematics bestows its practitioners with intellectual status" and consequently serves as a gatekeeper to engineering education. He is of the view that mathematics, when detached from engineering, runs the risk of alienating students (Winkelman 2009). Lynch and Walsh (2010) observed that first preference applications for level 8 engineering degree courses in Ireland have fallen from nineteen per cent of the total student cohort in 2000 to nine per cent in 2010. In their longitudinal study of secondary school students, they noted a significant shift away from engineering careers as students progressed through second level school. They observed that engineering was the only career sector to show such a drastic decline in popularity across second level. A significant finding of the study was that the minimum mathematics requirements for entry into engineering education contributed to students' hesitancy to pursue engineering degree courses. It was also found that students' "interest in and self-efficacy in regard to, a particular second level subject had a significant influence on their decision to apply for their chosen third level course". It was noted that male students opted for courses that they perceived had better career prospects while female students noted personal interests and occupational status as their main career influencers (Lynch & Walsh, 2010). However an analysis of the 2009 Irish education statistics shows that "in 2009, out of 8,420 students sitting the higher-level Leaving Certificate exam, approximately 6,800 students scored either grade A[10], B[11] or C[12]. By contrast, only an estimated 1,500 CAO places requiring this result were filled in third level colleges, with 1,200 of these places in engineering and technology. It appears that Ireland, in 2009, produced 5,300 students with Leaving Certificate maths achievements that are redundant, from a career perspective (notwithstanding indisputable general education value)" (Devitt and Goold 2010).

2.5 MATHEMATICS IN ENGINEERING EDUCATION

It is anticipated that the findings from this study and new knowledge in relation to engineers' mathematics usage in practice will inform engineering educators. It is therefore necessary to review the research literature concerning engineering education and more specifically the treatment of mathematics in engineering education.

It is asserted that general engineering education is "attempting to educate 21st-century engineers with a 20th-century curriculum" (Duderstadt 2008). Wulf and Fisher from the National Academy of Engineering in the U.S. assert that "many of the engineering students who make it to graduation enter the workforce ill-equipped for the complex interactions, across many disciplines, of real-world engineered systems" (Wulf and Fisher 2002). While much has been written about the need to reform engineering education, McMasters (2006) states that most of this literature has been written from "an academic rather than industry or employer perspective" (McMasters 2006). Trevelyan (2009) presents that the literature on engineering practice is rarely mentioned in contemporary writing on engineering education, "possibly because it is widely dispersed, hard to find, and often written for non-engineering audiences" (Trevelyan 2009). Given the perceived disconnect between engineering education and engineering practice, there are many calls for reforms in engineering education in order to prepare engineers for a rapidly changing world. For example, a U.S. report on engineering for a changing world, highlighting some

[10] Grade A: ≥85%
[11] Grade B:<85%, ≥70%
[12] Grade C: <70%, ≥55%

difficulties in engineering education, presents that: the "applied science" nature of engineering curricula is dated; the broader curricular experience where many different areas of knowledge are integrated ("big think") is favoured over specialisation ("small think"); passive learning environments are preferred to active learning approaches that engage problem solving skills and team building; faculty are rewarded for generating new knowledge rather than for application of knowledge (as in the case of medicine); engineering curricula are overloaded with knowledge that has a short shelf life; engineering students are forced to specialise early and engage in heavy workloads thus yielding 45% student attrition rates; there is no relation between early stages of curriculum and career; and lack of professional skills education (Duderstadt 2008).

There is also a view that social issues such as communications and team work contribute significantly to the gap between engineering education and engineering practice (Tang and Trevelyan 2009). Studies show that engineering graduates lack communication and problem solving skills required in engineering practice (Nair et al. 2009). One study of established engineers, with between five and twenty years of engineering experience, identify "communication, teamwork, self-management and problem-solving" as critical competencies required for their work (Male et al. 2010; Male et al. 2009). Another study of engineers who had been practising for no more than ten years, reveals the strong need for integrating "managerial, leadership, teamwork, creativity and innovation skills, as well as knowledge of business policies in classroom activities" into engineering education. The engineers also indicate the need for additional emphasis on project activities, summer training and closer links between engineering industry and academic institutions (Baytiyeh and Naja 2010). However given engineering graduates' needs to obtain a socially aware and technically oriented education for a business environment, Williams (2003) is of the view that "all the forces that pull engineering in different directions - toward science, toward the market, toward design, toward systems, toward socialization - add logs to the curricular jam" (Williams 2003).

A major problem currently facing engineering educators is attracting and retaining students. While engineering has evolved significantly in the past twenty years, the general academic backgrounds of students entering engineering degree programs have declined. It is reported that mathematics is one of the main factors contributing to student dropout in engineering education (James and High 2008). Croft and Grove (2006) highlight the high attrition rates in many engineering programmes and they state that there is widespread recognition that good achievement in school-level mathematics no longer guarantees a comfortable transition into first-year engineering courses at university (Croft and Grove 2006). Some U.S. colleges claim that as much as 60% of freshman engineering students are not calculus ready (Flegg et al. 2011; Gleason et al. 2010; Klingbeil et al. 2004). Engineering students are generally challenged by more complex mathematics delivered at a faster rate than what they experience in school (Irish Academy of Engineering 2004; Manseur et al. 2009; Manseur et al. 2010a; Manseur et al. 2010b). One of the biggest challenges facing engineering educators is the mathematics proficiency of students as evidenced by the availability of bridging courses and drop-in mathematics clinics for engineering students (Buechler 2004; Croft and Grove 2006; Fuller 2002; Gleason et al. 2010; Henderson and Broadbridge 2007; Henderson and Broadbridge 2008; Irish Academy of Engineering 2004; King 2008; Masouros and Alpay 2010; Reed 2003). Educators say that it is becoming increasingly difficult to engage engineering students in mathematics and to demonstrate the relevance of mathematics to an increasingly diverse student body (Henderson and Broadbridge 2007; Manseur et al. 2010a; Sheppard et al. 2009). Sheppard, Macatangay, Colby and Sullivan (2009) present that engineering students "generally find it difficult to relate math to real objects around them or to engineering practice". They say that the students "struggle to make the connection between mathematical representation and the real-world manifestation of the concept" (Sheppard et al. 2009).

In Ireland the teaching of mathematics to engineering students is usually associated with large class sizes and teachers are not recruited for their expertise in engineering mathematics but rather for their own specialised areas of research. As in many countries there is a division in service departments between mathematics and engineering and it is believed that this creates barriers in the students' minds with respect to mathematics and engineering applications. The Irish Academy of

Engineering note that the downside of an overly abstract approach to mathematics in engineering education is detachment from physical situations and confusion over mathematical notations, leading to uncertainty in students' minds (Irish Academy of Engineering 2004).

While there is no consistent research-informed view of "how, what, when and by whom mathematics should be taught to engineering students" (Flegg et al. 2011), there is a strong view that the engineering curriculum is overcrowded and that engineers should no longer be taught mathematics as if they were mathematicians (Flegg et al. 2011; Lesh and English 2005; Manseur et al. 2010a). There are also beliefs that mathematics is of limited use in graduate engineers' professional life. For example, Kent and Noss (2002) present one engineer's view of mathematics usage: "once you've left university you don't use the maths you learnt there, 'squared' or 'cubed' is the most complex thing you do. For the vast majority of the engineers in this firm, an awful lot of the mathematics they were taught, I won't say learnt, doesn't surface again" (Kent and Noss 2002). Chatterjee (2005), a professor of mechanical engineering, asserts that engineers solve technological questions as opposed to scientific or mathematical questions. He maintains that "the process of training an engineer to answer such questions requires a study of engineering models and the mathematical techniques used to analyse them. These models though approximate, require correspondence with reality in their conception, and precision in their description. And those mathematical techniques, like all mathematical techniques, require practice, sophistication and rigour. In this way, the technological world of an engineer builds up from the purer disciplines of mathematics and the sciences, but is not contained in them" (Chatterjee 2005). Wood (2010) reports that communication with mathematics can be problematic for students and her research reveals that no graduate believed that they had studied mathematics communication at university (Wood 2010).

Innovative ways proposed for the teaching of mathematics to engineering students include problem based learning (PBL), multidisciplinary approach, computer based methods and active learning methods (Coupland and Gardner 2008; Henderson and Broadbridge 2007; Henderson and Broadbridge 2008; Manseur et al. 2009; Manseur et al. 2010a). While there is little consensus on how reform of mathematics education in undergraduate engineering should take place, key issues of concern include: the "one-size-fits-all" approach to engineering mathematics which leads to teaching more mathematics than is required by specific disciplines; applied mathematics is of greater interest to engineers compared to theoretical mathematics; and teaching computational methods given the availability of powerful computing and design tools (Manseur et al. 2010b).

Challenges to the engineering science approach to engineering education, where engineering is taught after a strong foundation in science and mathematics, have resulted in the introduction of major design projects in many engineering degree courses. It is claimed that design pedagogy and project-based learning have advantages of improving student retention and motivation (Doppelt et al. 2008; Du and Kolmos 2009; Knight et al. 2007). Dym, Agogino, Eris, Frey and Leifer (2005) are of the view that engineering education should graduate engineers "who can design effective solutions to meet social needs" (Dym et al. 2005). They contrast the epistemological approach in engineering education where knowledge is applied to analyse a problem to reach "truthful" answers (convergent thinking) with conceptual design thinking where design solutions do not have a "truth value" (convergent-divergent thinking). They claim that engineering education does not teach divergent inquiry well and it is not acceptable for engineering students to present multiple concepts that do not have a truth value in their answers to exam questions. They say that system design and systems thinking skills include: thinking about system dynamics (anticipation of "unintended consequences emerging from interactions among multiple parts of a system"); reasoning about uncertainty (dealing with "incomplete information" and "ambiguous objectives" and application of probability and statistics); making estimates (one challenge of design is that as the number of variables and interactions grows, the system stretches beyond the designers' capability to grasp all of the details simultaneously and good system designers are usually good at estimation); and conducting experiments (design requires use of empirical data and experimentation) (Dym et al. 2005). They also present that engineering curricula underemphasise the application of probability and statistics and they also note that engineering graduates are generally not good at estimation (Dym et al. 2005). Winkelman (2009) also contrasts the "open-endness" of design processes, where there are a multiplicity of possible solutions for a given problem, with undergraduate engineering mathematics where "a single correct answer is generally assumed" (Winkelman 2009).

Many researchers are calling for a shift in approach from teaching mathematical techniques to teaching through modelling (Kent and Noss 2003; Lesh and English 2005). Winkelman asserts that engineering "is neither mathematics nor science, nor a combination of the two. Instead he sees mathematics as "abstract, based on the manipulation of symbols according to certain rules", which he says is disassociated from the "real world". Winkelman is of the view that mathematics "enters the real world through modelling" and that design should be taught alongside mathematics and not after mathematics (Winkelman 2009). Lesh and English (2005) are of the view that relevant ways of thinking in "real life" need to "draw on ways for thinking that seldom fall within the scope of a single discipline or textbook topic area and that attention should shift beyond "mathematics as computation" to "mathematics as conceptualisation, description and explanation" (Lesh and English 2005). "Solutions to non-trivial problems tend to involve a series of modelling cycles in which current ways of thinking are iteratively expressed, tested and revised; and, each modelling cycle tends to involve somewhat different interpretations of givens, goals and possible solution steps." Lesh and English assert that it is "possible for average ability students to develop powerful models for describing complex systems that depend on only new uses of elementary mathematical concepts that are accessible to middle school students" (Lesh and English 2005).

The debate about mathematics in engineering education, while driven by the need to improve student retention and success is also considering the mathematics skills required by future practising engineers (Coupland and Gardner 2008; Sheppard et al. 2009). Sheppard, Colby, Macatangay, & Sullivan (2009) advocate that engineering education should be centred on professional practice and the "demands on the new-century engineer." They are of the view that engineering schools are often influenced by academic traditions that do not always support the professions' needs. They say that in engineering the first professional degree is the undergraduate degree and that "the tradition of putting theory before practice and the effort to cover technical knowledge comprehensively, allow little opportunity for students to have the kind of deep learning experiences that mirror professional practice and problem solving" (Sheppard et al. 2009). There is a general support in the research literature for problem solving based learning strategies where students are required to engage in learning tasks that are relevant to engineering practice (Flegg et al. 2011). Janowski, Lalor and Moore (2008) from the University of Alabama are of the view that applying mathematics to solve complex engineering problems is an essential but often missing skill for young engineers. They support the idea of teaching mathematics in the context of engineering with a focus on: "the development of thinking and understanding; the development of engineering and mathematical language; the development of the confidence required to tackle large engineering projects and persist in finding solutions" (Janowski et al. 2008). Kent and Noss say that the engineering science "first principles" approach to mathematics in engineering education is being challenged by the "spectrum of mathematical competence" required in engineering practice" (Kent and Noss 2003). In Ireland Jane Grimson is also of the view that the science based approach to engineering education should be re-examined in the light of the needs of the 21st century engineering (Grimson 2002). While engineers in the past often had to resort to first principles, Grimson says that "problem solving today takes place at a higher level combining approaches and partial solutions and applying them to the problem in hand". Given the "vast array of modern problem solving tools and methodologies" available to engineers, Grimson calls on engineering educators to encourage students to "exploit the power of engineering tools in order to tackle real-world problems" (Grimson 2002). Similarly in Australia where several practising engineers say that their university mathematics was a 'waste of time', many engineers stressed the importance for engineers to understand the "mathematics and scientific fundamentals behind the software tools and techniques they use and the ability to validate quantitative outcomes of simulations" (King 2008). The Australian Learning and Teaching Council found that modelling, data analysis, statistics and risk assessment are deemed necessary for engineering practice (King 2008).

There are many calls for engineering curricula to better incorporate mathematics-oriented critical thinking skills including analytic skills, problem-solving skills and design skills (National Academy of Engineering 2005). Radzi, Abu and Mohamad (2009) maintain that with the current advancement in knowledge and technology, engineers are required to be increasingly critical in "discerning information and making decisive judgments when confronting unexpected situations and novel problems" (Radzi et al. 2009).

In an investigation of university students' conceptions of mathematics, Reid, Petocz, Smith, Wood and Dortins (2003) found that students experience mathematics in three different ways: components (toolbox of components and procedures); modelling (building and using models); and life (mathematics as an approach to life) (Petocz and Reid 2006; Reid et al. 2003). However for many students, the nature of a career involving mathematics is not at all clear (Petocz et al. 2007). While Petocz and Reid (2006) found that students' perceptions of mathematics in their future profession influence their approach towards learning mathematics in university (Petocz and Reid 2006), Wood found that "the use of mathematics within the job of engineer is not necessarily self-evident to an undergraduate student" (Wood 2008; Wood et al. 2011). Furthermore adjusting to the workforce can be problematic for many students as they discover what they learned in university needs to be contextualised for work (Wood 2010). In a study of first year engineering students in an Australian university, Flegg, Mallet and Lupton (2011) found that students generally regarded mathematics as relevant to their future career and study. In particular, the students noted specific benefits of mathematics education that include: ways of thinking (82%); ideas (79%); mathematical skills (76%); communicating using mathematical arguments (94%); and formulating and solving engineering problems (59%) (Flegg et al. 2011).

Thomas Romberg has another different perspective on mathematics education, he maintains that rather than "passing on a fixed body of mathematical knowledge by telling students what they must remember and do … society today needs individuals who can continue to learn, adapt to changing circumstances, and produce new knowledge" (Romberg 1992).

2.6 ENGINEERING PRACTICE

It is asserted that a lack of understanding about engineering limits the number of students entering and persisting in engineering education (Courter and Anderson 2009), thus it is interesting to explore what engineering is.

2.6.1 What is Engineering?

The Oxford English Dictionary defines an engineer as "one who contrives, designs or invents; an author, designer; also an inventor, a plotter, a layer of snares" (Oxford English Dictionary 1989). The U.S. Department of Labour describes engineering as the application of "the principles of science and mathematics to develop economical solutions to technical problems". It also says that engineers' "work is the link between scientific discoveries and the commercial applications that meet societal and consumer needs". Engineers work in design and development and in testing, production, or maintenance and engineers use "computers extensively to produce and analyse designs; to simulate and test how a machine, structure, or system operates; to generate specifications for parts; to monitor the quality of products; and to control the efficiency of processes" (U.S. Department of Labor website 2010-11). Wulf and Fisher from the National Academy of Engineering in the U.S. say that what engineers do is "design under constraint". They say that "engineering is creativity constrained by nature, by cost, by concerns of safety, environmental impact, ergonomics, reliability, manufacturability, maintainability – the whole long list of such 'ilities'" (Wulf and Fisher 2002). Sheppard, Colby, Macatangay and William (2006) present that there are three perspectives of engineering practice, these are: studies of individual and organisations engaged in engineering work; researchers who observe work of engineers and develop generalised understanding of engineering practice; and faculty and students engaged in engineering education. Their view is from research and engineering education perspectives. They say that engineering is, "at its core, problem solving" where formulating the problem and technical and non-technical requirements are key components. They say that engineers are able to engage in problem solving because they have mastered a specialised body of knowledge. However it is the integration of the problem solving process and specialised knowledge along with the available analytic and physical tools, the constraints and the requirements that comprise engineering practice (Sheppard et al. 2006).

However there is a view that there is an inadequate body of work on engineering practice and there are misconceptions as to what engineers actually do (Anderson et al. 2010; Cunningham et al. 2005; Tilli and Trevelyan 2008). Research also shows that students and teachers generally lack an understanding of what engineers do (Courter and Anderson 2009; National Academy of Engineering 2008). Chatterjee (2005) maintains that engineers have done a poor job defining who they are. He says that engineers who design are called scientists, engineers who develop new products are called entrepreneurs, engineers who program computers are called IT professionals and engineers who work in industry are called managers (Chatterjee 2005). Panitz (1998), in a study of the U.S. workforce, found that only about one third of engineering graduates work as engineers. The others worked as engineering managers, entrepreneurs, financial analysts, salespeople, educators and a variety of other positions (Panitz 1998). Chatterjee's view is that engineering's "broad sweep encompasses physics, chemistry, biology, mathematics, economics, psychology and more … it is the name for activity geared towards the purposeful exploitation of the laws, forces and resources of nature, not merely towards uncovering further esoteric truths but towards a direct improvement of the human condition" (Chatterjee 2005). Rosalind Williams from the Massachusetts Institute of Technology (2003) argues that engineering is undergoing an "identity crisis". She says that engineering has evolved into "an open-ended profession of everything in a world where technology shades into science, art, and management, with no strong institutions to define an overarching mission" and that "engagement with technology has far outgrown any one occupation" (Williams 2003).

A common theme in the literature describing engineering is associated with the conception of the term global engineer where the role of the engineer has become quite broad (Chatterjee 2005; Lohmann et al. 2006). Accordingly there are a number of different perspectives on what engineering practice is: it is “design process” (Eckert et al. 2004); “engineering practice is, in its essence, problem solving” (Sheppard et al. 2009); “the application of the theory and principles of science and mathematics to research and develop economical solutions to technical problems … the link between perceived social needs and commercial applications” (U.S. Department of Labor 2007); “a decision-making process (often iterative), in which the basic sciences and mathematics and engineering sciences are applied to convert resources optimally to meet a stated objective” (ABET Engineering Accreditation Commission 2010); and "the process of integrating knowledge to some purpose. It is a societal activity focused on connecting pieces of knowledge and technology to synthesize new products, systems, and sciences of high quality with respect to environmental fragility" (Bordogna 1992).

There is a view that engineering practice worldwide is changing. Many of the studies of engineering practice focus on the social relationships within a range of different engineering contexts. Sheppard, Macatangay, Colby and Sullivan (2009) say that historically the engineer was a “disengaged problem solver” because the engineer’s perspective was outside the problem whereby the engineer would “model the problem in “objective, mathematical terms” and devise a technical solution. They say this practice is outmoded and that there has been a shift from the outside to the inside perspective of “complex social, physical, and information interconnections that enable modern technologies to function” and engineers are now “immersed in the environment and human relationships from which perception of a problem arises in the first place” (Sheppard et al. 2009). Engineering is a highly collaborative process (Bucciarelli 2002; National Academy of Engineering 2005). Crawley, Malmqvist, Östlund and Brodeur say that modern engineers work in teams and that engineers exchange “thoughts, ideas, data and drawings, elements and devices” with other engineers around the world (Crawley et al. 2007).

In their study of engineers working in six different engineering firms, Anderson, Courter, McGlamery, Nathans-Kelly and Nicometo (2010) found that: engineers see real engineering work as technical problem solving while emphasising the importance of the coordinated efforts of a group of people; engineers identify a nuanced set of communication and coordination skills as the most important skills within their work; engineers say the most significant constraints on their work are organisational business practices relating to time and budgets; and engineering identity is a complex equation of problem solving, teamwork, lifelong learning and personal contributions where engineers value the thrill of solving a challenging problem (Anderson et al. 2010).

James Trevelyan (2009) is also of the view that engineering is both a technical and a social system. He found evidence that “engineers coordinate other people to deliver the products and services for which they are ultimately responsible” (Trevelyan 2009). In a longitudinal study of engineering graduates’ perceptions of their working time, Tilli and Trevelyan (2008) found that engineers spend 60% of their time explicitly interacting with other people (Tilli and Trevelyan 2008). In another study of engineers, Trevelyan (2010) also found that social interactions lie at the core of engineering and that engineering “relies on harnessing the knowledge, expertise and skills carried by many people, much of it implicit and unwritten knowledge” (Trevelyan 2010b). Trevelyan asserts that engineering practice is based on “distributed expertise” and engineering is a combined performance involving a range of people such as clients, suppliers, manufacturers, financiers and operators and as such a large proportion of engineers’ time is spent on social interactions. Engineering performance is time, information and resource constrained. Seldom is there complete information available and the available information has some level of uncertainty. A major part of engineers’ work is to explain, often at a distance and through intermediaries, how the products of their work need to be designed, built, used and maintained effectively (Trevelyan 2010a). Trevelyan observes that every engineering venture follows a similar sequence: engineers attempt to understand and shape clients’ perceptions of their needs; engineers conceive different ways to meet requirements economically; engineers collect data and create mathematical models to predict the technical and commercial performance of different solutions; engineers prepare plans, designs and specifications of work to be performed; engineers coordinate and manage work; and engineers arrange for decommissioning, removal and reuse and recycling at the end of a product’s life span (Trevelyan 2010a).

Trevelyan says that engineering practice relies on applied engineering science, tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience) and an ability to achieve practical results through other people. He adds that building a deep understanding of engineering practice into the curriculum has the potential to greatly strengthen engineering education (Trevelyan 2010a).

In a study of the early work experiences of recent engineering graduates Korte, Sheppard and Jordan (2008) hold that the social context of engineering in the workplace is a major driver of engineering work and they call on engineering educators to better prepare students for the social context of their future work by specifically offering industry-relevant learning experiences to students. In their study the new engineers defined their work as a "problem-solving process or way of thinking" where they tried to "organise, define, and understand a problem: gather, analyse, and interpret data: document and present the results: and project-manage the overall problem-solving process". The engineers presented that the "workplace problems often lacked data and were more complex and ambiguous with far more variables" compared to school problems. One problem for engineers was that workplace problems often had multiple and conflicting goals and multiple solutions. Another problem for the engineers was their "not knowing the "big picture" in which a problem was grounded". The engineers found that their lack of understanding of the big picture contributed to the uncertainty and ambiguity in their understanding of their work and to the value of their work in the organisation. Interpreting data was a new experience for many engineers. One engineer said he was "learning more about how to present my data to other people". A challenge for many new engineers was the accuracy of their methods which often depended on other people's judgement rather than as derived from data. The engineers presented that their work involved "a large amount of social interaction and social influence". They had to learn the constraints of the social system within their work groups and the new engineers "relied on their co-workers and managers to learn the subjective aspects of their work". The engineers say that "learning from co-workers was the primary method of learning on the job" (Korte et al. 2008).

Wood (2010), who investigated mathematics graduates transition to the workforce in terms of their communications skills, found that graduates generally felt they knew more mathematics than was required for their work positions. She also found that most engineers associated logical thinking with their work. The graduates noted that their education did not teach them to use standard computer products such as Excel (spread sheet software), Visual Basic (programming language) or SAS (Statistical Analysis System software). The graduates found that they had to change their ideas of working as a mathematician and how mathematics is used in the real world particularly where assumptions are relaxed. Prior to working the graduates had not considered the use of mathematics to communicate ideas. In the workplace, graduates are often the only ones who can speak the mathematical language and many graduates are unable to release the strength of their mathematics because they do not know how to communicate mathematically (Wood 2010).

Trevelyan (2011) says that in Australia, most companies assert that it takes up to three years for a novice engineer to become reasonably productive in a commercial context. While medical educators have embraced extensive clinical practice, Trevelyan argues that it is not possible for engineering educators to do the same given the diversity of engineering career settings and the complexity of engineering environments. He notes that the scarcity of systematic research on engineering practice makes it difficult for educators who wish to design learning experiences to enable students to manage the transition into commercial engineering contexts more easily (Trevelyan 2011).

The increasing availability of computerised tools and resources is contributing to the changing nature of engineering where IT tools are dominating modern engineering practice (Anderson et al. 2010). Grimson (2002) says that "the engineer today has at his or her disposal a vast array of modern problem-solving tools and methodologies, which can be applied without detailed knowledge of the underlying techniques" (Grimson, 2002). Crawley, Malmqvist, Östlund and Brodeur say that "modern engineers design products, processes and systems" that are sometimes state-of-the-art technology but engineering is mostly "applying and adapting existing technology to meet society's changing needs" (Crawley et al. 2007).

2.6.2 The Engineering Profession

Despite the growing importance of engineering practice to society, the engineering profession is held in low esteem compared to other professions. Duderstadt (2008) attributes this poor image to the "undergraduate nature" of the curriculum and to the "evolution of the profession from a trade" and the way that industry all too frequently tends to view engineers as "consumable commodities, discarding them when their skills become obsolete or replaceable by cheaper engineering services from abroad". So too, the low public prestige of the engineering profession is apparent both in public perception and in the declining interest of students in engineering careers relative to other professions such as business, law, and medicine. "Today's engineers no longer hold the leadership positions in business and government that were once claimed by their predecessors in the 19th and 20th century, in part because neither the profession nor the educational system supporting it have kept pace with the changing nature of both our knowledge-intensive society and the global marketplace. In fact the outsourcing of engineering services of increasing complexity and the off shoring of engineering jobs of increasing value raise the threat of the erosion of the engineering profession in America and with it our nation's technological competence and capacity for technological innovation" (Duderstadt 2008).

In a study of perceptions of engineers and engineering, the National Academy of Engineering (NAE) found that there is no readily identifiable "public face" of engineering. They also noted that it takes a "powerful awareness" to be able to see engineering even though it is everywhere. The NAE found that some engineers can "be very hard on themselves" and that they see themselves as "nerds and geeks". One of the study participants says "people who are not in it [the field] have a hard time grasping what we do [and] we don't do a good job of explaining it either. It [engineering] is seen as a bunch of technical things they can't grasp … and boring, too". The NAE say that the perceived difficulty of technical aspects of engineering, especially mathematics and science, contributes to difficulties communicating engineering (National Academy of Engineering 2008).

Jane Grimson (2002) holds the view that the context-free approach of engineering science is not readily adaptable to solving real world problems and that engineers' failure to realise the importance of the context-sensitive view undermines the engineering profession. She is of the view that society values engineers who can apply their skills across disciplines and she notes the importance of engineers communicating effectively with non-technical people. She says that engineers should have the ability to explain technical problems. Given the speed of development of new engineering knowledge Grimson stresses the requirement for the engineering profession to keep up to date and to develop business, financial, marketing and management expertise (Grimson 2002).

In "Educating the Engineer of 2020", the National Academy of Engineering (NAE) in the United States say that "practising engineers seek to maintain a professional identity that they can carry with them, irrespective of who is their current employer". Professional bodies are the primary avenues for engineers to support their identities as professional engineers and for identifying opportunities for continuing professional education (National Academy of Engineering, 2005). The NAE is of the view that engineers' engagement in public policy issues is poor and that this is damaging the image of the profession. The NAE says that "it is critical to try to improve public understanding of engineering, so that the public can appreciate the value and consequences of new technology and meaningfully participate in public debates where technology is a critical factor" (National Academy of Engineering 2005).

2.7 MATHEMATICS USAGE IN ENGINEERING PRACTICE

In light of the points highlighted in this literature review, it is unfortunate that there is limited published research on practising engineers' mathematics usage. These points include: the diversity of mathematics as a subject; students' disaffection with and difficulty learning mathematics; the declining interest in engineering careers; the perception that students' difficulties with mathematics is a major factor in the declining choice of engineering careers; the need to reform engineering education; the rapidly changing nature of engineering practice; and the role of engineering in the global economy. There is a need for research to develop a measurement of mathematics usage in general professional engineering work. This includes engineers' usage of specific mathematics topics, concepts, contexts and levels of complexities, ways mathematics is used and required in engineering practice and engineers' motivation to use mathematics in work. This knowledge is required to inform engineering mathematics education.

2.7.1 Investigating Engineers' Mathematics Usage

Burkhard Alpers (2010) notes the significance of researching the mathematics used by engineers in their work. He says that in order to provide "a mathematical education of engineering students which is relevant for their later work as engineers, one needs studies that try to capture the mathematical expertise of engineers" (Alpers 2010b). According to Alpers there are only a few studies of engineers' usage of mathematics because "they are not easy to conduct". Of the studies conducted, researchers have concentrated on specific branches of engineering rather than investigate the work of practising engineers generally and some studies have investigated engineering students' mathematics usage. Research methods used to investigate engineers' mathematics usage include ethnography, interviews and investigation of tool usage. Studies focus on usage of school mathematics, mathematical understanding and hidden mathematics. Alpers is of the view that investigating engineering students' work is "unrealistic" because students, unlike engineers, have "no time pressure" in their work, the students do not have to fit into any organisational structure and specific student tasks are not representative of broad engineering practice. However students are far more accessible than practising engineers to participate in studies. Another potential limitation of investigating engineers' mathematics usage is that a lack of familiarity with engineering work could restrict researchers' identification of mathematics usage (Alpers 2010b).

Given the perceived importance of mathematics knowledge and skills in the engineering curriculum, research literature concerning the type of mathematics used in engineering practice is sparse. Monica Cardella, from the University of Washington

Seattle is one of the few people who have researched the role of mathematics in engineering practice and she has found that only “few papers include empirical evidence for the role and the importance of mathematics in engineering” (Cardella 2007).

Cardella (2007) notes that “while many educators believe that mathematics is important for engineering students, there is a belief among some practising engineers that the mathematics they learned in college is not applicable to their daily work” (Cardella, 2007). Two British academics, Kent and Noss (2003) identify “different uses of mathematics in engineering practice: the direct usefulness of mathematical techniques and ideas to practice” and the “indirect usefulness - the ways in which mathematics contributes to the development of engineering expertise and judgment” (Kent and Noss 2003). An Irish academic, Jane Grimson, maintains that while engineering education produces graduates who “have a deep understanding of the scientific and mathematical principles underpinning their particular discipline …. the constraints on engineering problem-solving today are increasingly not technical but rather lie on the societal and human side of engineering practice” and “the engineer today has at his or her disposal a vast array of modern problem solving tools and methodologies, which can be applied without detailed knowledge of the underlying techniques …. problem solving today takes place at a higher level, combining approaches and partial solutions and applying them to the problem in hand” (Grimson 2002) .

Contemporary authors of published research on the subject of engineers’ usage of mathematics include: Monica Cardella (United States of America): Cynthia Atman (United States of America); Burkhard Alpers (Germany); Elton Graves (United States of America); Peter Petocz (Australia); Anna Reid (Australia); Julie Gainsburg (United States of America); Philip Kent (United Kingdom); Richard Noss (United Kingdom); Mike Ellis (United States of America); Brian Williams (United States of America); Habib Sadid (United States of America); Ken Bosworth (United States of America); Larry Stout (United States of America); Zlatan Magajna (Slovenia); John Monaghan (United Kingdom); Chrissavgi Triantafillou (Greece); Despina Potari (Greece); and Jim Ridgway (United Kingdom).

2.7.1.1 Monica Cardella and Cynthia Atman

Studies by Cardella and Atman focus on engineering students rather than on practising engineers' mathematics usage. In their study of industrial engineering students' use of mathematics, Cardella and Atman interviewed and observed five industrial engineering students' using mathematics while they worked on their capstone projects. They also conducted interviews with four engineering students from other areas (Cardella and Atman 2004). Their data was analysed according to the five aspects of mathematical thinking described by Schoenfeld (Schoenfeld 1992). Cardella and Atman found that: the students thought about mathematics in terms of core knowledge rather than as a thinking process; they used "guess and verify" and problem decomposition mathematical problem solving strategies; they used mathematical tools e.g. Excel and MapPoint and experts' advice; they recognised multiple approaches to solving problems; they viewed mathematics as content knowledge; they expressed a belief that mathematics is equivalent to a set of tools; they looked at problems with a mathematical perspective; and they struggled to deal with uncertainty. The authors note that the students were unable to apply many mathematical skills they had learned. Because the students grappled with "tension between estimation and precision", the authors say that the students had an "incomplete understanding of mathematical thinking". The authors are of the view that mathematics courses benefit engineering students by the material and the thinking processes and strategies learned. They also note that students might not be aware of their use of mathematics but "if engineering students believed that mathematics was more about a way of thinking than about particular content knowledge, they might value mathematics more, be more motivated to learn mathematics and might be more predisposed to apply mathematical thinking" (Cardella and Atman 2005).

In another study, Cardella and Atman observed and listened to senior and freshman engineering students who were asked to design a playground. The authors found that mathematical thinking plays a large role in engineering design and they say that engineering design problems “motivate and accentuate mathematics learning” (Cardella and Atman 2007).

Cardella (2007) interviewed one industrial engineering undergraduate and one mechanical engineering graduate student about their perceptions of what they had learned from their mathematics courses as well as their use of mathematics and mathematical thinking in their design projects. She found, that in addition to the content knowledge, the students “learned to frame problems, apply mathematics to engineering topics, discern what information is relevant to a particular problem or project, use mathematical software and work with peers on homework.” While the students did not remember all mathematical content knowledge, they did develop a foundation that prepared them to “relearn” the material if needed. The students also developed beliefs and affects about and towards mathematics. Cardella found that “recognising the value of mathematics as a tool likely prepares students to use mathematics in appropriate contexts” (Cardella 2007).

In further work, Cardella (2008) interviewed nine students representing five engineering disciplines who had worked on a 5-month long capstone design project. She asked the students what they learnt from their mathematics courses and if they gained anything else from them. She also interviewed four mechanical engineering graduate students who worked with an industry client. She found that both groups engaged in mathematical thinking activities. Cardella offers the opinion that engineering students should learn the following: problem solving strategies; mathematical “software important to engineering practice”; how to communicate with “others who can provide mathematical expertise”; how to “access social and material resources”; how to “manage their use of resources”; how to “plan their process for finding and solving problems” and how to “monitor their progress in accomplishing their goals”. She proposed that the “full space of mathematical thinking – the mathematical knowledge base as well as problem-solving strategies, resources, use of resources, beliefs and affects and mathematical practices” should be considered in engineering students’ mathematical education (Cardella 2008).

Cardella (2010) observed and interviewed five industrial engineering undergraduates and four mechanical engineering masters degree students. Using grounded theory methodology, mathematical modelling emerged as another theme in addition to Schoenfeld's five aspects of mathematical thinking. Cardella found that mathematical modelling is central to engineering practice and a valuable tool for engineers. She states that examples of how engineers use mathematics can provide context and motivation for learners and she also notes that "some undergraduate engineering students can become frustrated by the ambiguity and uncertainty that are normal for authentic engineering tasks" (Cardella 2010).

2.7.1.2 Burkhard Alpers

Burkhard Alpers (2010) also studied students. In his study, he hired two mechanical engineering students during their last semester to work on CAD (Computer Aided Design tool) and FEM (Finite Element Method tool) tasks that reflect practical work of junior engineers. Together with a colleague he worked with the students, studied their work notes and interviewed the students. He observed that "engineers using mathematical objects predominately think in application terms". Alpers found that while computational tools permeate engineering work that an understanding of the mathematical concepts at the interface of tools is necessary for engineers' work. He noted that "a mathematical expectation of results" is required to check the output of computer tools. He noticed that students encountered "breakdown situations" where tools produce unexpected results. In breakdown situations, where the underlying mathematics is too complicated for the design engineer, the user has to find a way to work around the situation or ask an expert. Alpers noticed that the students often used "quick solutions" and "qualitative reasoning" where "more precise quantitative models" might have been more efficient. Alpers' investigation showed that while "most of the mathematical concepts and procedures are "buried in technology," for reasonable usage of the interface, mathematical knowledge and understanding is still necessary" (Alpers 2010a; Alpers 2010c).

2.7.1.3 Elton Graves

A study of senior engineering students in Rose-Hulman Institute of Technology in Idaho State University found that the "concepts learned in the calculus, differential equations and statistics courses were regularly used by the students in their engineering courses". While the students might not always remember a mathematical concept, they knew where to go to review any forgotten material. The students believed that mathematics is important, useful and would be a tool that they would use when they leave college (Graves 2005).

2.7.1.4 Peter Petocz and Anna Reid

Petocz and Reid (2006) used phenomenography (qualitative approach to research how people experience, understand and ascribe meaning to a specific phenomenon) to investigate recent graduates' views of using mathematics in the workplace. They found that graduates view mathematics in three different ways: mathematical techniques; applying the idea to a broader range of work problems; and a way of understanding the world. They also noted that what remains when the mathematics has been forgotten is their ability to solve problems and think logically (Petocz and Reid 2006).

2.7.1.5 Julie Gainsburg

Julie Gainsburg studied the mathematics behaviour of structural engineers at work (Gainsburg 2006). In an ethnographic study Gainsburg observed engineers from two different firms as they engaged in four work tasks. She found that mathematical modelling was central to and ubiquitous in the engineers' work whereby the structural engineers transformed "hypothetical structures into mathematical or symbolic language for the purpose of applying engineering theory." Gainsburg defines mathematical modelling as "translating a real-world problem into mathematics, working the math, and translating the results back into the real-world context". She noted that the engineers use, adapt and create models of various representation forms and degrees of abstraction. She found that structural engineers' proposed design "must be informed by an analysis of the design behaviour but analysis cannot occur until there is a design to analyse." Thus the structural engineers model "hypothetical entities" as a means for generating data. Another difficulty noted is the engineers' difficulty of keeping track of the various models based on varying assumptions. She observed that the engineers chose a model that was inadequate because they could justify their design decisions. Gainsburg maintains that engineering modelling is "context dependent and context specific" and that the mathematical methods are "always subordinate to the engineers' judgment about their use" (Gainsburg 2006).

Gainsburg lists the benefits of mathematical modelling in the classroom. She says "modelling experiences are expected to enhance students' ability to transfer mathematical tools to novel problem-solving situations" and she notes that "computer-based technologies are assumed to have reduced the need for workers to perform routine calculations but increased the requirement to solve more complicated, non-routine problems that involve analysing, interpreting, and finding patterns in data as well as constructing, describing, explaining, and manipulating complex systems – all activities associated with modelling." Gainsburg contrasts structural engineers' modelling of "physically non-existent or inaccessible phenomena" with classroom modelling of "existing phenomena". She says that real-world problem solving would push students' reasoning and justifying to higher levels and compel students to weave non-mathematical ideas and resources into that reasoning". If the goal is real-world problem solving then Gainsburg calls for "constructivist, process-oriented curricula" rather than "content and procedural proficiency" (Gainsburg 2006).

2.7.1.6 Philip Kent and Richard Noss

Two mathematics educators Kent and Noss interviewed and observed civil and structural engineers working in a large engineering design consultancy in London (Kent and Noss 2002). They found that younger engineers do most of the analysis, especially computer-based analysis while older engineers do the broader design tasks. One engineer, who was of the view that engineers learn by "apprenticeship", said:

> "At the start of their careers, engineers are unable to deal with everything in a project, and they begin by being given straightforward things to do. They get introduced to all aspects of a structure bit-by-bit, and no one person actually ends up designing the whole structure. So, as an engineer grows up, they may no longer be using the mathematics that they started out using, they are still using the understanding that they derived earlier in their experience, and some sort of this is difficult to describe as to the sort of knowledge it is".

Kent and Noss observed that, while mathematical analysis was done by black boxes, the engineer who uses the mathematical result is required to understand what's happening inside the black box. Consequently there is:

> "a lot of looking at the results, finding out where things aren't performing as you would expect. You need the knowledge of how and what you expect the answer to be, so that you can see where the problems are. There is this big cycle of you make the model, check it, look at the results, check it again, make the model again if necessary".

Kent and Noss observed that mathematics is used as a "communication tool" between the designer and the specialist whereby the "specialists" are able to:

> "synthesise complex problems down to something very small, which can be expressed mathematically … the specialist can give you a set of equations, which you can adjust, change the parameters. So the maths is used as a communication tool, he's digested a situation into a model which is accessible to the general engineer, with a general mathematical background".

Kent and Noss found that the use of software in engineering practice makes mathematics easier to use and understand, for example one engineer said:

> “you play around with a computer model of a bridge, overstress it and watch it collapse, underbrace it and watch it vibrate”.

Kent and Noss observed the “designer-specialist interface” in the engineering firm and they note that the engineers’ work is “less abstract” than the specialist mathematician and that an engineering design task has its own “complexities” of which mathematics is often a small but “crucial component”.

Kent and Noss found that geometry is a key element of structural understanding. They noted that engineers spoke about structural geometry in terms of “qualitative understanding” and they say that a “structural feel” or “a sense of qualitative is entwined with the notion of design in contrast to the quantitative calculations of analysis”. Kent and Noss say that design involves using the results of analysis and it is not, “in the way most engineers think about it, a quantitative, mathematical activity”. However they are of the view that the “structural feel” is intuitive and that it is learnt by experience, part of which is learning mathematics in school and using mathematics in engineering practice.

Kent and Noss say that the fact that the majority of design engineers can work without having to do advanced mathematics is due to mathematical expertise in the form of computer programs and analytical specialists, in engineering practice. They suggest that due to the ubiquity of mathematical technology that the “balance between explicit analytical skills and “qualitative” appreciation” is radically shifting and they suggest that the challenge facing “undergraduate service mathematics” is about “questioning the interfaces between engineering and mathematical knowledge as differently experienced by practising and student engineers” (Kent and Noss 2002).

Kent and Noss are of the view that while “the role that mathematics plays in professional practice has changed radically in the last 30 years,” there is clear agreement that undergraduate engineering students continue to need to know and learn mathematics. They say the “fundamental question is what kind of mathematics is needed and when.” They “found that some aspects of engineering mathematics remain crucial: the possession of a mental sense of ‘numbers’; the ability to approximate scales and orders of magnitude; the ability to perform approximate mental calculations; and the 'application' of engineering principles based on mathematical ideas – and how all these contribute to professional engineering judgement" (Kent and Noss 2003).

Kent and Noss' perception of civil engineering practice is that confidence at a certain basic level of mathematics is the most important thing for the majority of engineers.

2.7.1.7 Mike Ellis, Brian Williams, Habib Sadid, Ken Bosworth and Larry Stout

Ellis, Williams, Sadid, Bosworth and Stout (2004) conducted a survey of Idaho State University's College of Engineering Advisory Board and recent alumni of the College of Engineering to determine if the engineers use topics on the engineering mathematics curriculum. While the authors are concerned about their participant sampling process, they did find that "at least a conceptual understanding of majority of math topics is required to perform their job functions even though the survey indicates that the actual usage of these same calculation techniques is significantly less" (Ellis et al. 2004).

2.7.1.8 Zlatan Magajna and John Monaghan

Magajna and Monaghan (2003) observed the use of mathematics in a computer aided design and manufacturing setting by six "technicians"[13] over three weeks. They noticed "an evident discontinuity between the school mathematics used and the observed mathematical practices". Although the technicians did not consider their work was related to school mathematics, Magajna and Monaghan found evidence that in making sense of their practice, the technicians resorted to a form of school mathematics, this they call mathematical thinking. It was also found that the role of technology in the technicians' mathematical activity was crucial (Magajna and Monaghan 2003).

[13] Technician: In Ireland, technicians have a diploma (level 6) qualification while engineers have a degree (level 8) qualification. Unlike level 8 engineering education entry requirements, students entering level 6 engineering courses are not required to have a grade of C3 (55- 59.9%) or higher in higher level Leaving Certificate mathematics.

2.7.1.9 Chrissavgi Triantafillou and Despina Potari

In Triantafillou and Potari's investigation of technicians' use of mathematics in a telecommunications organisation in Greece, they adopted a sociocultural perspective where mathematics is embedded in the work context and is mediated through the tools. In their ethnographic study, Triantafillou and Potari found that all the technicians in the study trusted the instruments and tools they used in their work and only the expert group were aware of the need to "go more deeply into the way they operate". The expert group of technicians also acknowledged that they needed mathematics to better understand their work particularly in breakdown situations. The technicians were observed to have used basic mathematical ideas from statistics, algebra and geometry (Triantafillou and Potari 2006).

2.7.1.10 Jim Ridgway

A study of the mathematical needs of engineering apprentices using ethnography, interviews and psychometric testing revealed that mathematical challenges of engineering differ from mathematics taught in school. Ridgway found that the apprentices' work required "high levels of precision" and included "a good deal of practical problem solving". Ridgway suggests that learning is dependent on context and that learning mathematics in school, then applying it to a rather unfamiliar industrial context is likely to require relearning (Ridgway 2002).

2.7.2 Summary

While there is no consistent research-informed view of "how, what, when and by whom" mathematics should be taught to engineering students (Flegg et al. 2011), research concerning the mathematical expertise that is in fact used in engineering practice is sparse (Alpers 2010b; Cardella 2007; Trevelyan 2009). It could be argued that the studies of mathematics expertise required and used by practising engineers are scattered in that only a minority of engineering types have been studied and also only in the context of certain aspects of mathematics usage. Of the studies that do exist, most take a qualitative approach and they are thus confined to small numbers of engineers. It is engineering students, structural engineers working in two different firms, civil and structural engineers working in one large engineering design consultancy and technicians that are represented in the available literature concerning engineers' use of mathematics. These types do not adequately represent modern professional engineering practice which comprises many different branches of engineering (e.g. civil, electronic and mechanical). Furthermore, in these studies, mathematics is mostly confined to mathematical thinking and the use of computer tools. However mathematical activity has a greater scope. For example, Ernest (2010) lists various types of mathematics, including: functional numeracy; practical work-related knowledge; advanced specialist knowledge; mathematical knowledge and powers in both posing and solving mathematical problems; being confident in one's personal knowledge of mathematics; and being able to identify and critique the mathematics embedded in social, commercial and political systems (Ernest 2010).

There is currently no broad picture of the mathematical expertise required or used by practising professional engineers. In order to prepare engineers for engineering practice, there is a need to investigate the role of mathematics in engineering practice generally.

2.8 SUMMARY

This chapter contains a review of literature about mathematics education, career choice, engineering education and engineering practice. The purpose of this chapter is to establish the current available knowledge about the role of mathematics in engineering practice and also research knowledge concerning students' experiences with school mathematics and its role in engineering career choice. Included in this chapter are: an exploration of what mathematics is; the different general learning theories relating to mathematics learning and teaching; career choice factors and the selection of engineering careers; a review of mathematics in engineering education; a discussion about engineering practice; and a summary of research concerning engineers' use of mathematics.

It is shown that mathematics has great variety, depth and uses. There is some evidence to suggest that mathematics is a special subject compared to other school subjects and that a "mathematics problem" (Smith 2004) exists whereby there is a real disaffection in many students towards mathematics. In particular, students' difficulty with higher-level school mathematics is considered to be a major contributor to the declining number of entrants to engineering degree courses worldwide. Research literature shows that women's mathematical self-efficacy is significantly lower than men's perceptions of their mathematics capability and that this is a major influence on career choice. The National Council of Teachers of Mathematics (NCTM) in the U.S. is one active initiative aimed at reforming school mathematics and it provides principles and standards to guide teachers who seek to improve mathematics education in their classrooms and schools. Vygotsky's theory of social constructivism indicates that understanding and social interaction are key components of effective mathematics learning (Vygotsky 1978).

Research literature shows that the mathematical ability of students entering engineering education is a concern and there is an on-going debate about the need to reform engineering education. Given that there is little research on mathematics used by practising engineers generally and that the work that does exist takes a qualitative approach and involves small samples of engineering students, there is a need to enhance the published research on practising engineers' mathematics usage and the relationship between students' experiences with school mathematics and their choice of engineering careers, which is the object of this study.

CHAPTER 3: RESEARCH DESIGN

3.1 INTRODUCTION

"The purpose of research is to enhance knowledge, to in some way enable us to know more" (King and Horrocks 2010). In this study, the main aims are to develop new knowledge about the two main research questions:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

Prior to discussing the specific design, this chapter starts with a background theory based framework for the research design. This chapter is organised as follows:

Page number

3.2 BACKGROUND THEORY BASED FRAMEWORK FOR THE RESEARCH DESIGN

The background theory is presented in two parts:

Page number

3.2.1 Measuring Engineers' Mathematics Usage

Measuring mathematics usage is a major part of this study. As presented in Chapter 2, mathematics is a very diverse subject and is viewed differently by different people, in different situations and in different time periods. A key theme underpinning this study is the question whether there is a mismatch between mathematics taught in schools and universities and the mathematics required for engineering practice. For example, one view in the research literature is that engineering schools are often influenced by academic traditions that do not always support the professions' needs and the "the tradition of putting theory before practice and the effort to cover technical knowledge comprehensively, allow little opportunity for students to have the kind of deep learning experiences that mirror professional practice and problem solving" (Sheppard et al. 2009).

Robyn Zevenbergen (2000) distinguishes between research work that mainly tries to detect school mathematics in the workplace and real ethnographical studies which try to capture hidden mathematics. She holds that studies conducted "through the eyes of school mathematics" only recover "frozen mathematics". Instead she says that there are three forms of mathematics used in the workplace: formal mathematics (what mathematicians use); school mathematics; and everyday mathematics (ethnomathematics). She contends that in the workplace people develop contextualised strategies for resolving everyday problems that have little resemblance to school mathematics. Zevenbergen's views are in the context of workers who use a range of school mathematics including: workers who use high level of school mathematics (e.g. engineers); workers who use medium levels of school mathematics, which is modified to the context (e.g. bankers); and workers that do not use school mathematics such as workplaces that are mechanised (e.g. fast food outlets). She also recognises that technology influences how mathematics is used in both high level and low level mathematics workplaces (Zevenbergen 2000). While Zevenbergen is critical of studies that search for school mathematics in workplaces, Julie Gainsburg (2005) is of the view that information concerning school mathematics usage in the workplace is of value to engineering educators. Gainsburg contends that "our current knowledge about the actual mathematical requirements of today's workplace is far from complete" and that "we have hardly begun to explore how (and whether) learning math in school contributes to adult problem-solving proficiency". She presents that the gap between formal (school mathematics) and informal mathematics (context-dependent) has become an accepted paradigm in ethnographic studies of mathematical behaviour and that this is "highly problematic" because location-dependent definitions of formal and informal mathematics limit research studies that investigate workplace mathematics. Instead Gainsburg is of the view that there are many kinds of mathematical behaviour, "displaying various degrees of formality, generality, and precision" which are not only exhibited within single settings but by single practitioners in response to varying conditions". She recommends that investigations of mathematical behaviour in the workplace should emphasise individual behaviour rather than distributed activity and focus on a level of mathematical activity relevant to school mathematics programmes, or problem-solving behaviour that workers substitute for such mathematical activity (Gainsburg 2005).

Given that one aim of this study is to generate new knowledge of mathematics usage in engineering practice so as to inform engineering educators, all workplace mathematics, including usage of school mathematics is considered. The initial task was to represent such mathematics. While the researcher could not find any complete representation of workplace mathematics in the literature, representations of school mathematics competence that include real-world applications of mathematics were studied. This included work by Romberg (1992) who suggests that school mathematics assessments should include the following principles:

identification of a set of specific and important mathematical domains; construction of a variety of tasks that reflect the typical procedures, concepts and problem situations for each domain; administration of tasks via tailored testing (sample of items following certain rules); student scores for a particular domain should result from a logical combination of the complexity of the tasks and the students' responses to these tasks for each domain (Romberg 1992). As discussed in Chapter 2, the NCTM's mathematics standards focus on content and process standards (National Council of Teachers of Mathematics 2000); Niss (2002) presents mathematics as eight competencies (Niss 2003); PISA mathematics assessments focus on content, competencies and situations (Organisation for Economic Co-Operation and Development 2009); and TIMSS classifies mathematics into "content domains" and "cognitive domains" (International Association for the Evaluation of Educational Achievement 2011).

3.2.1.1 De Lange's Mathematics Assessment Pyramid

The *curriculum mathematics*[14] usage instrument developed in this study is a derivation of de Lange's mathematics assessment pyramid. This has some similarities with the PISA mathematics assessment and it comprises of three levels of mathematical thinking and understanding. De Lange's mathematics assessment pyramid arose from Realistic Mathematics Education (RME) which was introduced in the Netherlands in the nineteen eighties. RME is based on an epistemological view of mathematics as a human activity and this education system was designed to reflect how users of mathematics "investigate a problem situation, decide on variables, build models relating the variables, decide how to use mathematics to quantify and relate the variables, carry out calculations, make predictions and verify the utility of the predictions" (De Lange and Romberg 2004). Unlike the traditional approach of learning mathematics, with RME mathematics is introduced in the context of carefully chosen problems, where in the process of trying to solve these problems students

[14] Curriculum mathematics: Term devised in this study to represent engineers' mathematics education at school and university.

develop mathematical ability. Teachers employ a method of guided reinvention, by which students are encouraged to develop their own informal methods for doing mathematics. Students, while working on context problems, develop mathematical tools and understanding. First, students develop strategies closely connected to the context of problems, then they develop models for solving other but related problems and eventually, the models give the students access to more formal mathematical knowledge.

The RME approach to mathematics assessment is closely aligned with instruction whereby mathematics assessments focus on the ways in which students identify and use concepts and skills to model, solve and defend their solutions with respect to increasingly complex tasks. Monitoring student progress involves the use of open tasks, "through which students relate concepts and procedures and use them to solve non-routine problems, in contrast to conventional tasks that require the reiteration of procedures learned to solve problems that merely mimic the content covered" (De Lange and Romberg 2004). Jan de Lange of the Freudenthal Institute in the Netherlands developed the mathematics pyramid assessment model for mathematics education whereby every assessment question can be located in a pyramid according to three dimensions: the mathematical content; and the degree of difficulty and the level of thinking shown in Figure 3-1.

De Lange distinguishes three components of mathematics education and assessment which are located on the three pyramid axes. These are: (i) domains of mathematics (e.g. algebra and geometry); (ii) the complexity of assessment questions (e.g. easy or difficult); and (iii) levels of mathematical thinking and understanding (lower, middle and higher); (De Lange 1994).

Central to de Lange's mathematics assessment pyramid are three levels of mathematical thinking and understanding. Level 1, which is usually referred to as reproducing, includes: reproducing facts; recalling properties; performing routine procedures; applying standard algorithms; and dealing with statements that contain standard symbols and formula. Level 2 is also called connecting and at this level students start making connections within and between the different domains in mathematics; integrate information in order to solve simple problems; have a choice of strategies; and have a choice of mathematical tools. Level 3 is mathematising and at this level students are required to recognise and extract the mathematics embedded in a situation and use mathematics to solve a problem (that may involve multiple answers); analyse; interpret; develop models and strategies; and make mathematical arguments, proofs, and generalisations (de Lange and Romberg, 2004, de Lange, 1999).

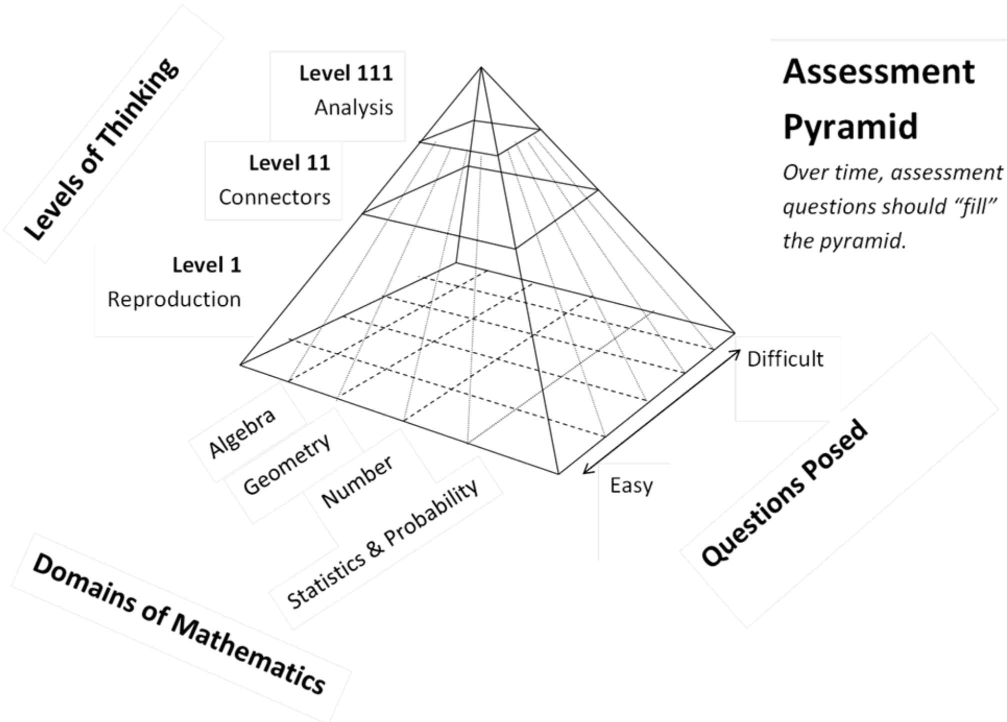

Figure 3-1: De Lange's assessment pyramid (De Lange and Romberg 2004).

De Lange's assessment pyramid with its three levels of thinking is very similar to both PISA where there are six proficiency levels and TIMMS where there are three cognitive domains: knowing; applying; and reasoning. In de Lange's assessment pyramid, a connection is made between the levels of competence students are expected to have in order to solve a particular problem, the degree of complexity and the difficulty of the content of the problem and the degree of complexity which is caused by the way the question is posed. In a balanced test there should be questions in all content domains, of varying degrees of difficulty and at all levels of thinking. The reason the model is pyramidal in shape is that as the level of thinking required increases, it becomes harder to distinguish mathematical content domains and also the range between easy and hard questions becomes smaller (De Lange 1994; De Lange 1999; De Lange 2001; De Lange and Romberg 2004; Verhage and De Lange 1997).

De Lange's mathematics assessment approach was chosen in this study because it provided a foundation for developing an instrument to measure school and university mathematics usage in engineering practice. In this study the term *curriculum mathematics* was devised to represent engineers' mathematics education at school and university. The three dimensions of *curriculum mathematics* are content, academic level and usage type. The three dimensional model of *curriculum mathematics* also provided a means to visually represent various aspects of mathematics which the researcher believes works particularly well for engineers generally.

3.2.1.2 Project Maths

One dimension of de Lange's assessment pyramid is mathematics domains. This research coincided with a major revision of the school mathematics curriculum in Ireland and it was decided to incorporate the mathematics domains in the revised curriculum into this study. The new initiative called "Project Maths" is an on-going initiative to change how mathematics is taught and learned in post-primary schools by showing how mathematics connects with real-life problems, and about how skills developed in mathematics can be used in other subjects, in the workplace and at home. The rationale behind Project Maths is that by teaching mathematics in contexts that allow learners to see connections within mathematics, between mathematics and other subjects and between mathematics and its applications to real life, learners develop a "flexible" and "disciplined" ways of thinking and also an "enthusiasm to search for creative solutions". Project Maths syllabi comprises five strands: statistics and probability; geometry and trigonometry; number; algebra; and functions. Within Projects Maths there are five key skills central to teaching and learning mathematics, these are: information processing; being personally effective; communicating; critical and creative thinking; and working with others (National Council for Curriculum and Assessment 2010b).

Draft syllabi for Junior Certificate (ordinary level and higher level) and Leaving Certificate (foundation level, ordinary level and higher level) are organised according to topics and corresponding learning outcomes for each of the five strands (National Council for Curriculum and Assessment 2010b).

In this study, the Project Maths draft syllabi, available at the time of this research, were used to reflect as accurately as possible the mathematics domains and topics of interest in mathematics generally.

3.2.1.3 Measuring Mathematics Usage in Engineering Practice

The methodology used to measure *curriculum mathematics* in this study is based on de Lange's mathematics assessment pyramid (De Lange 1999; De Lange and Romberg 2004). Mathematics usage is measured with reference to three dimensions: (i) Domain, this refers to the five mathematics domains specified in the new "Project Maths" syllabi (National Council for Curriculum and Assessment 2010b); (ii) Usage type, the three main usage types are reproducing, connecting and mathematising (De Lange 1999; De Lange and Romberg 2004); and (iii) Level, this refers to academic levels. This methodology is developed further in Chapter 4.

A second type of mathematics of interest in this study is *mathematical thinking* (*thinking*) which is defined in Chapter 2 as "the ability to interpret information presented in a mathematical manner and to use mathematics accurately to communicate information and solve problems (Radzi et al. 2009). According to Schoenfeld, mathematical thinking includes: the knowledge base; problem solving strategies; effective use of resources; mathematical beliefs and affects and engagement in mathematical practices (Schoenfeld 1992).

3.2.2 Measuring Engineers' Feelings about Mathematics

In addition to measuring *curriculum mathematics* and *mathematical thinking* usage, *engaging* usage, which is engineers' motivation to take a mathematical approach, is also measured in this study.

While the main emphasis in mathematics education has generally been placed on the cognitive aspects of learning mathematics, since the late 1980s considerable research attention has been directed towards the affective domain of mathematics education (Fennema 1989; Hannula 2006; McLeod 1989; McLeod 1992; McLeod and Adams 1989; Zan et al. 2006). In Chapter 2 it is reported that there is a real disaffection in students towards mathematics and, by extension, other numerate studies. In mathematics education research literature it is often held that many students are not motivated to learn mathematics and that they often engage with mathematics in a state of boredom or anxiety (Sedig 2007). However motivation is the process whereby goal-directed activity is instigated and sustained and it is central to learning and performance generally (Schunk et al. 2010). Students who are motivated to learn are likely to expend greater mental effort during instruction and employ cognitive strategies they believe will promote learning such as organising and rehearsing information, monitoring level of understanding and relating new material to prior knowledge. When students attain learning goals, they believe that they possess the requisite capabilities for learning and these beliefs in turn motivate them to set new and challenging goals. While motivation cannot be directly measured, it can be inferred from behavioural indicators: choice of tasks, effort, persistence and achievement (Schunk et al. 2010). There is little, if any, literature available concerning engineers' motivation to take a mathematical approach in their work.

3.2.2.1 Motivation Theory

There are two types of motivation: intrinsic (motivation to engage in an activity for its own sake); and extrinsic (motivation to engage in an activity as a means to an end). Intrinsically motivating activities challenge students' skills, present new information to students, provide students with a sense of control over outcomes and involve learners in fantasy (Schunk et al. 2010).

Csikszentmihalyi (1992) describes intrinsic motivation as "flow" or a state of optimal psychological experience when people engage in activities, feel a sense of enjoyment, feel a sense of accomplishment and develop a desire to repeat the experience. Individuals experiencing flow are so intensely involved with a task that they may lose awareness of time and space. Central to Csikszentmihalyi's theory of flow is the balance between the challenge perceived in a task and the skills a learner brings to a task. The challenge for a teacher is to keep the ratio between the learner's skills and the challenge within a range called the "flow channel" so that the learner experiences neither boredom nor anxiety (Csíkszentmihályi 1992). Csikszentmihalyi's theory, while in the affective domain, bears some resemblance to Vygotsky's theory of the zone of proximal development in the cognitive domain of mathematics learning which is defined as "the distance between the actual development level as determined by independent problem-solving and the level of potential development as determined by problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky 1978). It would thus appear that there is both an optimum cognitive level and an optimum affective level for presenting learning challenges to students (Csíkszentmihályi 1992).

In his social cognitive theory, Bandura (1986) advanced a theory that individuals possess self-beliefs that enable them to exercise a measure or control over their thoughts, feelings and actions. He presented that individuals are influenced more by how they interpret their experience than by their attainments (Bandura 1986). Social cognitive theory focuses on how people acquire knowledge, rules, skills, strategies and emotions through their interactions with and observation of others. Bandura's social cognitive theory posits that behaviour represents an interaction of an individual with the environment and it assumes a triadic relationship between personal factors, behaviours and environmental influences as they interact with and affect one another. Although learning occurs enactively (by doing), human learning is greatly expanded by the capacity to learn vicariously, whereby individuals are exposed to modelled influences. Modelling refers to behavioural, cognitive and affective changes that result from observing models. According to Bandura, motivation is goal-directed behaviour instigated and sustained by expectations concerning anticipated outcomes of actions and self-efficacy for performing those actions. Self-efficacy is "people's judgements of their capabilities to organise and execute courses of action required to attain designated types of performances" (Bandura 1986). Self-efficacy strongly influences the choices people make, the effort they expend and how long they persevere in the face of challenge. Bandura's (1997) self-efficacy theory posits four principal sources of information: performance accomplishment; vicarious experiences; social persuasions; and physiological states through which individuals acquire and modify their self-efficacy beliefs (Bandura 1997). Bandura also presents that constructs such as self-construct, perceived usefulness and anxiety all influence individuals' actions (Bandura 1986). While self-concept relates to general confidence, self-efficacy is task specific and many studies show that mathematics self-efficacy, mathematics self-concept, mathematics anxiety and perceived usefulness of mathematics are strong predictors of mathematics performance (Pajares and Miller 1994). Ferla, Valcke and Cai (2009), in their study of almost 9,000 15-year old students, found that students' academic self-concept strongly influences their academic self-efficacy beliefs (Ferla et al. 2009).

The area of causes internal to a person that drives their behaviours is called the affective domain and it includes attitudes, feelings, beliefs, confidence and values. In the context of mathematics education, McLeod (1992) identified three components of affect: beliefs; attitudes; and emotions (McLeod 1992). According to Goldin (2002) the affective domain has four components: emotions (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context); attitudes (moderately stable predispositions toward ways of feelings in classes of situations, involving a balance of affect and cognition); beliefs (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured); and values, ethics and morals (deeply-held preferences, possibly characterised as "personal truths", stable, highly affective as well as cognitive, may be highly structured) (Goldin 2002).

Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation posits that predictors of achievement behaviour are: expectancy (am I able to do the task?); value (why should I do the task?); students' goals and schemas (short- and long-term goals and individuals' beliefs and self-concepts about themselves); and affective memories (previous affective experiences with this type of activity or task), Figure 3-2 (Schunk et al. 2010; Wigfield and Eccles 2002). Students enter tasks with different personal qualities, prior experiences and social support which influence their initial sense of self-efficacy for learning. Expectancy-value research has substantiated that students with positive self-perceptions of their competence and positive expectancies of success are more likely to perform better, learn more and engage in an adaptive manner on academic tasks by exerting more effort, persisting longer and demonstrating more cognitive engagement. Task perceptions refer to students' judgments of the difficulty of the task and these are influenced by students' perceived causes of outcomes and also how students perceive their social and cultural environments. The purpose of instruction, content difficulty, instructional presentation, performance feedback, goals, rewards and attributional feedback all influence task engagement. Students who value and are interested in academic tasks are more likely to choose similar tasks in the future. Interest refers to the liking and wilful engagement in an activity. Interest can be: personal (personal enjoyment or importance of specific activities or topics); situational (interestingness of the context e.g. novel versus textbook) or psychological (heightened interest when personal interest interacts with situational interest) (Schunk et al. 2010; Wigfield 1994; Wigfield and Eccles 2000; Wigfield and Eccles 2002).

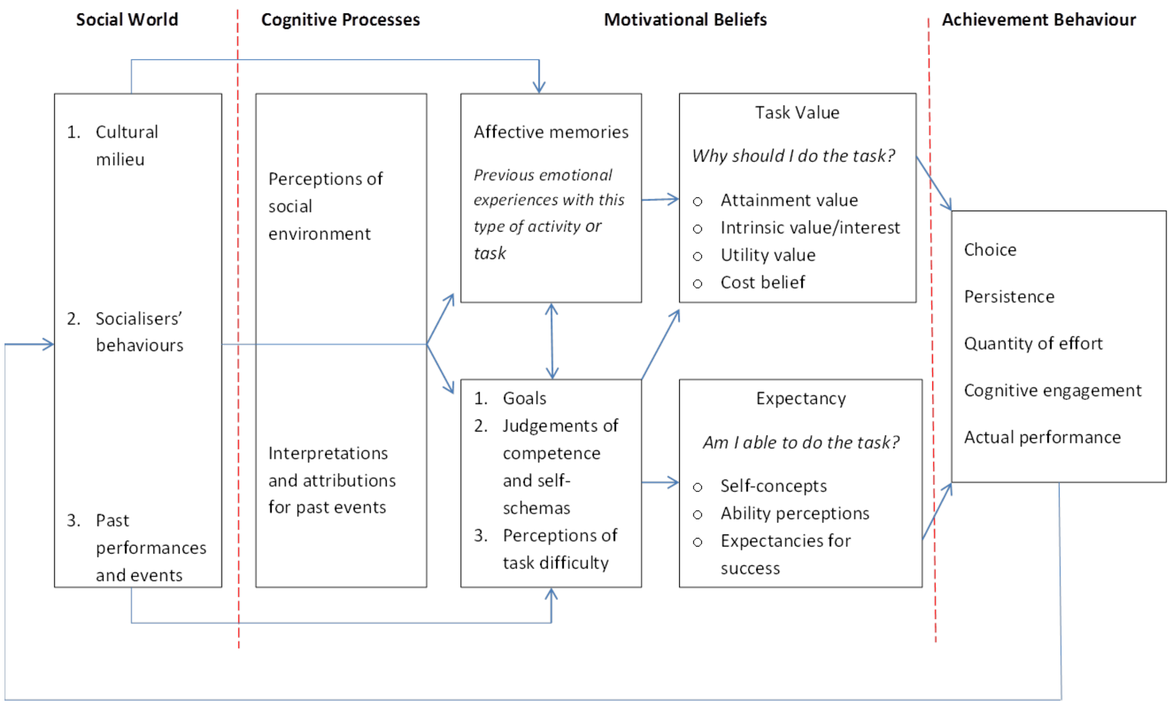

Figure 3-2: A social cognitive expectancy-value model of achievement motivation (Schunk et al. 2010).

After expectancy and task value, students' goals and self-schemas (short- and long-term goals, beliefs and domain-specific self-concepts about themselves) as well as affective memories predict student achievement. McLeod and Adams 1989 noted that observations of students carrying out problem solving tasks demonstrated that student reactions were mostly emotional (McLeod and Adams 1989).

Goal setting is a key motivational process and learners with a goal and a sense of self-efficacy for attaining engage in activities they believe will lead to attainment. There are two general goal orientations that students can adopt towards their academic work: a mastery orientation with the focus on learning and mastering the content and a performance orientation with the focus on demonstrating ability, getting good grades or besting other students. Goals can be positive (lead individuals toward desired end-states) or negative (lead individuals to move away from (avoid) undesired end-states) (Schunk et al. 2010).

Attributions are perceived causes of outcomes and they are important influences on achievement behaviours, expectancies and affects. There are occasions when attributions are not necessary and students' motivation is more a function of their efficacy and value beliefs for the task. However if the situation is a novel one for students, the probability increases that they will make attributions for their performance. Attribution also increases when the outcome is unexpected. Students' attributions for success and failure fall into two general categories: environmental and personal. Environmental factors are: specific information (e.g. teacher's direct attribution); social norms (e.g. others' performance) and situational features (distinctiveness, consensus and consistency of cues). Personal factors are: causal schemas (structures for understanding and inferring causality from events); attributional bias (heuristics that individuals may use to infer causality); prior knowledge (one's past performance on the task) and individual differences (various styles of making attributions). Ability and effort are the most frequently used attributions. Diverse attributions can be grouped along three basic dimensions of locus, stability and controllability and these provide the psychological and motivational force in attribution theory. Dimensions of locus, stability and controllability are linked to different emotions e.g. pride, shame and guilt. If students experience success and attribute it to an internal cause, they are likely to take pride in the success while a failure that is attributed to internal causes lowers self-esteem. Teachers may help influence students' self-esteem by suggesting that poor performance resulted from an external factor e.g. exam was difficult. If a cause is seen as controllable, the individual is deemed responsible and vice versa, for example ability is classified as uncontrollable and the individual will feel shame, embarrassment or humiliation which, in the case of a student, could lead to an avoidance of the subject. In contrast if a student's failure is due to low effort, which is deemed controllable, then the student is likely to feel guilty. This guilt can be harnessed to increase effort and to a better subsequent performance. Attributions to stable causes for failure should result in affects (feelings) of hopelessness, while attributing failure to poor preparation for an exam can still leave the student hopeful about future exams because the effort can be increased (Weiner 1994). Effort feedback can help to raise motivation and achievement, especially among students who have previously encountered learning difficulties. As students gain skills, switching to ability feedback may have better effects because students should not have to work as hard to succeed. Teacher feedback can have an important influence on students' attributions and expectancy beliefs. By better understanding students' behaviours, teachers can help them formulate achievement beliefs that enhance motivation (Schunk et al. 2010).

Social cognitive theory provides a theoretical basis for self-regulated learning. Self-regulation is the process whereby students activate and sustain cognitions, behaviours and affects that are systematically oriented toward attainment of their goals. Self-regulated learning is a cyclical process whereby students set goals and plans, monitor progress and use feedback from prior experiences to adjust their current learning methods (Zimmerman 2000). Pintrich (1999) showed that there are strong relationships between motivation and self-regulated learning (Pintrich 1999). Interest and affect influence goal setting which in turn influences self-regulation (setting goals and assessing goal progress). Students, who are motivated to attain a goal, engage in self-regulatory activities they believe will help them. Very often, the nature of schooling limits the degree of self-regulation and learning is regulated externally to the student. Social cognitive theory views self-regulation as comprising of three processes: self-observation (attention to aspects of one's behaviour), self-judgement (comparing current performance with one's goal) and self-reaction (behavioural, cognitive and affective responses to self-judgements). Anticipated consequences of behaviour enhance motivation and actual accomplishments enhance self-efficacy (Schunk et al. 2010).

Sociocultural influences from peers, families, cultures and communities play an important role in students' development, achievement and motivation. Family influences are critical in children's development and motivation. Strict parenting can negatively affect children's motivation and achievement in school. Children benefit from authoritative parenting practices that provide guidance and limits while helping children to regulate and take responsibility for their behaviours. Mothers' beliefs about their parenting efficacy including education, communication and general parenting and fathers' involvement in the academic lives of their children relate positively to academic motivation. Motivation is enhanced when parents allow children to have input into decisions, state expectations as suggestions, acknowledge children's feelings and provide children with choices. Parental involvement in the academic lives of their children relates positively to motivation. Children from lower socioeconomic backgrounds tend to display lower achievement and motivation while homes that are rich in interesting activities stimulate children's motivation to learn (Schunk et al. 2010).

Peer networks can heavily influence individuals' academic motivation. Peer networks are large groups of peers with whom students associate. Students often select their peer group on the basis of some similarity in values, attitudes or beliefs. Within the groups these values are reinforced and individuals' academic motivation and students in networks tend to become similar which can lead to more or less engagement in school activities. Students with high academic motivation are likely to belong to highly motivated groups and receive group approval for academic behaviours while students with lower motivation tend to belong to groups with low motivation and approval for positive academic behaviours comes from teachers rather than peers. Students in networks tend to become similar over time. The desire for peer approval can affect goal choice. Peer pressure can emanate from friends and groups; it rises during childhood and peaks when parental involvement in children's activities declines and consequently adolescents are more vulnerable. Students who associate with academically inclined peer networks make a better transition from elementary school to high school. School dropout is associated with low involvement in school activities and negative influence from peers. Community involvement in education has a positive effect on student motivation. Cultural differences are often found in motivation variables (Schunk et al. 2010).

Teachers are a huge influence on students' motivation. Teachers' decisions about what activities students will work on and decisions about grouping affect student motivation. When teachers teach well-structured content, they engage in practices that are consistent with principles of contemporary cognitive learning which enhance motivation. Models provide vicarious information for learners to use appraising their self-efficacy and motivating them to try the task for themselves. A major teaching function is to provide different forms of feedback (performance, attributional and strategy) to students. An important type of teacher expectation is teacher self-efficacy or teachers' beliefs about their capabilities to help students learn. Efficacious teachers are more likely to plan challenging activities, persist in helping students learn and overcome difficulties, and facilitate motivation and achievement in their students. Research suggests that constructivist teaching (theory contending that individuals construct much of what they learn and understand through individual and social activity) changes the focus from controlling and managing student learning to encouraging student learning and development (Schunk et al. 2010).

Classroom and schools' structure and organisation impact student motivation. Classroom organisation refers to how activities are set up, how students are grouped, how authority is established and how time is scheduled. Similarly schools' culture and organisation can have strong effects on students' motivation (Schunk et al. 2010).

3.2.2.2 Feelings about Mathematics

While many researchers regard affect as the single greatest factor impacting the learning process generally, it is an exceptionally complex construct that is difficult to quantify. Chamberlin (2010) suggests that affect in mathematics is at the intersection of mathematics, psychology and education (Chamberlin 2010). Studies show that emotions are very much part of problem solving in mathematics classrooms (Op 't Eynde et al. 2006; Op 't Eynde and Hannula 2006). For the past forty years many mathematics educators and educational psychologists have looked at how to measure affect. Early instruments mostly focused on one component of affect such as student attitudes or mathematics anxiety. One exception is the Fennema-Sherman mathematics attitudes scale (1976) which is a quantitative instrument comprising nine different scales measuring attitudes, self-confidence, parents' and teachers' perceptions, effects of motivation, success, anxiety, usefulness and mathematics as a male domain and this instrument is still used by many current researchers (Fennema and Sherman 1976). Chamberlin questions the validity and/ or reliability of many instruments and emphasises the need to create affective instruments that can be used to monitor student affect in mathematics classrooms (Chamberlin 2010).

Ernest (2011) lists attitudes to mathematics (confidence, anxiety, liking mathematics), beliefs (about self and mathematics, teaching and learning mathematics), appreciation of mathematics, perception of mathematics classroom climate and other aspects (values, feelings) as belonging to the affective domain. He maintains that failure at mathematics reinforces fear and dislike of the subject, damages self-confidence and self-image resulting in a "self-perpetuating cycle of failure". On the other side, success at mathematical tasks leads to pleasure and confidence and a sense of self-efficacy, the resultant improved motivation leads to more effort and persistence. Ernest states that "a conceptual foundation on which mathematical learning is to build, through tapping into meaningful out of school experiences and knowledge," is motivational, because out of school activities are "purposive and goal directed" (Ernest 2011).

The National Council of Teachers of Mathematics (1988) includes two affective goals in their Curriculum and Evaluation Standards for School Mathematics. These are "learning to value mathematics" and "becoming confident in one's own ability" (National Council of Teachers of Mathematics 1988). While many students suffer from "mathephobia" and research literature regularly associates the terms anxiety, boring and difficult with mathematics, the researcher is not aware of any other subject where students' feelings are as strong. For example, in Chapter 2 it is noted that a study found that Junior Certificate students in Ireland perceive mathematics as one of the most difficult and least interesting subjects (National Council for Curriculum and Assessment, 2007). According to Ernest (2011), it may be that student feelings are stronger in mathematics than in other subjects because "in mathematics more than any other subject there is the possibility that they [learners] will experience absolute failure at the tasks they are given" (Ernest 2011).

There is no overestimating the significance of the affective domain in mathematics education. For example, a study investigating Australia's capacity to produce a critical mass of young people with the requisite mathematical background and skills to pursue careers in science, technology, engineering and mathematics, identified five areas contributing to students' decisions not to continue with higher level mathematics. These are: self-perception of ability; interest and liking for higher-level mathematics; perception of the difficulty of higher-level mathematics subjects; previous achievement in mathematics; and perceptions of the usefulness of higher-level mathematics (McPhan et al. 2008).

The Cockcroft report (1982) maintains "it is not easy to pick out points which summarise all the research on attitudes to mathematics. Strongly polarised attitudes can be established, even amongst primary school children, and about 11 years seems to be a critical age for this establishment. Attitudes are derived from teachers' attitudes (though this affects more intelligent pupils rather than the less able) and to an extent from parents' attitudes (though the correlation is fairly low). Attitude to mathematics is correlated with attitude to school as a whole (which is fairly consistent across subjects) and with the peer group's attitude (a group attitude tends to become established). These things do not seem to be related to type or size of school or to subject content. Throughout school, a decline in attitudes to mathematics appears to go on, but this is part of a decline in attitudes to all school subjects and may be merely part of an increasingly critical approach to many aspects of life" (Cockcroft 1982).

Research on affect in mathematics has been traditionally associated with low mathematical achievements and with gender differences in mathematics performances, differences between female's and males' mathematical self-efficacy, attitudes about mathematics, perceived usefulness of mathematics and causal attributions for success or failure. Studies have consistently shown that students' self-perception of ability and expectancies for success are the strongest predictors of subsequent grades in mathematics and are even better predictors of later grades than are previous grades (Schunk et al. 2010).

3.2.2.2-1 Mathematics Self-Efficacy

Mathematics self-efficacy is an individual's perception of their ability to successfully complete a specific mathematics problem. Mathematics self-concept is an individual's perception of their general mathematics competence. Fennema & Sherman (1978) found that the confidence in one's ability to learn mathematics is correlated with mathematical achievement at about the 0.45 level which is significant (Fennema and Sherman 1978). Research has consistently found that boys have higher self-perceptions of mathematics ability than girls even when there are no actual differences in results (Correll 2001; Fennema 1989; Fennema and Sherman 1978; Jacobs et al. 2002; Schunk et al. 2010). Fennema (1989) found that "males who have more confidence in their ability to do mathematics, report higher perceived usefulness and attribute success and failure in mathematics in a way that has been hypothesised to have a more positive influence on achievement" (Fennema 1989). Correll (2001) claims that since males tend to overestimate their mathematical competence relative to females, males are also more likely to pursue activities leading down a path toward a career in science, mathematics and engineering (Correll 2001).

Gender studies show that girls tend to attribute extrinsic and unstable factors such as good effort contributing to success and while they do not attribute their successes to ability they do attribute failures to intrinsic causes such as lack of ability. On the other hand boys attribute their success in mathematics to stable and intrinsic causes such as skill and ability and their failures to extrinsic and unstable causes such as lack of effort (Burton 1984; Fennema 1989; Leder 1984; Middleton and Spanias 1999).

Research has consistently shown a decrease in the mean level of self-perceptions of mathematics ability as children move into adolescents (Wigfield et al. 1996).

While engineering students' self-efficacy beliefs are strongly tied to their successful navigation of the engineering curriculum, research investigating self-efficacy influencers in college mathematics courses is sparse (Brown and Burnham 2012). Brown and Burnham hold that the predominant use of quantitative methods of measuring self-efficacy and other motivational constructs are restricted by their numerical outputs. They say that the interpretative nature of qualitative studies limits population sizes and consequently the generalisability of research findings. They advocate a mixed methods approach that allows researchers "to simultaneously ask confirmatory and exploratory questions and therefore verify and generate theory in the same study." Brown and Burnham's case study approach to studying mathematics self-efficacy in the context of an engineering mathematics course employed a mathematics self-efficacy survey developed by Betz and Hackett (Betz and Hackett 1983) and semi-structured interviews. They found that positive and negative mastery experiences (interpretation of past performances) were the most prominent source of self-efficacy over the course of a freshman engineering mathematics course. While students' mathematics problem solving self-efficacy improved, the same was not the case for mathematics courses self-efficacy. Correcting students' previous misunderstandings and increasing student involvement in challenging learning environments impacted positively on students' self-efficacy (Brown and Burnham 2012).

3.2.2.2-2 Mathematics Task Value

Students' perceptions of the importance, utility and interest in mathematics are strong predictors of their intentions to continue to take mathematics courses (Wigfield and Eccles 1992). Fennema and Sherman (1978) also reported a positive correlation between perceived usefulness of mathematics and mathematical achievement (Fennema and Sherman 1978). Wigfield and Eccles (1992) found that male and female adolescents differed in the relative value they attached to various subjects and that boys valued mathematics more than girls (Wigfield and Eccles 1992). Fennema and Sherman (1977) showed that by middle school, boys began to rate mathematics as more useful than did girls (Fennema and Sherman 1977). Girls' perceptions of mathematics usefulness decline throughout high school. Jacobs, Lanza, Osgood, Eccles, and Wigfield (2002) show that while students' value perceptions of mathematics, language arts and sports declined in high school, mathematics declined most rapidly. Explanations for students' declining task value beliefs range from attributing poor performance to low ability, students becoming interested in social comparisons and the mismatch between the students' developmental needs and the organisation of the school (Jacobs et al. 2002) .

3.2.2.2-3 Self-Regulated Mathematics Learning

Self-regulation is a “crucial characteristic of effective mathematics learning” (De Corte et al. 2000). Schoenfeld (1992) describes self-regulation as “resource allocation during cognitive activity and problem solving.” He associates self-regulation with metacognition which concerns one’s knowledge of one’s own cognition processes. As children get older, they get better at planning tasks and learn from the experience of earlier attempts at similar tasks. Schoenfeld demonstrated the importance of self-regulation during problem solving in a study where he contrasted an inexperienced student’s attempt to solve an unfamiliar problem with that of an experienced mathematician. The inexperienced student engaged only in unreflective exploration of the problem while the expert mathematician engaged in six levels of problem solving: reading the problem; analysing the problem; exploring the problem (transforming the problem into a routine task); planning; implementing the solution plan; and verifying the solution. Schoenfeld shows that self-regulation is particularly relevant to problem solving given that humans have a limited working memory that can only hold in the region of seven pieces of information at a time. His work shows that, an initial wrong decision, unless it is reconsidered and reversed, will result in failure to solve an unfamiliar problem in mathematics, while on the other hand, a period of structured exploring allows the problem solver to pursue interesting leads and abandon useless paths and ultimately solve the problem. Schoenfeld’s work also shows that students’ problem solving performance is enhanced when engaging in self-monitoring and controlling activities. While there is little work on the effectiveness of teaching problem solving strategies to students, Schoenfeld’s work demonstrates that teacher interventions (for example asking students what they are doing, why are they doing it and how does it help them) can raise the level of metacognitive activity and effectiveness in problem solving among students (Schoenfeld 1992). According to Ernest (2011) metacognition is about “management of thinking” whereby when solving mathematical problems the student is encouraged to take more control over the way he or she is attempting to solve the problem. Metacognitive questions “focus the attention of the problem solver on reflecting on and controlling progress towards the problem goal”. For example, it might involve asking oneself “is this approach too hard or too slow”? Metacognitive activities include “planning, controlling and monitoring progress, decision making, choosing strategies, checking answers and outcomes and so on” (Ernest 2011). Zimmerman (2000) proposes three phases of self-regulated learning: forethought (students plan their behaviours by analysing tasks and setting goals); performance (students monitor and control their behaviour, cognitions, motivations and emotions); and self-reflection (students make judgments about their progress and alter their behaviour accordingly). He also presents evidence that self-regulated learners feel self-efficacious whereby self-efficacy beliefs influence goal setting and self-efficacious people set high goals and they also increase their efforts to maintain these goals (Zimmerman 2000).

In a study of seventh grade Finnish students (age 13), Malmivuori (2006) found that students’ self-confidence and affective responses play a significant role in self-regulation of mathematics learning and problem solving (Malmivuori 2006). De Corte, Verschaffel and Op ’t Eynde (2000) are of the view that self-regulation, in addition to metacognitive processes, also encompasses motivational and emotional as well as behavioural monitoring and control processes”. They list four essential components of self-regulation in the theoretical framework of learning mathematics: acquiring a mathematical disposition as the ultimate goal; constructive learning processes as the road to the goal; powerful teaching-learning environments as support; and assessment as a basis for control and feedback. From a review of the available research, De Corte, Verschaffel and Op ’t Eynde (2000) identify three components of instruction that foster self-regulation in mathematics classrooms: realistic and challenging tasks; variety of teaching methods and learner activities, including “modelling of strategic aspects of problem solving by the teacher, guided practice with coaching and feedback, problem solving in small groups and whole-class discussion focusing on evaluation and reflection concerning alternative solutions as well as different solution strategies”; and classroom climate that is conducive to the development in pupils of “appropriate” beliefs about mathematics (De Corte et al. 2000). Pape, Bell and Yetkin (2003) maintain that self-regulated learners are active participants in their own learning whereby they are able to select from a repertoire of strategies and they are able to monitor their progress using these strategies toward a goal. Multiple representations and rich mathematical tasks (opportunities to engage students' thinking); classroom discourse and learning to think mathematically (probing students' thinking); environmental scaffolding of strategic behaviour (connecting strategies to grades); and varying needs for explicitness and support (differential support required for individual students) are crucial to the development of self-regulation in mathematics learning (Pape et al. 2003).

3.2.2.2-4 Sociocultural Influences on Mathematics Learning

According to Zeldin and Pajares (2000), students who are exposed early to mathematics-related content by relatives who work in mathematics based fields often find this domain comfortable and familiar. Their vicarious experiences with family members create a positive self-efficacy perception in the mathematics and science areas. Zeldin and Pajares also found that girls who receive encouragement from parents and teachers to persist and persevere in male-dominated academic domains will develop higher mathematics self-efficacy perceptions in the midst of academic and social obstacles (Zeldin and Pajares 2000).

In "Everybody Counts: A Report to the Nation on the Future of Mathematics Education", the National Research Council in the U.S. maintain that there is little difference between boys' and girls' mathematics ability, effort and interest until adolescents. However in adolescents "as social pressures increase, girls tend to exert less effort in studying mathematics, which progressively limits their future education and eventually their career choices". The report also presents that gender differences in mathematics performance result from the accumulated effects of sex-role stereotyping perpetrated by families, schools and society and that such stereotypes cause females to drop out prematurely from mathematics education (National Research Council 1989).

Schoenfeld (1992) also believes that societal beliefs influence children's learning of mathematics. He states that parents in the U.S. are more likely than Japanese parents to believe that "innate ability" is a better predictor of children's mathematics success than is effort. Thus U.S. parents are less likely to encourage their children to work hard on mathematics. In contrast to the U.S., mathematics teachers in Japan and China allow more time for students to understand mathematics concepts and solve mathematics problems (Schoenfeld 1992). The U.S. National Research Council (1989) in their “Everybody Counts” report to the Nation state that mathematics is more than what society generally believes is “theorems and theories,” instead “mathematics offers distinctive modes of thought which are both versatile and powerful, including modelling, abstraction, optimisation, logical analysis, inference from data and use of symbols. Experience with mathematical modes of thought builds mathematical power – a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives. Mathematics empowers us to understand better the information-laden world in which we live” (National Research Council 1989).

3.2.2.2-5 Teachers’ Beliefs about Mathematics

In his social cognitive theory, Bandura (1986) presents that behaviour represents an interaction of an individual with the environment and that learning is greatly expanded by the capacity to learn vicariously. As such mathematics teachers are role models and their attitudes, emotions, beliefs and values about mathematics impact their students’ learning (Bandura 1986).

According to Lampert (1990), students acquire beliefs about mathematics through years of watching, listening and practising mathematics in the classroom (Lampert 1990). Koehler and Grouws (1992), in their model of mathematics learning, maintain that mathematics learning is based on students’ behaviours which are influenced by their beliefs about themselves, their beliefs about mathematics, teachers’ knowledge of mathematics, and by teachers’ attitudes and beliefs about mathematics knowledge and teaching mathematics (Koehler and Grouws 1992). Smith, Hollebrands, Parry, Bottomly, Smith and Albers (2009) found that “students’ perceptions of their teachers’ perceptions of their ability to do mathematics decreases as the students progress from elementary to high school” (Smith et al. 2009). In another study, Yara

(2009) found that students' positive attitude could be enhanced by teachers' enthusiasms, resourcefulness and behaviour, thorough knowledge of subject matter and by making the subject interesting. The attitude of the teacher and the teacher's disposition to mathematics "could make or unmake" students' attitudes towards the learning mathematics (Yara 2009).

Ernest (2011) addresses the subject of "mathematical myths" which, he claims, result in false impressions about how mathematics is done. Myths suggest that there are gender differences in mathematical ability, others imply that "mathematics is a logical, rigid and hierarchical subject," more suggest that "there is a fixed way of getting the right answer" and another view is that "memory and effort are important in doing mathematics." Ernest holds that classroom experiences are decisive in developing children's views of mathematics. He reports on a study where students often distinguish mathematical topics as "hard-easy" and "useful-not useful" and another study where most children viewed mathematics as computation. Ernest claims that "experiences in school mathematics form the basis for the conceptions, appreciation and images of mathematics constructed by learners, especially negative ones". According to Ernest many learners experience a "Dualistic" view of mathematics where teachers give students a "myriad of unrelated routine mathematical tasks which involve application of memorised procedures and by stressing that every task has a unique, fixed and objectively right answer, coupled with disapproval and criticism of any failure to achieve this answer." These teaching methods create images of mathematics as "cold, absolute, inhuman and rejecting". Ernest (2011) calls for more research on the "human face" of mathematics. He states that "children construct powerful stereotyped images of mathematics for themselves based on their classroom learning experiences." He claims that the teachers' views of the nature of mathematics affect mathematics teaching and he suggests that mathematics teachers should ask themselves, "what is mathematics" (Ernest 2011). Similarly, Schoenfeld (1992) states that mathematics instruction should provide students with a sense of "what mathematics is and how it is done" and that as a result of their instructional experiences, students should learn to "value mathematics and feel confident in their ability to do mathematics". One of Schoenfeld's aspects of mathematical thinking is mathematical beliefs whereby he presents that individuals' beliefs and affects toward mathematics will impact how and when they use mathematics and engage in mathematical thinking (Schoenfeld 1992).

Teachers' beliefs are important in that they determine the nature of the classroom environment which in turn shapes students' beliefs about the nature of mathematics. Schoenfeld suggests that teachers' beliefs are formed by their own schooling experience and the same beliefs are apparent in successive generation of teachers, which Schoenfeld calls a "vicious pedagogical/ epistemological circle" (Schoenfeld 1992). The U.S. National Research Council (1989) in their "Everybody Counts" report to the Nation claim that all young children like mathematics and they do mathematics naturally. However as children become "socialised by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. Later, as parents, they pass this conviction on to their children. Some even become teachers and convey this attitude to their students." The report goes on to state that "self-confidence built on success is the most important objective of the mathematics curriculum" and that the ability of individuals to cope with mathematics, wherever it arises in their later lives, depends on the attitudes toward mathematics conveyed in school and college classes. The report states that mathematics curricula must avoid leaving a "legacy of misunderstanding, apprehension, and fear" (National Research Council 1989).

3.2.2.2-6 Schools' and Mathematics Classrooms' Structure and Organisation

Research findings suggest that girls do better in mathematics when boys are not in the classroom (United Nations Educational Scientific and Cultural Organisation 2007). In a review of the research literature on mixed and single-gender classrooms, Forgasz, Leder and Taylor (2007) note that benefits for girls in single-sex settings include: greater positive self-concept; less gender stereotyping; and views that the learning environment is more comfortable. Research indicates that girls in single-sex settings benefit with respect to confidence and achievement in mathematics (Forgasz et al. 2007). Some reasons favouring single-sex school include: single-sex schools reduce influences of adolescent subcultures that distract students' attention from academic learning; coeducational schools restrain academic achievement whereby girls do not want to lose their appeal to boys by being good at mathematics; and girls in single-sex classrooms have a sense of ownership of their class while boys dominate coeducational classrooms (Park et al. 2011). In their study, Tully and Jacobs (2010) found that females attending single-gender secondary schools display the highest self-perception of mathematical ability compared to both females from co-educational schools and males. They found that interactive, relaxed and collegial classrooms where 50% of class time was devoted to problem solving activities impacted positively on students' self-concept and self-efficacy. Female students particularly benefitted from teacher encouragement and contextual applications of mathematics problems. Tully and Jacobs found that both male and female students preferred an interactive environment for mathematics learning (Tully and Jacobs 2010).

3.2.2.3 Measuring Practising Engineers' Feelings about Mathematics

One aim of this study is to investigate the relationship between students' experiences with school mathematics and their choice of engineering as a career. Another aim is to investigate factors influencing practising engineers' engagement with mathematics in their work. In this study engineers' feelings about mathematics include: their feelings about school mathematics experiences; the degree that their feelings about mathematics impacted their choice of engineering as a career; the value of higher level Leaving Certificate mathematics in the context of their current work; and their engagement with mathematics in work. Measurement of engineers' engagement with school mathematics and engineers' motivation to take a mathematical approach in their work is based on Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation. This theory posits that predictors of achievement behaviour are: expectancy (am I able to do the task?); value (why should I do the task?); students' goals and schemas (short- and long-term goals and individuals' beliefs and self-concepts about themselves); and affective memories (Schunk et al. 2010; Wigfield and Eccles 2002).

3.3 RESEARCH DESIGN

In this section the specific research design is considered and is organised as follows:

Page number

3.3.1 Research Frameworks

According to Collis and Hussey (2009), positivism is about measuring social phenomenon whereas interpretivism is based on the belief that social reality is shaped by our perceptions. Positivism involves quantitative methods based on mathematical proof and researchers focus on large samples with measurable outcomes and generalisability of results. Interpretivism involves qualitative methods and researchers seek to describe or assign meaning to phenomena in the social world by exploring a small number of cases in depth (Collis and Hussey 2009). Each research type represents a different inquiry paradigm and researchers' choice of methodology is often based on their familiarity with one type or on the nature of their research. The researcher here is an engineer whose previous M. Eng. research area, involving the design and evaluation of electronic circuitry, was inherently a positivist paradigm.

Researchers have long debated the relative value of qualitative and quantitative inquiry. Quantitative research tests hypothetical generalisations and produces objective knowledge which is unbiased by the research/ researcher process. On the other hand qualitative data generates rich descriptions of the research phenomena and is seen as "highly subjective" (Collis and Hussey 2009; King and Horrocks 2010). In engineering education research, research thus far has generally favoured quantitative approaches. Probably, this is because the audience for engineering education research comprises mostly engineers who have more experience interpreting quantitative results (Borrego et al. 2009). The fact that quantitative methods dominate engineering education research has implications for this study for a number of reasons; the audience is likely to comprise of engineers and engineering educators whose work is mostly based on logical or mathematical proof; and the research participants are engineers who also may be more comfortable with quantitative approaches rather than descriptive approaches when participating in studies. On the other hand, a qualitative approach to engineering education research offers a new perspective.

While the philosophical framework guides how the research should be conducted, the credibility of any new knowledge produced in a study is based on reason and argument. The quality of research findings is dependent on a rigorous and methodical approach within the chosen research paradigm. Research quality is broadly measured in terms of reliability and validity. Quantitative results generally have high reliability and low validity and qualitative results have low reliability and high validity (Collis and Hussey 2009). Reliability refers to the degree to which the findings of a study are independent of accidental circumstances or whether or not some future researchers would come up with the same results and interpretations if the research was repeated. Validity refers to the extent to which research accurately represents the social phenomena studied. The concept of validity originated in quantitative research with a type 1 error (rejecting a true null hypothesis) and a type 2 error (accepting a false null hypothesis). In qualitative research the impact of the researcher on the research setting, the values of the researcher and the truth of the respondent's account all impact on validity (Silverman 2010).

While positivism was once the dominant research paradigm, there is much recent criticism of using a measurable approach rather than investigating the inner experience of the individual. The subjective approach of dealing with the direct experiences of people in specific contexts is "currently preferred by many" (Cohen et al. 2008). According to Ernest, both styles of research have value and "together the two kinds of data combine to give a better picture" (Ernest 2011).

3.3.1.1 Choice of Research Framework

The research methodology employed should be appropriate to the research question and the nature of the context and knowledge sought. The research questions in this study concern practising engineers' mathematics usage and the relationship, if any, between school mathematics experiences and engineering career choice. These questions concern measurement of mathematics usage which is suited to a quantitative approach and exploring engineers' experiences in engineering practice and their previous experiences with school mathematics which is suited to a qualitative approach. The knowledge sought is also a mix of: objective knowledge (measuring engineers' mathematics usage) and subjective knowledge (interpreting engineers' experiences with mathematics).

Combining quantitative and qualitative data produces a "very powerful mix" (Creswell 2005). Mixed methods research is "the type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches (e.g. use of qualitative and quantitative viewpoints, data collection, analysis, interference techniques) for the broad purpose of breadth and depth of understanding and corroboration" (Johnson et al. 2007). Mixed methods are used when researchers build from one phase of research to another. Sequential mixed methods are used to elaborate on the findings of one method with the other method. For example, a study may begin with a quantitative method in which a concept is tested, followed by a qualitative method involving detailed exploration with a few cases. Using two complementary research methods has the advantage of offsetting weaknesses in each. While each method gives a distinctive contribution to the investigation of the research questions, findings from one set of data could be compared with the findings from the other. A qualitative component gives validity to a study and the "uniqueness and idiosyncrasy of situations, such that the study cannot be replicated is considered a strength of qualitative research" (Cohen et al. 2008). The use of multiple methods simultaneously is described as "triangulation", which involves using different or independent methods to research the same issue and has the advantage of improving the quality of the conclusions drawn from the different types of data.

Quantitative measurements of engineers' mathematics usage and the role of mathematics in engineering practice, while generating new knowledge, do not adequately explain why these findings might arise. Qualitative research is generally suited to exploring the "why" type questions, and involves a degree of subjective interpretation by the researcher. Employing a mixed methods approach captures both the objective and subjective data. Given the breadth and variety of engineering practice objective data is required to generate knowledge about engineering practice generally. Subjective data, which is based on engineers' personal experiences, is an important aspect of this study as the researcher perceives engineers to comprise a fairly silent profession particularly given the dearth of research literature investigating engineers' usage of mathematics (Alpers 2010a; Alpers 2010b; Cardella 2007). Giving voice to engineers' views with respect to the research questions adds significant value to this study in the context of generating new knowledge. Together both viewpoints of mixed methods studies are considered to give a fuller picture and a deeper understanding of the research topics compared to using a single approach. A further advantage is that corroboration of findings in a mixed methods approach enhances the credibility of the findings (Johnson et al. 2007).

Explanatory mixed methods consist of first collecting quantitative data and then collecting qualitative data to help explain the quantitative results (Creswell 2005). This study employs a sequential explanatory strategy mixed methods design which is the collection and analysis of quantitative data followed by the collection and analysis of qualitative data building on the results of the initial quantitative data, as illustrated in Figure 3-3.

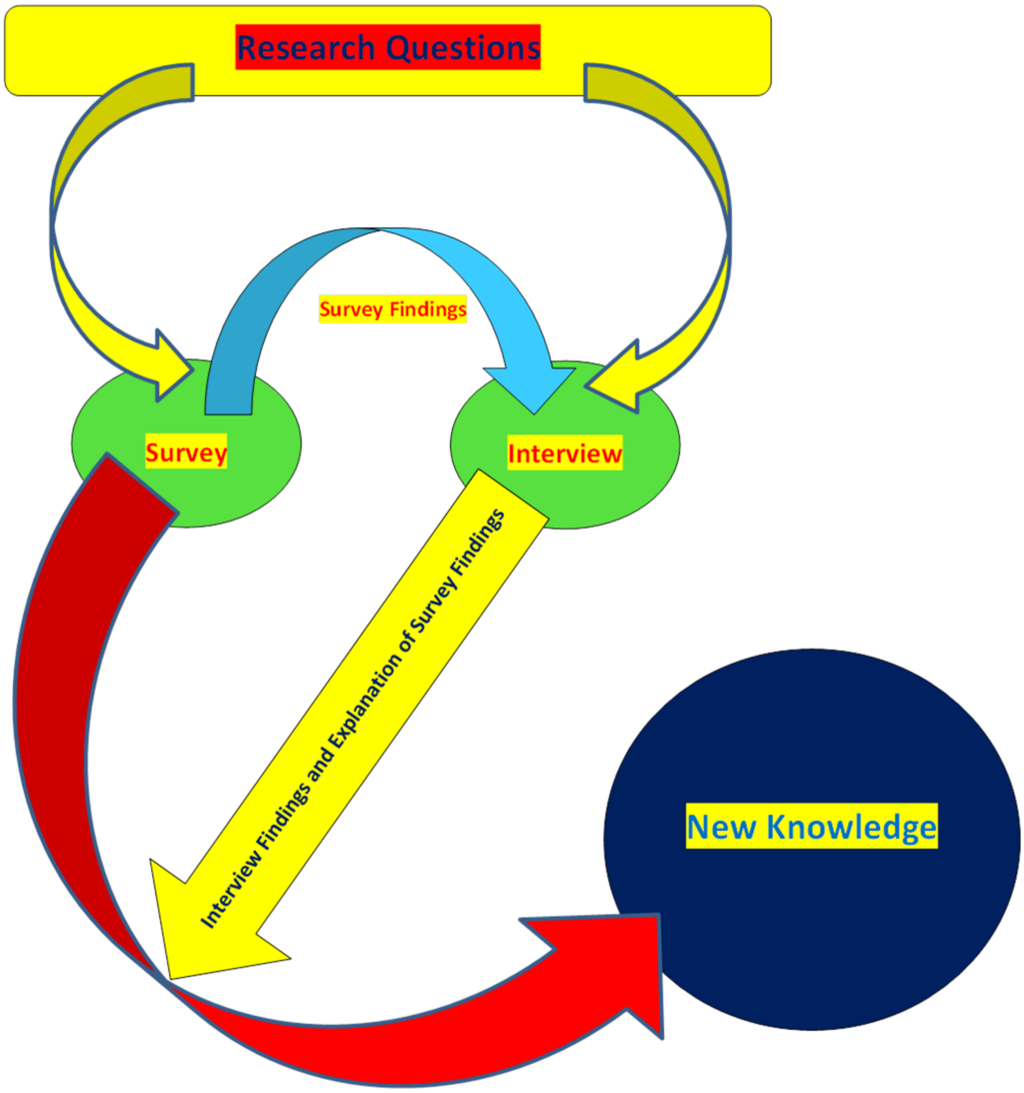

Figure 3-3: Sequential explanatory strategy mixed methods design.

In the mixed methods sequential explanatory approach, secondary qualitative data collection and analysis is required to achieve a deeper understanding about the research topics and to give greater meaning to the findings discovered in the initial quantitative phase. The decision to employ both quantitative and qualitative approaches in this study was driven by a number of factors: (i) to capture both the objective (measuring mathematics usage) and subjective (exploring individual engineers' feelings about mathematics) nature of the research questions; (ii) to engage engineers both quantitatively and qualitatively in the study; (iii) to capture the breadth and depth of the research phenomena; and (iv) to give greater reliability and validity to the research findings. While objective measurements of mathematics usage and mathematics affinity are goals of this study, developing an understanding as to why engineers use specific mathematics and determining whether and why there is a relationship between students' experiences with school mathematics experiences and engineering career choice are also goals. The sequential explanatory strategy mixed methods design is a thorough approach to measuring mathematics usage and to generating an understanding of mathematics feelings in both school and engineering practice and whether there is a relationship between school mathematics feelings and engineering career choice.

3.3.2 Data Collection Methodologies

The main research methodologies used in positivism are experimental studies, surveys, cross-sectional studies and longitudinal studies. Hermeneutics, ethnography, participative enquiry, action research, case studies, grounded theory and gender/ ethnicity studies belong to the interpretivist paradigm (Collis & Hussey, 2009). The data collection instruments chosen in this two-phase mixed methods research study are a survey questionnaire and semi-structured interviews.

A survey questionnaire approach is chosen as the most effective method for collecting quantitative data from a large population; all participants are asked the same questions. Survey questionnaires are suited for on-line administration and automatic data collection from a large number of participants. Given that the quantitative phase of this study is primarily about measuring engineers' current mathematics usage and their motivation to take a mathematical approach in both their career decision and their work, experiments, cross-sectional studies or longitudinal studies are not suited.

In qualitative research, interviews are an effective method for eliciting information about participants' actions, thoughts and feelings about a specific topic. Interviews give participants the opportunity to express their views on issues that are important to them whilst affording the researcher the flexibility to explore, in depth, topics relating to the research questions. Mathematics affinity is a core theme in this research and interviewing is a productive method for collecting data on engineers' feelings in this regard. Qualitative interviewing uses open-ended questions that allow for individual variation of responses and it also allows the interviewer to explore and probe within inquiry areas relating to the research questions and the interviewees can respond in their own language. Audio recording of interviews allows the researcher to focus solely on the interview process while the entire interview can subsequently be transcribed and analysed. While the process of open discovery is the main strength of interviews, structured interviews where the questions are planned in advance often restrict the discovery of new knowledge. On the other hand unstructured interviews are very time-consuming and the questions can drift away from the research questions. In this study semi-structured interviews are chosen because they make interviewing multiple participants more systematic. An interview protocol can be used to guide the interviews; this is a list of questions and predetermined inquiry areas that the interviewer wants to explore during each interview. In this study such a protocol is deemed to make good use of engineers' often limited interview time and it also allows the researcher to focus attention on areas of particular importance as they emerge during the interviews. The interview protocol can be modified over the course of the interviews if required.

The main difference between a grounded theory methodology and the methodology used in this study relates to the data analysis; in grounded theory data is repeatedly collected and analysed in an attempt to saturate the findings. In this study, due to the diversity of engineers' disciplines, roles and work and in order to give consideration to the quantitative phase, the goal was not to saturate the findings but to give meaning to the findings and to allow new knowledge to emerge. It was considered that a single approach to collecting qualitative data that explored various engineers' mathematics usages and how engineers' relationships with school mathematics impacted their career choice and work was best suited to interviews given the diversity of the engineering population. Also given the diversity of engineers' disciplines, roles and engineering work generally, the research questions in this study are too broad for a case study methodology whereby a single-phenomenon is usually studied. Compared to qualitative interviews the main disadvantages of ethnography in this study are: engineers' workplaces are difficult to access; engineering workplaces are generally not representative of a diversity of engineering disciplines, roles and activities; and the mathematics used in such environments might not be visible to the researcher.

3.3.3 Study Population

Given that this research concerns the role of mathematics in engineering practice and whether there is a relationship between students' experiences with school mathematics and engineering career choice and also that mathematics education varies from country to country, it was decided to confine this study to professional engineers practising in Ireland.

Engineers generally comprise a very broad category of disciplines and in many reports "engineering" includes a variety of job and qualification types. In addition to the traditional engineering graduate disciplines e.g. civil, electrical, electronic, mechanical, chemical, computer and software etc. engineering often includes roles adopted by non-graduates. While there is no single standard group that identifies engineers, it is specifically professional engineers with level 8 engineering degrees or equivalent that are of interest in this study.

For the purpose of this study the research population is identified as engineers who meet the criteria of "Chartered Engineer" as determined by Engineers Ireland, the professional body representing the engineering profession in Ireland since 1835. In addition to supporting the engineering profession, Engineers Ireland's accreditation process assures the quality of engineering and engineering technology education programmes in Ireland is in line with international norms. There are two main grades of membership of Engineers' Ireland: ordinary member (MIEI) - usually achieved through an accredited level 7, 8 or 9[15] qualification and technician member (Tech IEI) - usually achieved through an accredited level 6 qualification. Engineers Ireland award professional titles to their members according to their qualification and these include:

[15] Level 7, 8 or 9 qualification: Ordinary Bachelor Degree (level 7), Honours Bachelor Degree (level 8) and Masters Degree (Level 9)

Chartered Engineer (CEng); Associate Engineer (AEng MIEI) and Engineering Technician (Eng Tech IEI). Of the 23,891 engineers registered with Engineers Ireland on 31st December 2010, 5,755 are Chartered Engineers (Engineers Ireland 2011). The "Chartered" (CEng) title is recognised internationally as the title to be used by Irish professional engineers and has the same status as professional engineering titles used in other countries. Chartered Engineers have at least a level 8 academic qualification (equivalent to an honours engineering degree) and a minimum of four years' relevant professional experience. Civil engineers make up about 9,000 of the membership and approximately a further 5,000 are mechanical engineers. The remaining members covers all engineering disciplines including include electrical/ electronic, bio-medical, software and chemical (Engineers Ireland 2011; Engineers Ireland 2012). Not all engineers are required to be members of Engineers Ireland or hold CEng title.

It is not feasible to study entire large populations and it is accepted research practice to study a sample of the population of interest. In quantitative inquiries, the dominant sampling strategy is probability sampling, which is the selection of a random and representative sample from the larger population (Collis and Hussey 2009). The advantage of random sampling is that subsequent generalisation of the research findings to the population can be made. The larger the sample size the better it represents the population. In order to confidently generalise from quantitative study results, statistical analysis requires a minimum sample size that reflects the entire population.

By contrast, purposeful sampling is the dominant strategy in qualitative research. Purposeful sampling seeks information-rich cases which can be studied in depth (Patton 2002). There are many variations of purposeful sampling and the one that is of greatest interest in this study is maximum variation sampling. This strategy aims at capturing and describing the central themes or principal outcomes that cut across a great deal of participants. There are no minimum sample sizes required in qualitative research because for interpretivists "the goal is "to gain rich and detailed insights of the complexity of social phenomena ... therefore they [researchers] can conduct their research with a sample of one" (Collis and Hussey 2009).

Maximum variation sampling can yield detailed descriptions of each participant, but for small samples a great deal of heterogeneity can be a problem, because individual cases are so different from each other. However Patton (1990) presents that the maximum variation sampling strategy turns this weakness into a strength because "any common patterns that emerge from great variation are of particular interest and value in capturing the core experiences and central, shared aspects or impacts of a program" (Patton 2002). In the qualitative phase maximum variation sampling is used to select a sample of Chartered Engineers representing a diversity of engineering types.

3.3.4 Initial Quantitative Phase

An initial quantitative phase, using a survey questionnaire, to measure mathematics usage in engineering practice was chosen for the following reasons:

(i) the professional engineering population in Ireland is large (there were 23,891 engineers registered with Engineers Ireland on 31st December 2010 of whom 5,755 are chartered (Engineers Ireland, 2011))

(ii) the engineering population in Ireland comprises a diversity of engineering disciplines, roles and functions working in many different types of organisations

(iii) there is no prior measurement of engineers' mathematics usage for engineers practicing in Ireland or indeed elsewhere

(iv) defining mathematics in the context of measuring mathematics usage is somewhat complex

(v) there is little prior knowledge on the relationship between school mathematics affinity and engineering career choice

- A large sample size and a system of measuring both mathematics usage and engineers' feelings about mathematics is required because of the diversity of engineer' work, the diversity of mathematics and the dearth of previous research about engineers' use of mathematics and the degree to which engineers' feelings about mathematics impact their career choice. The initial quantitative measurement of the role of mathematics in engineering practice and in the formation of engineers, in addition to generating knowledge about the engineering population in Ireland also informs the qualitative phase of this study.

The quantitative phase addresses: (i) how mathematics usage in engineering practice is measured; (ii) how engineers use mathematics in their work; (iii) what motivates engineers to engage, or not, with mathematics; (iv) engineers' experiences and feelings about their school mathematics; and (v) the influence of students' feelings about mathematics on their choice of engineering as a career. The initial quantitative findings inform the secondary qualitative data collection process and the subsequent qualitative data analysis builds on the results of the quantitative phase. While weight is given to the quantitative data due to the reliability given by a large sample size, the qualitative phase is used to explain and interpret the quantitative results.

3.3.5 Secondary Qualitative Phase

In the second phase of the sequential explanatory strategy mixed methods design, semi-structured interviews are chosen as the research instrument to collect qualitative data about the research questions and also about the findings in the initial quantitative phase. The research questions in this study are about the role of mathematics in engineering practice and the relationship between school mathematics and engineering career choice.

3.3.6 Quality Considerations

The quality of research is largely judged on the credibility of the research findings. When judging research, Eisner (1991) asks "does the story make sense? How have the conclusions been supported" (Eisner 1991)? In this study, the combination of several approaches helps to overcome the weakness, biases and limitation of a single approach. Another advantage of a mixture of approaches is that the combined data is more comprehensive and robust (Cohen et al. 2008). Using two different or independent methods to research the same issue facilitates triangulation, checking the outcomes from one set of observations with the outcomes from another (Cohen et al. 2008).

The concepts of reliability, validity and generalisability provide a framework for conducting and evaluating research. Reliability concerns the consistency of data collection and the repeatability of results by future researchers. Validity is the extent to which the research findings reflect the phenomena under study. Generalisability is the extent that the results from a sample apply to the population (Collis and Hussey 2009). Another factor in assessing the quality of studies is the researcher's bias when interpreting the data. These are discussed below.

3.3.6.1 Reliability

Quantitative data collection is based on precise measurements of research variables and generally has high repeatability or reliability. However in qualitative data collection where the researcher's subjectivity influences the research, reliability cannot be used as a measure of research quality. To enhance the quality of the qualitative phase, a thorough process of interpreting the qualitative data is required. It is recommended that "researchers should present sufficient detail of the processes of their data collection and analysis so that a reader can see how they might reasonably have reached the conclusions they did" (King and Horrocks 2010).

3.3.6.2 Validity

In qualitative research where the researcher has direct access to the participants and the opportunity to explore the phenomenon in depth, the validity can be high. In quantitative research where the data collection process does not reflect the phenomena in the research questions the validity may be low or uncertain. It is recommended that researchers should ensure that the tests or measures "do actually measure or represent what they are supposed to measure or represent" (Collis and

Hussey 2009). In-depth individual interviews that collect personal detailed experiences enhance the validity of mixed methods studies. To enhance the validity of the quantitative phase importance is placed on the design of the survey instrument and particularly the clarity of measurable quantities contained in the survey questionnaire and their relevance to the survey questions.

3.3.6.3 Generalisability

Generalisation of research findings is when researchers can say, with confidence, that what they have learned about a sample is also true of the population. If a sample size is small or is narrowly defined the usefulness of the findings may be limited. The larger the sample size the more representative the sample findings are of the population and hence the greater the population generalisability.

When using a large random sample any differences in data profile between the sample and the population are small and likely to occur by chance rather than bias on the part of the researcher (Fraenkel and Wallen 2008).

While generalisability usually refers to quantitative studies, Collis and Hussey (2009) contend that it is possible to generalise from a single qualitative case if the “analysis has captured the interactions and characteristics of the phenomena you are studying” (Collis and Hussey 2009).

In this study, generalisability is evaluated in terms of the number and selection of research participants in the quantitative phase and also from a comparative analysis of the quantitative and qualitative findings.

3.3.7 Researcher’s Role

Given a researcher’s role in collecting and analysing data and generating new knowledge, there is a concern that the researcher’s own biases, values and personal background might shape the interpretations of data. Researcher bias can occur in the data collection stage, data analysis phase and data interpretation phase in both quantitative and qualitative studies. Researcher bias is a greater threat to the integrity of qualitative research, because the researcher is usually the instrument for collecting data. In order to minimise bias when collecting data, survey questions and interview questions should not demonstrate a particular view. However Collis and Hussey (2009) say that in qualitative studies "it is impossible to separate what exists in the social world from what is in the researcher's mind … therefore the act of investigating social reality has an effect on it … interpretivists believe that social reality is subjective because it is socially constructed" (Collis and Hussey 2009). Creswell recommends that researchers should declare their own experiences and backgrounds so that readers can better understand the researcher's interpretation of the phenomenon (Creswell 2005). Johnson, Onwuegbuzie and Turner (2007) also suggest that researchers include a section in their research proposals titled "Researcher Bias" where they discuss "their personal background, how it may affect their research and what strategies they will use to address the potential problem" (Johnson et al. 2007).

Following this advice, I describe my experience and personal views. My perception of the role of mathematics in engineering practice and in the formation of engineers is primarily shaped by my education and my employment as a professional engineer in both the industrial and academic worlds. I was always comfortable dealing with mathematics in both primary and secondary school. Mathematics was my best subject in university where I took a level 8 degree course in electrical/ electronic engineering in the nineteen eighties. My entire undergraduate engineering class also achieved excellent results in mathematics subjects. My industry experiences includes engineering work in a well-established microelectronics design and manufacture organisation and in the start-up of a major multinational organisation in Ireland. My academic experience includes research using quantitative methods for level 9 masters of engineering qualification and lecturing in electronics subjects in both universities and institutes of technology. I have also managed European research projects promoting electronics amongst women and creating an awareness of technology amongst secondary school students (Goold 1999; Goold 2000).

Rather than having any identifiable biases about mathematics education, I do have an interest in discovering new knowledge that addresses students' difficulty with higher level Leaving Certificate mathematics and the declining number of entrants to engineering degree courses (Devitt and Goold 2010). At the beginning of my research studies, I surveyed 1,289 senior cycle students (50.3% boys, 49.7% girls) from all 29 secondary schools in county Kildare and I found that 33.7% planned to take higher level mathematics for the Leaving Certificate examination. This compares to just 16% nationally who take the higher level option. Of the students who did not choose higher level mathematics, 82% based their decision on the difficulty of the subject. 42% of the students who opted for higher level mathematics cited that they did so because they felt they were good at the subject. It is my view that higher level Leaving Certificate mathematics should not be too difficult, or perceived to be too difficult, for a majority of the national student population. I also engaged with two Transition Year[16] classes (one mixed class and one all-girls class) from two different schools in a practical technology learning environment. I conducted some focus group discussions where the students discussed their interest in mathematics, technology and careers. I observed that both mathematics and engineering ranked towards the bottom of a majority of the students' interests.

As an engineering educator I welcome any reform of mathematics education and at the beginning of this study I have an open mind regarding the new Project Maths syllabus. By engaging in both quantitative and qualitative methods and a rigorous process of analysing the data, I embark on the journey of discovery with an objective of contributing new knowledge to the type of mathematics required by engineers in their work and the relationship, if any, between students' feelings about mathematics and their choice of engineering as a career.

[16] Transition Year: an optional, one-year, standalone, full-time programme taken in the year after the Junior Certificate in Ireland that has a strong focus on personal and social development and on education for active citizenship - Jeffers, G. (2011). "The Transition Year Programme in Ireland. Embracing and Resisting a Curriculum Innovation." *The Curriculum Journal*, 22(1), 61-76.

3.3.8 Ethical Considerations

When conducting research, consent and cooperation of research subjects is required. Participants should engage voluntarily in any research study and they should fully understand the nature of the research project (Cohen et al. 2008). Furthermore all participants should be treated with respect and it is important that the researcher is courteous and that participants are not uncomfortable or indeed coerced into answering sensitive questions (Collis and Hussey 2009). In the case of students, it is necessary to consult and seek permission from teachers or other adults responsible for these subjects and children themselves must also be given a real and legitimate opportunity to refuse to participate in the study. In Ireland there is a child protection policy and a code of behaviour for working with children and young people and it is necessary to get official permission when working with people under the age of eighteen years.

While it is sometimes argued that it is necessary to be vague about the purpose of the research in order to achieve findings of value, according to ethical guidelines research participants need to know the purpose and aims of the study, the use of results and the likely consequences the study will have on their lives (Creswell 2005). It is also necessary to protect participants' anonymity and this often has the added advantage of encouraging more open responses from participants (Creswell 2005).

According to Punch (2005), it is important to identify research questions that will benefit individuals being studied and that will be meaningful for others besides the researcher (Punch 2005). Cohen, Manion and Morrison (2008) suggest that a selfish approach to the benefits of the research by the researcher is unethical and they ask "what will this research do for the participants and the wider community, not just for the researcher" (Cohen et al. 2008)?

Data should be reported honestly and findings should not be distorted to satisfy any particular interest group. When reporting research findings, credit should be given for material quoted from other studies with both an in-body citation and a bibliographic entry in the references section of the document (Creswell 2005).

While in education research it is considerably less likely to encounter ethical dilemmas compared to research in social psychology or medicine, the welfare of subjects should be kept in mind, even if it involves compromising the impact of the research (Cohen et al. 2008).

This research was conducted according to the recommended ethical guidelines. The preliminary work involving school students was authorised by the school principals. Engineers, participating in this study, were advised about the aims and purpose of the research and they participated willingly in the study. Interviewees consented to audio recording and all participants were assured anonymity. Data was analysed thoroughly and honestly and credit was given to material obtained from other sources. There were no major ethical concerns encountered in this study.

3.4 SUMMARY

The research methodology employed in this study is a sequential explanatory strategy mixed methods design which is the collection and analysis of quantitative data followed by the collection and analysis of qualitative data that ultimately builds on the results of the initial quantitative data. The corresponding data collection methods chosen are a survey questionnaire and semi-structured interviews. The instrument used to measure *curriculum mathematics* usage in engineering practice is a derivation of de Lange's mathematics assessment pyramid and it is also based on the new "Project Maths" syllabus. In addition to measuring *curriculum mathematics, mathematical thinking* usage and *engaging* usage (motivation to take a mathematical approach) are also measured in this study. Measuring engineers' feelings about mathematics and career choice is based on motivation theory.

The research population is selected as engineers who meet Engineers Ireland's criteria for "Chartered Engineer". The overall goal of this research is to contribute new knowledge to the type of mathematics required by engineers in their work and to determine whether there is a relationship between students' experiences with school mathematics and their choice of engineering as a career.

CHAPTER 4: SURVEY METHODOLOGY AND DATA ANALYSIS

4.1 INTRODUCTION

This chapter presents the methodology used for the collection and analysis of quantitative data from practising engineers in relation to the two research questions:

1. What is the role of mathematics in engineering practice?
    a) How can mathematics usage in engineering practice be measured?
    b) How do engineers use mathematics in their work?
    c) What motivates engineers to engage, or not, with mathematics?

2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?
    a) To what degree do students' feelings about mathematics influence engineering career choice?
    b) What factors in mathematics education influence students' affective engagement with mathematics?

This chapter is organised as follows:

Page number

4.2 SURVEY POPULATION

For the purpose of this study the research population is identified as engineers who meet the criteria of "Chartered Engineer" as determined by Engineers Ireland, the professional body representing the engineering profession in Ireland. There are 5,755 (424 women) Chartered Engineers registered with Engineers Ireland each of whom have a minimum of a level 8 academic qualification and four years' relevant professional experience (Engineers Ireland, 2011). Engineers who meet Engineers Ireland's requirements for Chartered Engineer and who are not registered as Chartered Engineers with Engineers Ireland are included in this study.

Within the spectrum of Chartered Engineers and for the purpose of this study, engineer types are classified according to their discipline e.g. agriculture and food, chemical, civil, electronic/electrical, mechanical, manufacturing/production, software etc. and roles e.g. basic research, design/development, education, maintenance and production etc. A typical career development path for an engineering graduate is to progress from graduate engineer to senior engineer and then onto engineering management. As engineers' careers develop many engineers opt for non-engineering career routes, some of these engineers continue to work in an engineering environment and sometimes they manage people as opposed to managing engineering projects and other engineers move to different industries. Thus, for the purpose of this research engineers are also classified according to their position e.g. engineer, senior engineer, engineering manager and former engineer.

4.2.1 Study Sample

Given the large population size, data is collected from a sample of the population and following appropriate statistical analysis inferences are extrapolated to the entire population. In order to support population-wide generalisations, the sample must be carefully chosen, according to the two criteria:

1. The sample size must be above a specified minimum, for precision

2. The sample must be randomly chosen, to prevent bias

While larger responses may give more precise results and enhance the reliability of quantitative studies, it is often difficult to get large numbers of people to respond to survey questionnaires. One difficulty in determining the response rate required in this study is the dearth of prior information available concerning the mathematics used by engineers in their jobs and the impact of engineers' feelings about mathematics on career choice. Theoretically the required sample size for any population based survey is determined by: (i) the estimated population proportion (ii) the desired level of confidence and (iii) the acceptable margin of error. For example, in the extreme case of a very large or infinite population, sample size can be calculated using the following formula:

$$n = \frac{1.96^2 \times P(1 - P)}{\delta^2} \qquad \text{(Reilly 2006)}$$

where n is the required sample size; 1.96 is the standard normal score associated with 95% confidence; P is the estimated population proportion, ± δ is the error margin (Reilly 2006). In this study the initial proportion is unknown and an initial estimate of 50% delivers a bigger sample size than any other value of P. Using this formula as a guide to estimating the sample size required in this study to estimate to within 10%, with 95% confidence, the proportion of engineers in any particular category, using a conservative initial estimate of 50%, assuming an infinite population of engineers, is:

$$n = \frac{1.96^2 \times P(1 - P)}{\delta^2} = \frac{1.96^2 \times 0.5(1 - 0.5)}{(0.10)^2} = 97$$

Despite being only a small fraction of the 5,755 Chartered Engineers that are registered with Engineers Ireland, this sample size is sufficient as it is based on the assumption of an infinite population. This sample size is for the estimation, to within 10%, with 95% confidence, of the proportion of all engineers who would answer "yes" to any question (or who would belong to any yes/no category). Sample sizes required for any other data type, such as Likert scales, or measurements, are typically smaller. The richest data generally consists of observations with potentially infinite possible outcomes. A response on a Likert scale, having five outcomes, is richer than a binary response (two possible outcomes). When determining the sample size required for a Likert scale with 5 outcomes, the formula for measurements used is:

$$\text{Sample size} = \frac{1.96^2 \times \sigma^2}{\delta} \qquad \text{(Reilly 2006)}$$

where ±δ = width of interval required and σ = standard deviation estimate. σ = √2 is a conservative estimate of sigma. The theoretical basis for this is as follows. If all responses were equally likely, the responses form a uniform discrete distribution with parameter k = 5, corresponding to 5 equally spaced response categories. The mean and variance of a uniform discrete distribution are $(k+1)/2$ and $(k^2-1)/12$ respectively. In practice, the actual distribution is unlikely to have a larger variance than a uniform discrete distribution, unless the responses are bimodal.

$$\text{Sample size} = \frac{1.96^2 \times 2}{0.1^2} = 768 \quad \delta = 0.1 \text{ (0.1 Likert units)}$$

$$= \frac{1.96^2 \times 2}{0.15^2} = 341 \quad \delta = 0.15 \text{ (0.15 Likert units)}$$

$$= \frac{1.96^2 \times 2}{0.2^2} = 192 \quad \delta = 0.2 \text{ (0.2 Likert units)}$$

$$= \frac{1.96^2 \times 2}{0.25^2} = 123 \quad \delta = 0.25 \text{(0.25 Likert units)}$$

$$= \frac{1.96^2 \times 2}{0.5^2} = 31 \quad \delta = 0.5 \text{ (0.5 Likert units)}$$

A $\delta = 0.1$, which represents 0.1 Likert units demands a very precise estimate of the mean and a very large sample size. While a delta value of 0.2 requires a smaller sample size, it is also a very good estimate.

In addition to the minimum sample size requirement, the population sample should be representative of the entire population and should be free from bias. Only random samples can be relied upon to be free from bias. A random sample is a sample selected in such a way that every unit has an equal chance of being selected. In an attempt to eliminate bias in this study, the survey questionnaire is electronically distributed to the entire population (5,755 Chartered Engineers registered with Engineers Ireland) using the same manner of distribution for each engineer. While every Chartered Engineer registered with Engineers Ireland is given the same opportunity to participate in the survey, it cannot be verified that engineers who volunteer to participate in the study comprise of a random sample. This is a weakness of this study and it cannot be verified that engineers with biased and strong opinions about the research topics in this study are not overrepresented in the sample. The survey analysis is conducted based on the assumption that the survey participants are a random sample.

4.3 SURVEY DESIGN

A survey questionnaire is generally the preferred method of quantitative data collection when the population is large. Survey design considerations include: the time required to complete the survey; the clarity of the survey questions and their relevance to the research questions; the administration of the survey; and the method of analysis. In survey design, it is standard practice to include a range of response options to survey questions where the participants can just tick the box beside their preferred answer and this is one way of reducing the time required to complete the questionnaire and such questions allow variables to be quantified and measured efficiently. The Likert format, where responses are based on a five point scale, ranging from “strongly agree” to “strongly disagree” or from “not at all” to “a very great deal”, were chosen as an efficient way of collecting quantitative data about engineers' mathematics usage in their work. Open-ended questions, while qualitative in nature, can also be included in survey questionnaires to explore areas where there is little prior knowledge and to collect qualitative type data such as engineers' feelings about mathematics. However such open-ended questions increase the time to complete the survey.

The first task was to design a survey questionnaire whereby practising engineers could provide information concerning (i) measurements of their mathematics usage in engineering practice; (ii) their own experiences and feelings about their school mathematics; (iii) factors that contribute to their interest in and learning of school mathematics; and (iv) the relationship between their experiences with school mathematics and their choice of engineering as a career.

The credibility of quantitative findings is highly dependent on the design of the research instrument and its content validity. While the research instrument must comprehensively cover the domain or items it purports to cover, Cohen, Manion, Morrison (2008) say it is not possible to address each item in its entirety without risking the respondents' motivation to complete, for example, a long questionnaire (Cohen et al. 2008). The survey design was given extra care to ensure good presentation, clarity of instructions and survey questions and automated data collection so as to maximise the response rate of survey questionnaires.

In this study there were many iterations of the survey design before it was ultimately deployed. Over a period of about three months, the survey questionnaire was repeatedly tested and revised by the researcher's engineering colleagues particularly with regard to the content, relevance to research questions, clarity of the instrument, time to complete and efficient operation of the software for distributing, completing and returning the questionnaires. A copy of the survey questionnaire is included in Appendix 1, volume 2 of this thesis.

For clarity and for ease of completion the survey content was divided into three parts:

1. Biographical information
2. Curriculum mathematics usage

3. Thinking usage and engaging with mathematics

4.3.1 Biographical Information

While one broad aim of this research is to develop a picture of how practising engineers generally use mathematics in their professional life, engineering practice comprises a diversity of engineers and engineering environments and these are likely to impact on mathematics usage. For example, one would not expect a research and development engineer working in a high technology environment to use and engage with mathematics in the same way as an engineer working as a project manager. One would also expect differences between different engineering disciplines e.g. civil engineers are likely to use more geometry and trigonometry than electronic engineers. One would expect that there are many factors that contribute to engineers' use of mathematics in the workplace, so a broader (rather than narrower) scope of exploration was appropriate. Hence biographical information, concerning engineers' gender, Chartered Engineer status, engineering discipline, engineering role, company size and current position, result and year of Leaving Certificate mathematics, degree that higher level Leaving Certificate mathematics is required for job, degree of enjoyment of school mathematics and an open question requesting the participants' views on how to improve young people's affective engagement with mathematics, was required to facilitate a thorough analysis of the data.

4.3.2 Measuring *Curriculum* Mathematics Usage

The methodology used to measure engineers' *curriculum mathematics*, the term devised in this study to represent engineers' mathematics education at school and university, usage is based on de Lange's pyramid of mathematics assessment as presented in Chapter 3 of this thesis (De Lange 1999; De Lange and Romberg 2004).

De Lange's method of assessing mathematics education is based on three dimensions: domains of mathematics, difficulty of questions posed and levels of thinking. In this study, the survey instrument was designed to measure *curriculum mathematics* with respect to three dimensions: domain, level and usage, Figure 4-1.

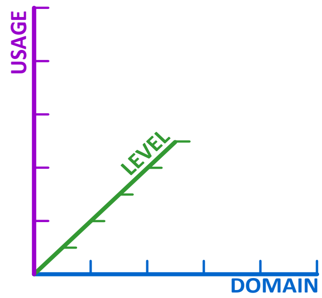

Figure 4-1: Three dimensions of *curriculum* mathematics.

Domain refers to mathematics topics. At the time of planning this study a new Junior Certificate and Leaving Certificate mathematics syllabus "Project Maths" was being introduced into secondary schools in Ireland. Both the Junior Certificate and Leaving Certificate Project Maths syllabi each comprise five strands and these same five strands were adopted to comprise the domain dimension in the survey instrument. The five strands are:

1. Statistics and probability
2. Geometry and trigonometry
3. Number
4. Algebra
5. Functions

Level refers to academic progression levels. In the context of mathematics usage, the level dimension distinguishes between mathematics at various different academic stages. In Ireland academic stages include: Junior Certificate (level 3); Leaving Certificate (levels 4 and 5); and honours bachelor degree (level 8). In the Junior Certificate and Leaving Certificate exams in Ireland most students choose between the ordinary level and the more advanced higher level options. Junior Certificate ordinary level mathematics is the first formal mathematics assessment in Ireland.

Students' decision to take either ordinary level or higher level Leaving Certificate mathematics is one that impacts the supply of prospective engineering students. In university there are two main types of mathematics education: engineering mathematics which is integral to engineering courses; and non-engineering mathematics, which students pursue in arts or science degrees study. The five levels of academic progression chosen in this study are:

1. Junior secondary (Junior Certificate ordinary level)
2. Intermediate secondary (Leaving Certificate ordinary level)
3. Senior secondary (Leaving Certificate higher level)
4. Engineering (B. E. / B. Eng.)
5. B. A. / B.Sc.

Usage refers to the type of mathematics usage and in this study there are five usage types. Three of these relate to *curriculum mathematics* usage and the other two usage types, *mathematical thinking* (*thinking*) usage and *engaging* usage are discussed in the next section. The three types of *curriculum mathematics* usage are similar to the levels of thinking in de Lange's pyramid of mathematics assessment. These are:

1. Type 1: Reproducing
2. Type 2: Connecting
3. Type 3: Mathematising

These three levels of *curriculum mathematics* usage, reproducing, connecting and mathematising are defined as follows:

1. Reproducing (type 1) is usage of mathematics through knowledge of facts and concepts, recalling mathematical properties, performing routine procedures, applying standard algorithms and operating with mathematics symbols and formulae. Users require knowledge of facts, concepts, definitions and routine procedures that have been memorized and previously practiced.

2. Connecting (type 2) is usage of mathematics by making connections within and between different mathematics topics and integrating information in order to solve problems, where there is a choice of strategies and mathematical tools. Users have to choose their own strategies and mathematical tools, and make connections between the different domains in mathematics.

3. Mathematising (type 3) is usage of mathematics by extracting the mathematics embedded in a situation and using mathematics to develop models and strategies; making mathematical arguments, proofs and generalisations to solve the problem; analysing; interpreting and translating mathematical models into real world solutions. Users have to recognise and extract the mathematics embedded in situations, develop new strategies and models, give arguments and proofs and implement solutions.

A representation of the methodology used to measure engineers' *curriculum mathematics* usage in the survey questionnaire is shown in Figure 4-2.

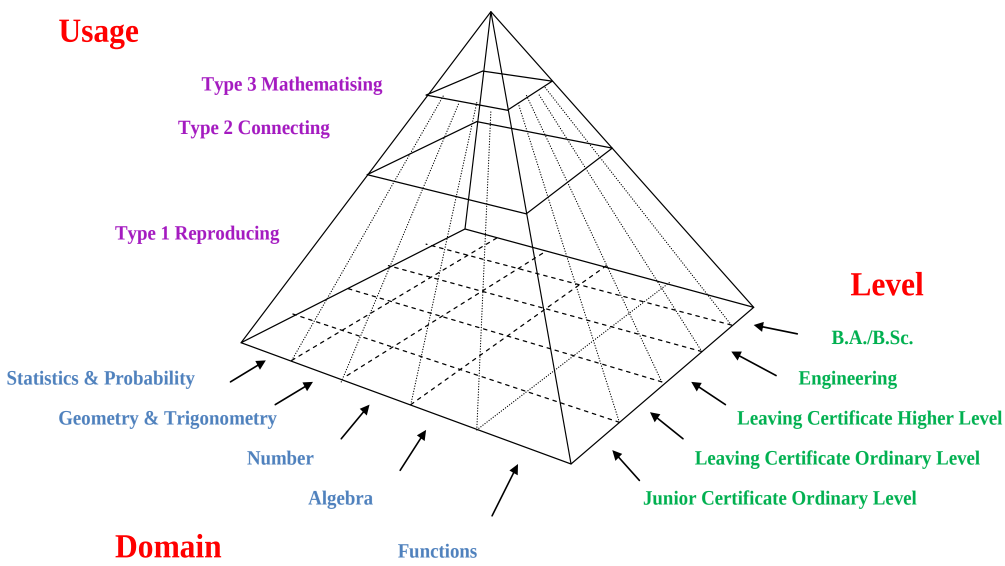

Figure 4-2: *Curriculum* mathematics assessment pyramid.

In the survey questionnaire, a five point Likert scale was used to measure engineers' *curriculum mathematics* usage with respect to mathematics domain; academic level and usage type as illustrated in Figure 4-3. The five points on the Likert scale are: 1 = Not at all; 2 = Very little; 3 = A little; 4 = Quite a lot; and 5 = A very great deal. For each of the five mathematics domains, five academic levels and three usage types, engineers were asked to rate their usage of *curriculum mathematics* in their work for the previous six months. Given that there is a total of seventy five domain-level-usage combinations, mathematics usage questions in the questionnaire are presented separately for each of the five domains: 1. Statistics and probability; 2. Geometry and trigonometry; 3. Number; 4. Algebra; and 5. Functions. For example, in the case of statistics and probability, participants were presented with the style of question shown in Figure 4-3.

QUESTION:

To what extent have you used Statistics & Probability in your work in the last 6 months?

INSTRUCTIONS:
Select your response from the options presented in EACH of the dropdown menus to indicate your usage of Probability & Statistics at each LEVEL and for each USAGE type. Usage types are mutually independent.

*For guidance, definitions and sample topics in Probability & Statistics, at various levels and usage types are provided in the support document attached, "**Survey INFO**."*

You should make an entry in ALL (yellow) answer boxes.

STATISTICS & PROBABILITY

USAGE TYPE		*e.g. facts or applying routine algorithms* Type 1 usage **Reproducing**	*e.g. use of different tools & problem solving strategies* Type 2 usage **Connecting**	*e.g. interpreting & developing models, translating into real world solutions* Type 3 usage **Mathematising**
SUBJECT LEVEL				
Junior – secondary				
Intermediate – secondary				
Senior – secondary				
Engineering				
B.A. / B.Sc.				

Figure 4-3: Measuring *statistics and probability* mathematics usage.

4.3.3 Measuring *Thinking* Usage and *Engaging* with Mathematics

In addition to the three types of *curriculum mathematics* usage, two other types of mathematics usage relating to the research questions are:

1. Type 4: *Thinking*
2. Type 5: *Engaging*

4.3.3.1 *Thinking* Usage

In the survey questionnaire *thinking* usage is also called type 4 usage. *Thinking* usage is usage of mathematical modes of thinking learned and practised through mathematics, e.g. methods of analysis and reasoning, logical rigour, problem solving strategies (e.g. problem decomposition and solution re-integration), recognition of patterns, use of analogy, and a sense of what the solution to a problem might be (Schoenfeld 1992).

Using the five point Likert scale (1 = Not at all; 2 = Very little; 3 = A little; 4 = Quite a lot and 5 = A very great deal), survey participants were asked to rate their *thinking* usage in work in: the previous six months; within 2 years of graduating; within 3 to 5 years after graduating; within 6 to 10 years after graduating and greater than 10 years after graduating, Figure 4-4.

In an open-question, participants were asked to identify the modes of *thinking* resulting from their mathematics education that influence their work performance.

QUESTION:
To what extent, with or without direct application of mathematics, did your mathematics training (with its associated modes of thinking and analysis) directly influence your approach to your work?

INSTRUCTIONS:
Select your response from the options presented in EACH of the dropdown menus to indicate your thinking usage of maths in the last 6 months, within 2 years of graduating and within 10 years of graduating.

*For guidance, definition and examples of THINKING USAGE are provided in the support document attached, "**Survey INFO.**"*

You should make an entry in ALL (yellow) answer boxes that represent your career.

THINKING USAGE

USAGE TYPE		*e.g. reasoning, logical techniques problem solving strategies, sense of solution etc.* Type 4 usage **Thinking**
WHEN		
in the last 6 months		
within 2 years of graduating		
within 3-5 years after graduating		
within 6-10 years after graduating		
greater than within 10 years after graduating		

Figure 4-4: Measuring thinking usage.

4.3.3.2 *Engaging* with Mathematics

Engaging usage (type 5 usage) relates to emotional relationships with mathematics. In the context of this study *engaging* usage is defined as the motivation and persistence to take, a mathematical approach to a problem as a result of one's attitudes, beliefs, emotions, goals, sense of value, interest, confidence, self-efficacy and sociocultural influences (Csíkszentmihályi 1992; McLeod and Adams 1989; Schunk et al. 2010).

Measurement of engineers' engagement with school mathematics and engineers' motivation to take a mathematical approach in their work is based on Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation whose theory posits that predictors of achievement behaviour are: expectancy (am I able to do the task?); value (why should I do the task?); students' goals and schemas (short-and long-term goals and individuals' beliefs and self-concepts about themselves); and affective memories (previous affective experiences with this type of activity or task) (Schunk et al. 2010; Wigfield and Eccles 2002). Engineers' engagement with mathematics is driven by their motivational beliefs in school, in university and in engineering practice, Figure 4-5.

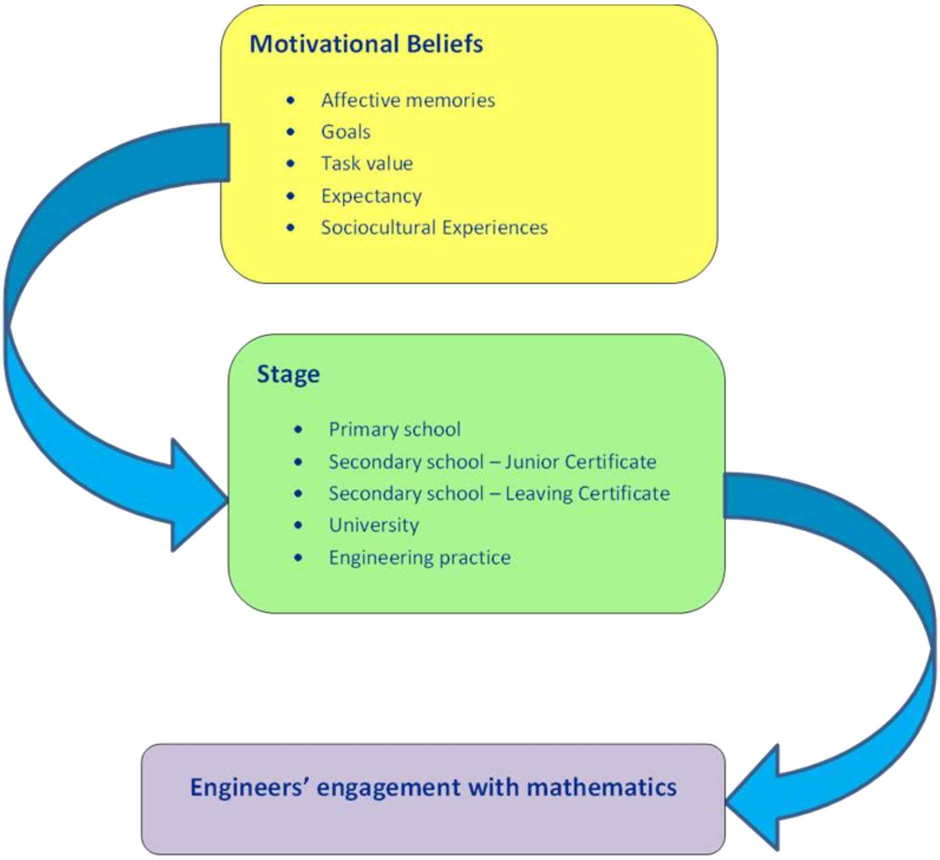

Figure 4-5: Representation of *engaging* usage.

To measure engineers' engagement with mathematics in their work and their feelings about mathematics in their job, survey participants were asked to use the five point Likert scale (1 = Not at all; 2 = Very little; 3 = A little; 4 = Quite a lot and 5 = A very great deal), as shown in Figure 4-6, to rate the following:

1. Degree a specifically mathematical approach was necessary
2. Degree engineers actively sought a mathematical approach
3. Degree engineers enjoyed using mathematics
4. Degree engineers felt confident dealing with mathematics
5. Degree engineers had a negative experience when using mathematics

QUESTION:
With regard to your work in the last 6 months, to what degree.....

***INSTRUCTIONS**:*
Select your response from the options presented in EACH of the dropdown menus to indicate your affective usage of maths in the last 6 months and state why in EACH of the corresponding text fields.

*For guidance, definition and examples of ENGAGING USAGE are provided in the support document attached, "**Survey INFO.**"*

You should make an entry in ALL (yellow) answer boxes.

ENGAGING USAGE

USAGE TYPE		*e.g. motivation, attitudes, beliefs, emotions, value, confidence and self-efficacy* Type 5 usage **Engaging**
QUESTION		
...was a specifically mathematical approach necessary?		
Why?		
...did you actively seek a mathematical approach?		
Why?		
...did you enjoy using mathematics?		
Why?		
...did you feel confident dealing with mathematics?		
Why?		
...did you have a negative experience when using mathematics		
Why?		

Figure 4-6: Measuring *engaging* usage.

In the context of *engaging* with school mathematics participants were asked, in an open question, to identify the events, experiences, aptitudes or other factors within and outside of school that contributed to their interest in and learning of mathematics, Figure 4-7.

QUESTION:
What events, experiences, aptitudes or other factors within and outside of school contributed to your interest in and learning of mathematics?

FACTORS WITHIN SCHOOL		
...primary school		
...secondary – Years 1 & 2		
...secondary – Junior Cert		
...secondary – Leaving Cert		
FACTORS OUTSIDE SCHOOL		
... primary school years		
... secondary – Years 1 & 2		
...secondary – Junior Cert		
...secondary – Leaving Cert		

Figure 4-7: Measuring factors that contribute to interest and learning of mathematics.

In the context of engineers' career choice, participants were asked to rate the degree their feelings about mathematics impacted their choice of engineering as a career using the five point Likert scale (1 = Not at all; 2 = Very little; 3 = A little; 4 = Quite a lot and 5 = A very great deal).

The survey questionnaire concluded with an open question inviting the participants to make additional comments.

4.3.4 Survey Support Document

To ensure high validity of the survey instrument, a separate “Survey INFO” document to accompany the survey questionnaire was designed to assist survey participants when completing the questionnaire. This survey INFO document describes and illustrates each of the five mathematics usage types. Sample topics for each of the five domains: (i) Statistics and probability; (ii) Geometry and trigonometry; (iii) Number; (iv) Algebra; and (v) Functions and for each of the five academic levels: (i) Junior secondary (Junior Certificate ordinary level); (ii) Intermediate secondary (Leaving Certificate ordinary level); (iii) Senior secondary (Leaving Certificate higher level); (iv) Engineering (B.E. / B. Eng.) and (v) B. A. / B.Sc. are included in the survey support document. A copy of the survey INFO document is included in Appendix 2, Volume 2 of this thesis.

4.4 ADMINISTRATION OF SURVEY

The survey and the survey INFO document were designed using Adobe Acrobat X Pro software. This is a PDF (portable document format) communications package that allowed the survey and the survey INFO document to be created as an interactive PDF and distributed to participants by email. The interactive Adobe Acrobat format allowed participants to view both documents simultaneously, to add their responses to the survey questionnaire and to return the completed questionnaire directly to the researcher by email.

Engineers Ireland kindly agreed to facilitate the administration of the survey to Chartered Engineers registered with the body. To encourage engineers to participate in the survey and to boost the response rate, all participants were entered into a draw for a prize donated by a luxury hotel located beside the university where the author conducted the research. On 11th February 2011, with the support and co-operation of Engineers Ireland, the survey questionnaire was distributed, by direct email, to 5,755 (424 women) Chartered Engineers. Engineers Ireland also created a direct link to the survey questionnaire in its weekly electronic newsletters on 9th and 16th March, 2011 which were emailed to its 21,700 members. Some engineers, whilst not registered Chartered Engineers, met the same academic requirements and professional experience criteria as Chartered Engineers registered with Engineers Ireland and were included in the survey. Copies of survey distribution emails and notices are included in Appendix 3, Volume 2 of this thesis.

In further attempts to boost the number of survey responses, the researcher distributed the survey questionnaire to organisations such as IBEC (Irish Business and Employers' Confederation), RIA (Royal Irish Academy), American Chamber of Commerce, Cork Electronics Industry Association and the IET (Institute of Engineering and Technology); to engineering companies (RPS Group, Eircom, ESB, Eirgrid, Elan, Pfizer, Ericsson, Bord Gais Eireann, Airtricity, Microsoft), to third level colleges (Trinity College Dublin, Dublin City University, University College Dublin, Cork Institute of Technology, Institute of Technology Tallaght, Waterford Institute of Technology, National University of Ireland Maynooth and National University of Ireland Galway) and to local authorities (city/ county councils). The IDA (Industrial Development Agency) and some major Irish multinational companies have strict policies whereby they do not support PhD students and they do not participate in survey studies.

4.5 SURVEY DATA COLLECTION

Survey participants sent their completed surveys by email directly to the researcher. All completed surveys were immediately acknowledged by a replying email. Some participants made direct contact with the researcher by either email or telephone seeking confirmation of the process for returning their completed surveys or clarification of the survey questions. The possibility of receiving duplicate responses was checked using the participants' email addresses. The Adobe Acrobat X Pro software allowed the PDF files to be directly converted into spread sheet format.

There were a total of 365 valid responses of which 39 were from women. This sample size is satisfactory for precision to within 0.15 units (on a Likert scale with five outcomes) and 95% confidence (probability that the findings from the survey questionnaire represent the population of Chartered Engineers in Ireland) as calculated in section 4.2.1.

In section 4.2.1 it is noted that this study makes the assumption that the survey participants are a random sample of Chartered Engineers. While all Chartered Engineers registered with Engineers Ireland were invited to participate in the survey, it may be that those who chose to participate have a strong interest in the research topic and are not representative of the entire population of Chartered Engineers in Ireland. The consequence of any sampling bias is that statistical conclusions are not valid for the entire population. In order to verify that the sample is random it is required to establish that no differences exist among study participants and non-participants. It is observed that women represent 10.7% of the survey participants and this compares similarly with the overall gender breakdown of Chartered Engineers where 7.4% of Chartered Engineers registered with Engineers Ireland are women (Engineers Ireland 2011). Engineers who participated in the survey represent a variety of engineering disciplines, roles and positions. Civil engineers are the greatest discipline represented in the survey sample with 44.5% of survey participants. Mechanical engineers represent 20.8% of participants, electronic and electrical engineers represent 21.4% and chemical engineers represent 4.4%, Figure 4-8. The breakdown of engineering disciplines amongst the survey participants is similar to that of Engineers Ireland registered Chartered Engineers where civil engineers are 45%, mechanical engineers are 19%, electronic and electrical are 19% and chemical engineers are 2.5% (in conversation with Engineers Ireland). Despite the similarity between the discipline breakdown in both the survey sample and Chartered Engineers registered with Engineers Ireland, there is insufficient data about the research topics in this study available for the non-participants and therefore it cannot be verified that a random sample of Chartered Engineers participated in the survey.

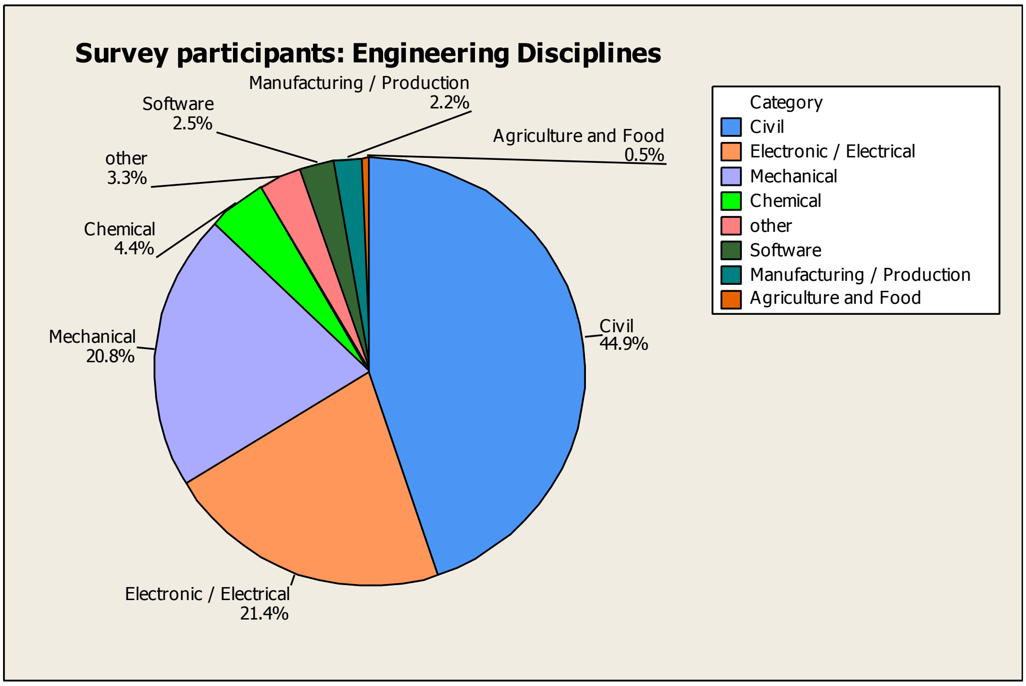

Figure 4-8: Survey participants by engineering discipline.

The majority of survey participants' roles are design / development (41.9%) and management / project management (28.2%), Figure 4-9.

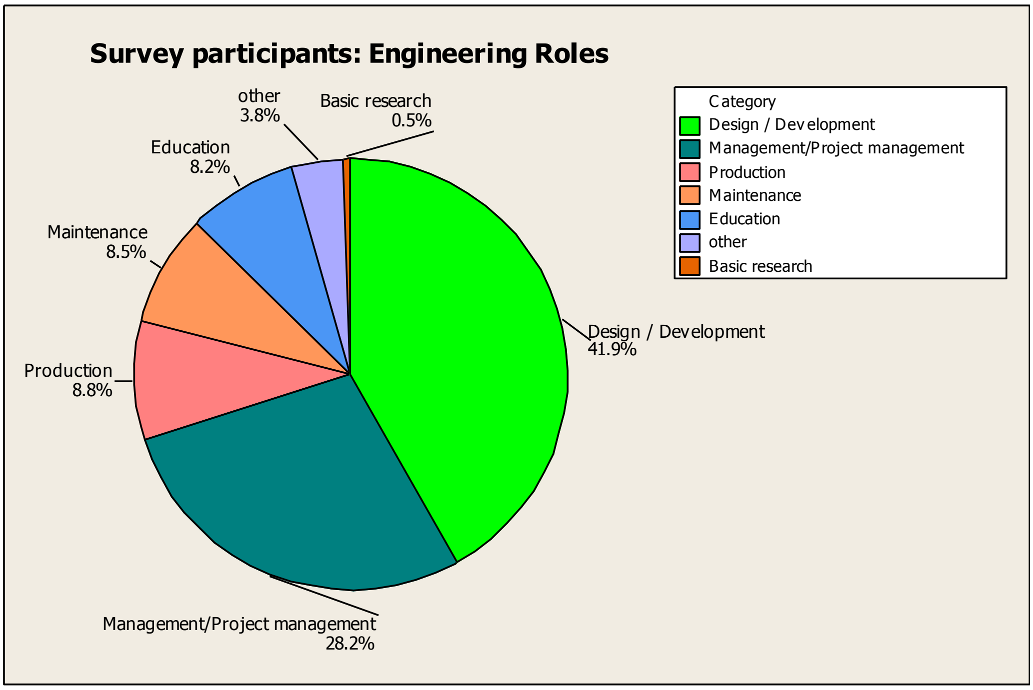

Figure 4-9: Survey participants by engineering role.

A profile of survey participants by engineering discipline and role are shown in Figure 4-10.

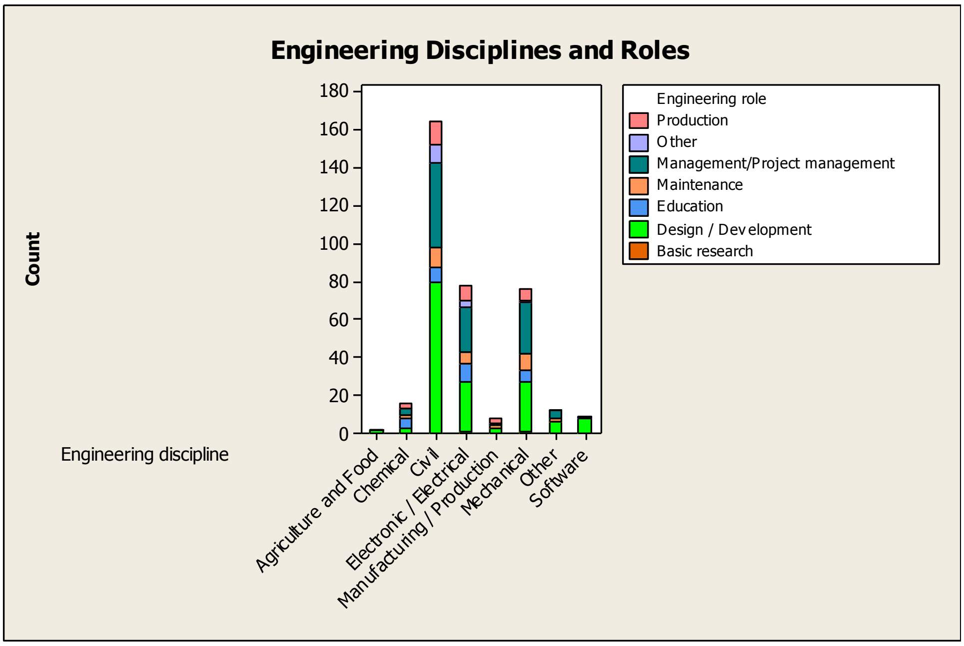

Figure 4-10: Survey participants by engineering discipline and role.

The majority of participants worked in large companies and multinational companies were well represented, Figure 4-11.

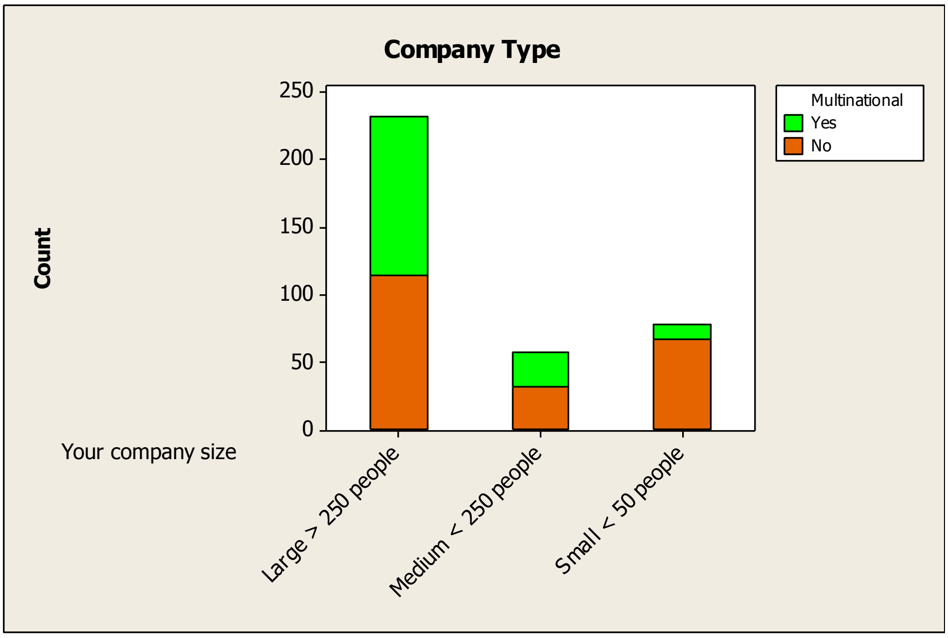

Figure 4-11: Participating engineers' company types.

Engineers, senior engineers, engineering managers and former engineers were all well represented, Figure 4-12.

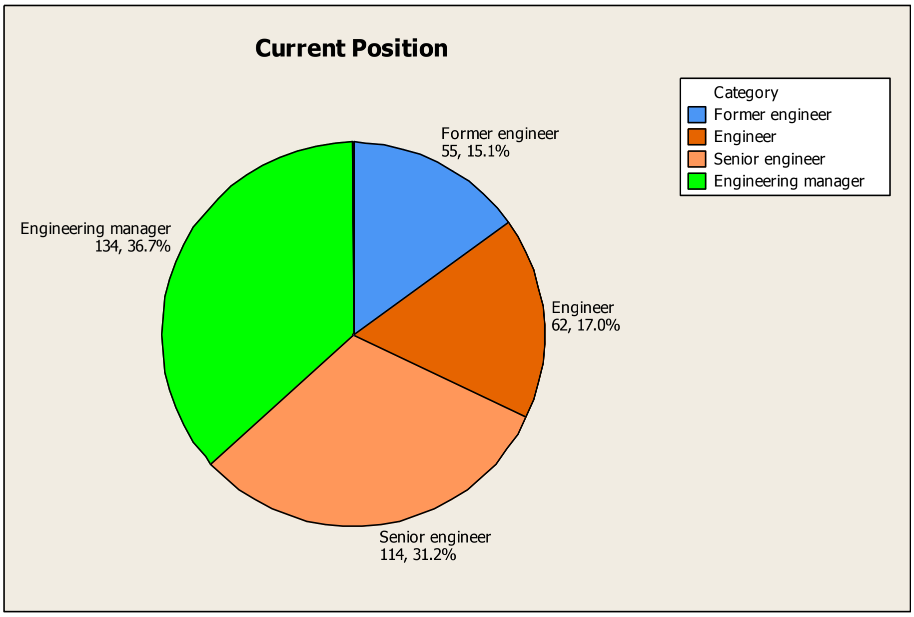

Figure 4-12: Participating engineers' current positions.

84% of engineers (308) completed higher level Leaving Certificate mathematics, Figure 4-13 and 98 engineers have A grades (≥ 85%), 116 engineers have B grades (≥70%, <85%) and 98 have C grades (≥55%, <70%), Figure 4-14.

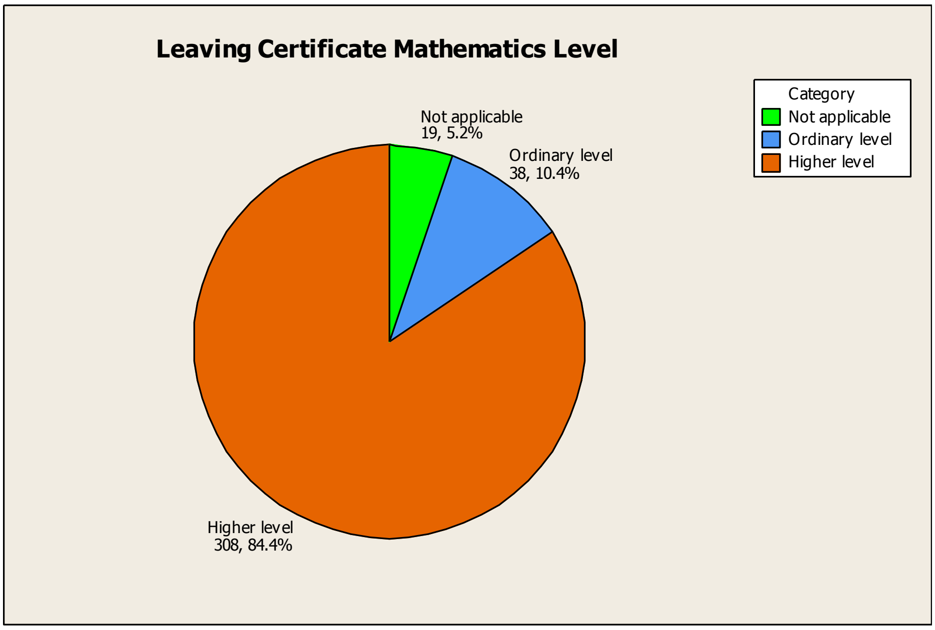

Figure 4-13: Participating engineers' Leaving Certificate mathematics levels.

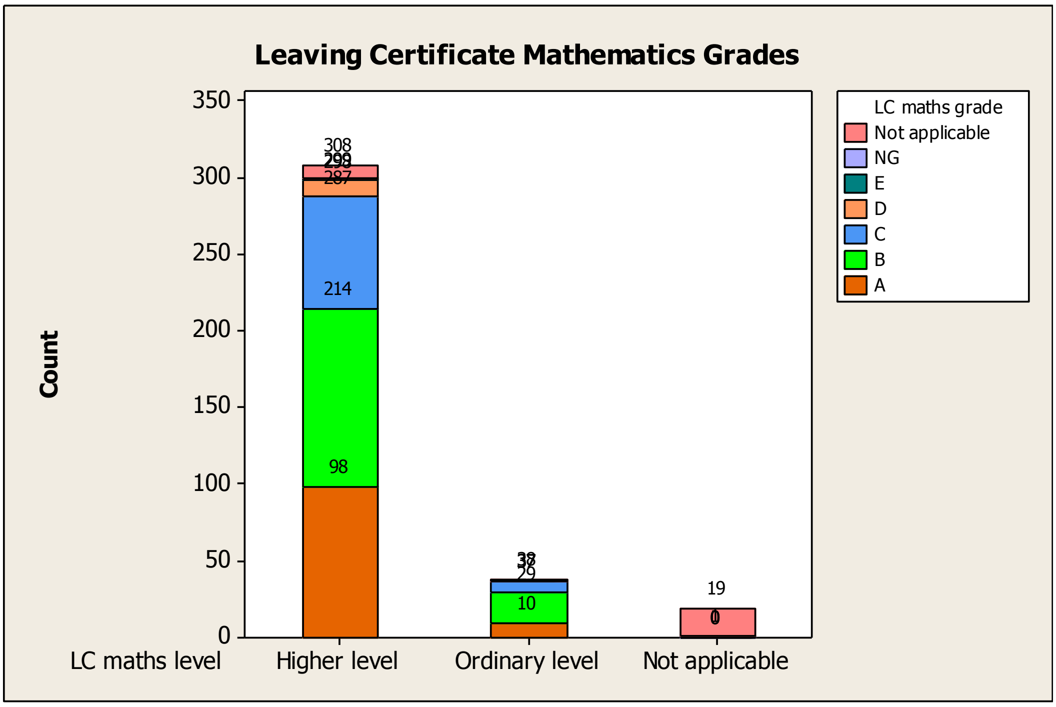

Figure 4-14: Participating engineers' Leaving Certificate mathematics grades.

4.6 SURVEY DATA ANALYSIS

Minitab statistical software (version 15) was used to analyse the quantitative data collected in this study. Minitab is one of a number of statistical packages that is widely used in industry and academia. While not unlike other statistical software packages, Minitab was chosen in this study primarily because of the availability of the software and the level of support available to the researcher. This section provides a brief overview of the statistical tools used in this study.

Minitab statistical software is used to analyse survey day in the following ways:

- Pie charts, bar charts and histograms are used to display data. For example, in this study the charts are used to illustrate the survey participants' background, their educational details, their feelings about mathematics, career decisions and engineering practice, their mean mathematics usage and also categories of participants' responses to open questions in the survey questionnaire.
- 95% confidence interval plots illustrate unknown population means with 95% probability, assuming the sample is representative of the population from which it

comes. For example, in this study 95% interval plots are used to illustrate engineers' mean mathematics usage.

- A paired t-test is used to determine if there is a difference between two sets of data for the same population. For example, in this study a paired t-test is used to test engineers' responses to two different questions for differences. A paired t-test determines if there is a difference between two population means by calculating the difference between pairs of variables and testing if the average difference is significantly different from zero. The t-value for a paired t-test is the mean of these differences divided by the standard error of the differences. A p-value, which is the probability of obtaining the data if the mean difference is zero, is calculated from the t-value. A p-value less than 5% confirms (with 95% confidence) that the mean difference between two sets of data is not zero and as well as establishing that there is a difference, the paired t- test also measures the 95% confidence interval for the mean difference between the population means. A p-value greater than 5% does not allow any assertion to be made because when a sample size is too small, it may not contain sufficient evidence to reject a false null hypothesis and the test lacks power. Power is the probability of correctly rejecting the null hypothesis when it is false. To confirm that there is no difference, a power of 95% and a value for the error margin that is the smallest difference of practical importance are required. According to Reilly (2006) a power of 80% is recommended for research and 95% for validation (Reilly 2006). The "power and sample size" feature of Minitab provides a means for checking if the sample size is sufficiently large.
- ANOVA (ANalysis Of Variance) is used to compare mean scores of more than two groups. ANOVA analysis determines if a specific factor has an effect on the results. For example, in this study ANOVA is used to test the effect of engineering discipline or role on mathematics usage. ANOVA tests the null hypothesis which is that the means among a number of groups are equal, under the assumption that the sampled populations are normally distributed. A single-factor experiment considers the effect of one factor on a response while excluding other factors that could impact the response. ANOVA involves comparing variability of observations within a group about the group mean with variability between the group means.

ANOVA calculates a ratio of mean squares between groups and mean squares within groups (F). If the null hypothesis is true, this ratio would equal 1 as both are estimates of population variance. A large F indicates that the sample means vary more than expected and this produces a small p-value (p-value is the probability of obtaining the data if the null hypothesis is true). If the p-value is less than 5%, the null hypothesis is rejected in favour of the alternative hypothesis that the factor does have an effect on the response. Minitab produces ANOVA tables showing Source (of variation), DF (degrees of freedom), SS (sum of squares of the deviations), MS (mean square deviation), F (variance ratio), p (probability of obtaining the data if the factor has no effect on the response) and R-Sq(adj) (proportion of the variation in the response that is explained by the model under consideration). As in the case of the paired t-test, in hypothesis testing, samples are used to make inferences about populations and this can lead to two different types of errors. A type 1 error occurs if the null hypothesis is rejected when it is true and a type 2 error occurs if the null hypothesis is not rejected when it is false. If the sample size is too small, it may contain insufficient evidence to reject a false null hypothesis and such a test lacks power. There is therefore a requirement to check if the sample size is sufficiently large using the "power and sample size" feature of Minitab. ANOVA is based on three assumptions: errors are independent of the factor; errors are normally distributed and errors have uniform variance. ANOVA is insensitive to violations of the normality assumption if the sample sizes are large. Reilly (2006) states that the requirement that errors are independent is "crucial", the other two assumptions are "less important" and ANOVA works well if the latter two are violated (Reilly, 2006). In this study errors are independent as the error variance does not depend on the factor level (engineering role or engineering discipline). While individual engineers may have different levels of mathematics usage, the variation is the same for all engineers. A two-factor experiment considers the effect of two factors on a response for example the effect of engineering discipline (civil, electronic, mechanical etc.) and engineering role (design, maintenance, production etc.) on mathematics usage. In Minitab, two-way analysis of variance is conducted using the General Linear Model analysis. In a two level factorial design, the General Liner Model calculates a p-value for each of the two factors and also for the interaction of the factors. If the interaction p-value is less than 5% then there is an interaction effect. When interaction is present it is not possible to conclude that one factor has a greater effect than another factor as the response depends on the value of the other factor.

The complete survey data analysis is included in Appendix 4, in Volume 2 of this thesis. The survey findings are discussed in Chapter 5.

4.7 SUMMARY

A survey questionnaire investigating the role of mathematics in engineering practice and whether there is a relationship between students' experiences with school mathematics and their choice of engineering as a career was designed using Adobe Acrobat X Pro software. Following testing for clarity of the instrument, relevance to the research questions, time to complete and efficient operation of the software the survey questionnaire and the supporting survey INFO document were distributed by direct email to 5,755 (424 women) Chartered Engineers by Engineers' Ireland. The response rate of 365 (39 women) engineers is within the sample size required for precision to within 0.15 units (on a Likert scale with five outcomes) and 95% probability that the findings from the survey questionnaire represents the population of Chartered Engineers in Ireland. Analysis of survey data was conducted using Minitab software and the validity of the statistical analysis is based on the assumption that a random sample of engineers participated in the survey.

CHAPTER 5: SURVEY FINDINGS

5.1 INTRODUCTION

This chapter presents the results of a survey of practising professional engineers. The purpose of the survey is to assist in answering the following questions:

1. What is the role of mathematics in engineering practice?
    a) How can mathematics usage in engineering practice be measured?
    b) How do engineers use mathematics in their work?
    c) What motivates engineers to engage, or not, with mathematics?

2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?
    a) To what degree do students' feelings about mathematics influence engineering career choice?
    b) What factors in mathematics education influence students' affective engagement with mathematics?

The role of mathematics in engineering practice concerns engineers' use of mathematics and engagement with mathematics in their work. In the survey questionnaire mathematics usage is categorised according to five usage types: *reproducing; connecting, mathematising; thinking; and engaging.*

As discussed in Chapter 4 *curriculum mathematics* is measured with reference to three dimensions: domain; level; and usage type. There are 5 domains, 5 levels and 3 usage types, Table 5-1. This corresponds to 75 domain-level-usage combinations for *curriculum mathematics*.

Five Content Domains	Five Academic Levels	Three Usage Types
Statistics and probability (D1)	Junior secondary (A1)	Reproducing (T1)
Geometry and trigonometry (D2)	Intermediate secondary (A2)	Connecting (T2)
Number (D3)	Senior secondary (A3)	Mathematising (T3)
Algebra (D4)	Engineering (A4)	
Functions (D5)	B.A./ B.Sc. (A5)	

Table 5-1: *Curriculum mathematics* dimensions.

Engineers' *thinking* usage is measured for different stages of their careers and in an open question engineers are asked to identify the modes of *thinking* they use in their work. Engineers' *engaging* usage is identified as the value of seeking a mathematical approach in engineering practice, engineers' self-efficacy in mathematics and their feelings about using mathematics in their work.

The relationship, if any, between students' experiences with school mathematics and their choice of engineering as a career concerns: the influence of the engineers' feelings about mathematics on their choice of engineering as a career; identifying engineers' experiences, aptitudes and factors within and outside of school that contributed to their interest in and learning of mathematics; and investigating how young people's affective engagement with mathematics could be improved.

The findings are presented in more detail under the following headings:

Page number

5.2 PERCEIVED VALUE OF HIGHER LEVEL LEAVING CERTIFICATE MATHEMATICS IN ENGINEERING PRACTICE

Question: Do you agree that you could perform satisfactorily in your current job without higher level Leaving Certificate mathematics?

Sample size: 365

5.2.1 Engineers' Work Performance without Higher Level Leaving Certificate Mathematics

Results: See results plots in Figures A4-1 and A4-2, Appendix 4, Volume 2.

Almost a third (32.2%) of the engineers who participated in the survey presented that they could perform satisfactorily in their current work without higher level Leaving Certificate mathematics to the degree of "strongly agree" and "agree". Over half (58.4%) disagree (disagree and strongly disagree points on the Likert scale) that they could do their work satisfactorily without higher level Leaving Certificate mathematics, Figures A4-1.

The 95% confidence interval plot for the mean value of the engineers' responses to the question if they agreed that they could perform satisfactorily in their current job without higher level Leaving Certificate mathematics is 3.45 Likert units[17], based on the 5 point Likert: scale 1 = "not at all"; 2 = "very little"; 3 = "a little"; 4 = "quite a lot"; and 5 = "a very great deal", Figures A4-2.

5.2.2 Impact of Engineering Discipline and Role on Perceived Value of Higher Level Leaving Certificate Mathematics in Engineering Practice

Sample size: See plot of engineering disciplines and roles, Figure A4-3, Appendix 4.

The three main engineering disciplines included in the sample size are civil, electronic/ electrical and mechanical. The five main engineering roles that comprise

[17] Likert units: Units on 5 point Likert scale, 1 = "strongly agree", 2 = "agree", 3 = "uncertain", 4 = "disagree", 5 = "strongly disagree".

these disciplines are production, management/ project management, maintenance, education, design/ development.

Results: See results plots in Table A4-1 and Figure A4-4, Appendix 4, Volume 2.

General linear model analysis shows that the interaction p-value is less than 0.05 indicating an interaction effect. This means that the effect of engineering discipline or engineering role depends on the other factor (engineering role and discipline respectively), Table A4-1.

Discussion:

Of the 365 engineers surveyed, almost a third (32.2%) of the engineers presented that they could perform satisfactorily in their current work without higher level Leaving Certificate mathematics. The overall mean value of the engineers' response in the 95% confidence interval plot is 3.45 Likert units. Overall engineers are between "uncertain" and "disagree" in their views that they could perform satisfactorily in their job without higher level Leaving Certificate mathematics, Figure A4-2.

An analysis by engineering discipline (e.g. civil, electrical/electronic and mechanical) or by engineering role (e.g. design/ development, education, maintenance, management/ project management and production) produced an interaction p-value of 0.035 (< 0.05) confirming that the effect of engineering discipline or engineering role on engineers' views about doing their job without higher level Leaving Certificate mathematics depends on the other factor, Table A4-1. In the case of mechanical engineers, it is seen in Figure A4-4 that their work performance without higher level Leaving Certificate mathematics is particularly dependent on their role with mechanical engineers working in production roles having especially low levels of response (degree they could do their work without higher level Leaving Certificate mathematics) and mechanical engineers working in education roles having especially high levels of response. Levels shown by engineers in other disciplines were more or less unrelated to their roles, Figure A4-4.

5.3 *CURRICULUM MATHEMATICS* USAGE IN ENGINEERING PRACTICE

Question: To what extent have you used *curriculum mathematics* in the last 6 months?

Sample size: 365

5.3.1 Engineers' Mean *Curriculum Mathematics* Usage

Results: See results plot in Figure A4-5, Appendix 4, Volume 2.

Discussion:

The engineers' *curriculum mathematics* usage was measured for each of: 5 domains (Statistics and probability, Geometry and trigonometry, Number, Algebra and Functions); 5 academic levels (Junior secondary, Intermediate secondary, Senior secondary, Engineering and B.A / B.Sc.); and 3 usage types (Reproducing, Connecting and Mathematising). Each engineer presents 75 domain-level-usage combinations of mathematics usage using a five point Likert scale (1 = Not at all; 2 = Very little; 3 = A little; 4 = Quite a lot; and 5 = A very great deal).

Engineers' mean mathematics usage for the 75 domain-level-usage combinations of *curriculum mathematics* is 2.73 Likert units[18]. For the entire *curriculum mathematics* spectrum (ranging from Junior Certificate ordinary level to level 8 engineering, arts and science mathematics,) practising engineers rate their *curriculum mathematics* usage is in the interval "very little" to "a little" as illustrated in the 95% confidence interval plot, Figure A4-5.

5.3.2 Engineers' *Curriculum Mathematics* Usage by Domain

Result: See results plot in Figure A4-6, Appendix 4, Volume 2.

Discussion:

[18] Likert units: Units on 5 point Likert scale, 1 = "not at all", 2 = "a little", 3 = "very little", 4 = "quite a lot", 5 = "a very great deal".

The *number* domain (D3) has the highest usage and *functions* (D5) has the lowest usage in engineering practice, Figure A4-6. Engineers rate their usage of the number domain slightly above “a little” and for each of the other four domains engineers rate their mean *curriculum mathematics* usage in the interval “very little” to “a little”.

5.3.3 Engineers’ *Curriculum Mathematics* Usage by Academic Level

Results: See results plots in Figures A4-7 and A4-8, Appendix 4, Volume 2.

Discussion:

Engineers’ usage of *curriculum mathematics* decreases by increasing academic level and usage of B.A./ B.Sc. mathematics is lower than for the other four academic levels. Average mathematics usage for all academic levels is in the range between “very little” and “a little”, Figure A4-7.

Further analysis of *curriculum mathematics* usage by academic level shows that 64.4% of engineers use higher level Leaving Certificate mathematics to the degree of “a little”, “quite a lot” or “a very great deal”. Correspondingly, 78.9% of engineers use Junior Certificate ordinary level mathematics; 64.7% of engineers use Leaving Certificate ordinary level mathematics; 57.3% of engineers use engineering mathematics; and 41.4% of engineers use B.A./ B.Sc. mathematics “a little”, “quite a lot” or “a very great deal”. The mathematics usage figure for higher level Leaving Certificate mathematics is in broad agreement with the result in Figure A4-1 whereby 32.2% of the engineers surveyed stated that they could perform satisfactorily in their current work without higher level Leaving Certificate mathematics.

5.3.4 Engineers’ *Curriculum Mathematics* Usage by Usage Type

Results: See results plot in Figure A4-9, Appendix 4, Volume 2.

Discussion:

Engineers' mean *curriculum mathematics* usage is in the interval "very little" to "a little" for each of the three usage types and decreases for increasing usage type, Figure A4-9.

5.3.5 Effect of Engineering Discipline and Role on *Curriculum Mathematics* Usage

Sample size: See sample size plot in Figure A4-10, Appendix 4, Volume 2.

Results: See results plots in Tables A4-2, A4-3, A4-4, A4-5, A4-6, A4-7, A4-8 and A4-9 and in Figures A4-11, A4-12, A4-13, A4-14, A4-15 and A4-16, Appendix 4, Volume 2.

Discussion:

The effect of engineering discipline and role on engineers' *curriculum mathematics* usage is tested for the three main engineering disciplines represented in the study: civil engineers; electronic/ electrical engineers; and mechanical engineers and also for the five main engineering roles: design/ development engineers; education engineers; maintenance engineers; management/ project management engineers; and production engineers, Figure A4-10.

General linear model analysis shows that the interaction of engineering discipline and role has an effect on: (i) mean overall *curriculum mathematics* usage (p-value = 0.032), Table A4-2; (ii) mean overall *statistics and probability* usage (p-value = 0.043), Table A4-3; (iii) mean overall *geometry and trigonometry* usage (p-value = 0.026), Table A4-4; (iv) mean overall *number* usage (p-value = 0.045), Table A4-5; and (v) mean overall *algebra* usage (p-value = 0.029), Table A4-6 as evidenced by interaction p-values less than 0.05 in each case. General linear model analysis of the effect of engineering discipline and role on *functions* usage shows no evidence that engineering, discipline, role or the interaction of engineering discipline and role has an effect on *functions* usage, Table A4-7. [With a p-value greater than 5% in Table A4-7, it was necessary to test for a type 1 error (the null hypothesis is rejected when it is true). The "power and sample size" feature of Minitab checks if the sample size is sufficiently large, Table A4-8. Using a power of 80% (0.8), it is calculated that a sample size of 11 for each discipline-role category is required to correctly reject the null hypothesis. Thus the sample size in this case is too small to state if engineering discipline and role have an effect on engineers' *functions* usage, Figure A4-10.] However given this test lacks power, the available sample size, based on a power of 80% is insufficient to confirm any effect of engineering discipline, or interaction of engineering discipline and role on engineers' *functions* usage, Table A4-8.

Thus the effect of engineering discipline and engineering role on engineers' overall mean *curriculum mathematics* usage, mean *statistics and probability* usage, mean *geometry and trigonometry* usage, mean *number* usage and mean *algebra* usage depends on the other factor. Analysis also shows that mean overall *curriculum mathematics* usage is dependent on the interaction of engineering discipline and role, Table A4-9.

Interaction plots show the impact of the interaction of engineering roles and disciplines on mathematics usage in Figures A4-11, A4-12, A4-13, A4-14, A4-15 and A4-16. Civil engineers, working in production roles, show especially low levels of overall mean *curriculum mathematics* usage compared to civil engineers working in other roles and also compared to other engineering disciplines, especially electronic/ electrical engineers working in production roles. For all disciplines mean overall *curriculum mathematics* is dependent on engineering roles, Figure A4-11. Both electronic/ electrical engineers, working in design and development roles, and civil engineers, working in production roles, show low usage of *statistics and probability* while mechanical engineers working in education roles show high usage of *statistics and probability*. Civil engineers' usage of *statistics and probability* is lower for engineers in production roles compared to civil engineers working in other roles and also compared to other engineering disciplines working in production roles, Figure A4-12. Civil engineers working in production roles show lowest usage of *geometry and trigonometry* while electronic/ electrical engineers working in production roles show the highest usage of *geometry and trigonometry*, Figure A4-13. *Number* usage is particularly dependent on civil engineers' discipline whereby civil engineers, working in both education and especially production roles, show lower usage compared to engineers from the other disciplines working in education and production roles respectively. Civil engineers, working in design/ development and maintenance, show higher *number* usage compared to engineers working in the other disciplines and in design/ development and maintenance roles respectively. It is also evident that both mechanical engineers' and electronic/ electrical engineers' *number* usage is dependent on engineering role, Figure A4-14. Civil engineers, working in production roles, show lowest usage of *algebra* and electronic/ electrical engineers, working in production roles, showed the highest usage of *algebra*. All engineering disciplines' usage of algebra is dependent on engineering role, Figure A4-15.

In another test of the effect of engineering discipline, engineering role and interaction of engineering discipline and role on higher level Leaving Certificate mathematics usage, general linear model analysis shows that the interaction of engineering discipline and role has an effect on engineers' use of higher level Leaving Certificate mathematics, Figure A4-16. The corresponding interaction plot shows that civil, electronic/ electrical and mechanical engineers' use of higher level Leaving Certificate mathematics is dependent on engineers' roles. For example, electronic/ electrical engineers, working in design/ development roles and production roles, show low and high usage levels respectively and mechanical engineers, working in education and maintenance, show high and low usage respectively. Civil engineers working, in production roles, show especially low levels of higher level Leaving Certificate mathematics compared to both electronic/ electrical and mechanical engineers working in production roles, Figure A4-16.

5.4 *THINKING* USAGE IN ENGINEERING PRACTICE

5.4.1 Engineers' Mean *Thinking* Usage

Question: To what extent, with or without direct application of mathematics, did your mathematics training (with its associated modes of thinking and analysis) directly influence your approach to your work?

- In the last 6 months?
- Within 2 years of graduating?
- Within 3-5 years after graduating?
- Within 6 – 10 years after graduating?
- Greater than 10 years after graduating?

Sample size: 365

Results: See results plot in Figures A4-17, Appendix 4, Volume 2.

Discussion:

Overall engineers rate their *thinking* usage in the previous 6 months as "quite a lot" (4.02). Over the lifetime of their engineering careers, engineers' mean *thinking* usage is highest when they are within 2 years of graduating (4.19 Likert units) and lowest when greater than 10 years since graduating (3.89 Likert units). *Thinking* usage reduces when the engineers are within 3 to 5 years since graduating and there are further reductions in *thinking* usage when the engineers are within 6 to 10 years since graduating and greater than 10 years since graduating, respectively, Figure A4-17.

5.4.2 Effect of Engineering Discipline and Role on Engineers' *Thinking* Usage

Sample size: See sample size plot in Figure A4-18, Appendix 4, Volume 2.

Results: See results plots in Tables A4-10 and A4-11, Appendix 4, Volume 2.

Discussion:

The effect of engineering discipline and role on engineers' *thinking* usage in their work (within previous 6 months of survey) is tested for the three main engineering disciplines: civil engineers; electronic/ electrical engineers; and mechanical engineers and also for the five main engineering roles: design/ development engineers; education engineers; maintenance engineers; management/ project management engineers; and production engineers, Figure A4-18.

General linear model analysis shows that neither engineering discipline, engineering role or the interaction of engineering discipline and role has an effect on thinking usage as evidenced by p-values greater than 0.05, Table 5-10 and a sufficiently large sample size, Table A4-11.

5.4.3 Engineers' Modes of *Thinking*

Question: What modes of *thinking*, resulting from your mathematics education, influence your work performance?

Sample size: 365

Results: See results plot in Figure 5-1 and Figure A4-19, Appendix 4, Volume 2.

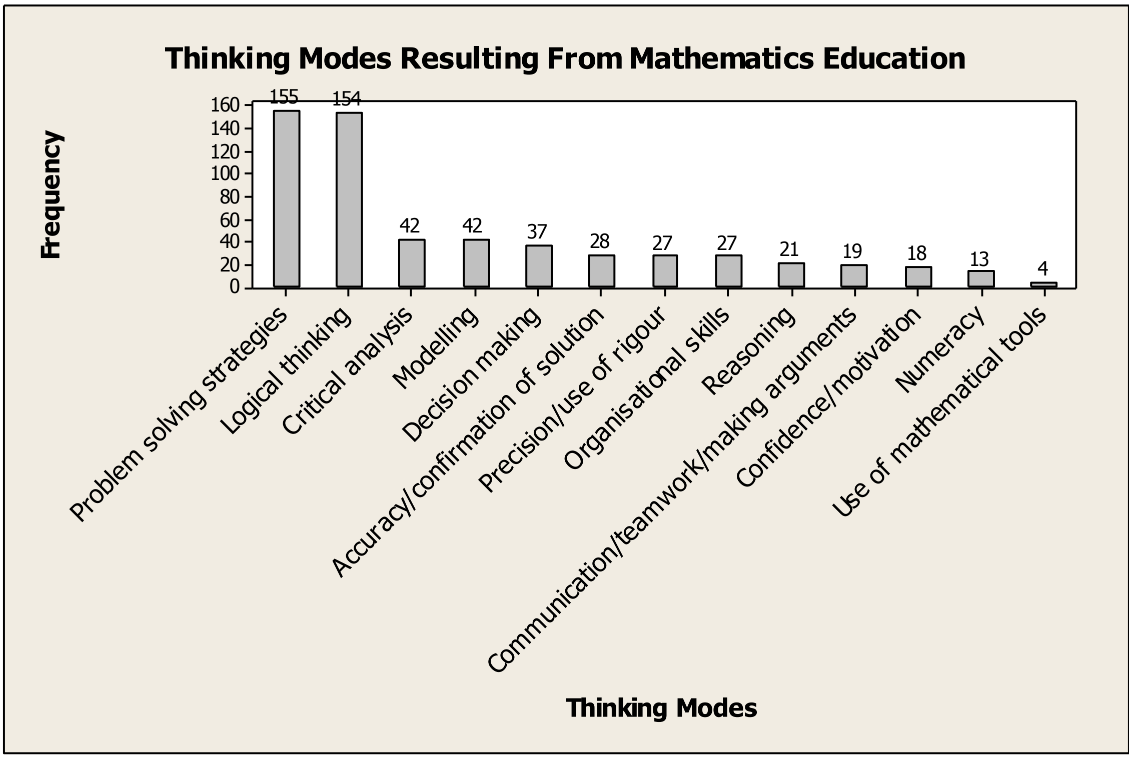

Figure 5-1: Engineers' modes of *thinking*.

Discussion:

An analysis of engineers' modes of *thinking*, resulting from their mathematics education, that influence their work performance, as identified by the engineers in an open question, shows that: problem solving strategies (26.4%), logical thinking (26.2%); critical analysis (7.2%); modelling (7.2%); decision making (6.3%); accuracy/ confirmation of solution (4.8%); precision/ use of rigour (4.6%); organisational skills (4.6%); reasoning (3.6%); communication/ teamwork/ making arguments (3.2%); confidence/ motivation (3.1%); numeracy (2.2%); and use of mathematical tools (0.7%) influence their work, Figure 5-1.

While the engineers identified modes of *thinking* they use in their work, their responses do not give an insight as to how these modes of thinking are used in engineering practice. This is further investigated in the qualitative phase.

5.4.4 Comparison of Engineers' *Thinking* and *Curriculum Mathematics* Usages

Questions:

- To what extent have you used *curriculum mathematics* in the last 6 months?
- To what extent, with or without direct application of mathematics, did your mathematics training (with its associated modes of thinking and analysis) directly influence your approach to your work?

Sample size: 365

Results: See results plot in Table A4-12, Appendix 4, Volume 2.

Discussion:

Paired t-test analysis shows that there is a difference, as evidenced by p-value = 0.000, between the average of engineers' *thinking* usage (within previous 6 months of survey) and the average of engineers' overall mean *curriculum mathematics* usage (also within previous 6 months of survey). The magnitude of this difference lies between 1.15 and 1.43 Likert units and it is the amount by which engineers' *thinking* usage is greater than their *curriculum mathematics* usage, Table A4-12.

5.5 *ENGAGING* WITH MATHEMATICS IN ENGINEERING PRACTICE

5.5.1 Degree a Specifically Mathematical Approach is Necessary in Engineers' Work

Question: With regard to your work in the last 6 months, to what degree was a specifically mathematical approach necessary?

Sample size: 365

Results: See results plots in Figures A4-20, A4-21 and A4-22 and Tables A4-13 and A4-14, Appendix 4, Volume 2.

Discussion:

Almost two thirds (64.6%) of the engineers present that a specifically mathematical approach is necessary in engineers' work either "quite a lot" or "a very great deal". A further 21.1% rate the need for a mathematical approach as "a little" and 7.4% of engineers say that they do not ("not at all") require a mathematical approach in their work, Figure A4-20. The overall mean rating for the degree a mathematical approach is necessary in engineering practice is 3.68 Likert units which is in the range "a little" to "quite a lot", Figure A4-21.

Paired t-test analysis shows that there is a difference between the average degree a mathematical approach is necessary in the engineers' work and both their average *curriculum mathematics* usage, Table A4-13 and their average *thinking* usage, Table A4-14 as evidenced by p-value = 0.000 in both comparisons. Engineers rate the necessity of a mathematical approach in their work considerably greater (by 0.80 to 1.11 Likert units) than their own *curriculum mathematics* usage and less than their *thinking* usage (by 0.25 to 0.45 Likert units).

Some of the reasons given by the engineers in an open question as to why a specifically mathematical approach is not necessary in their work relate to management roles not requiring a mathematical approach, the common sense nature of engineering, more human problems in engineering practice than mathematical problems, engineers not having enough time to take a mathematical approach and fear of alienating work colleagues. An example of a response relating to management roles is: "I operate generally in management now with limited use of maths". Other engineers present that: "engineering is mainly common sense"; "work is of a pragmatic nature"; "my business problems tend to be more human or business-process oriented"; "maths takes too much time"; and "sadly, an overtly mathematical approach may alienate the people you need to do the work".

Engineers who rate the necessity of a mathematical approach in their work as "a little" say that mathematics is "a small component of the overall work" and that "spread sheet modelling" and "computer aided network modelling software" perform much of the mathematics required in their work. For other engineers a mathematical approach is required to "understand reports" and to analyse "customer tender documents".

For the engineers who rate the necessity of a mathematical approach in their work as "quite a lot" or "a very great deal", mathematics enables them to obtain "objective" solutions to problems, to better support their arguments and to logically plan and execute projects. According to these engineers, mathematics is required: for "objective evaluation of a variety of parameters"; "to ensure that the results had sound mathematical reasoning behind them rather than a best guess"; "to calculate an absolute value in a rigorous manner that was open to public scrutiny"; "to build a convincing argument"; to ensure that a demonstrably fair decision was reached"; and "to determine the necessary and the correct quantities of materials required for a variety of different work types as accurately as possible, tens of millions of € involved". Many engineers present that data analysis and statistics are critical in their work. One engineer states that there are often "too many variables, each influencing others to varying degrees, to have managed and interpreted data without a mathematical approach".

A diversity of engineering types highlight the necessity of a mathematical approach in their work, some of these types include: Construction – "part and parcel of construction"; Electrical – "electrical load analysis required for most projects"; Structural – "all structural design requires the use of equations"; Biomedical – "development of mathematical models necessary to describe biological processes";

Production – "we work in sample rates, sample inspections, and then make inferences to the entire population"; Design – "although many design operations are carried out using computer software it is still necessary to understand how solutions are arrived at"; Engineering Management – "I supervise engineers involved in structural, mechanical, process and chemical disciplines, all of which involve numerical simulation, modelling and analysis"; Management – "compiling annual budgets and forecasting requirements"; and Business/ Financial – "detailed product licensing, use of ratios, redundancy calculations, special deals, products with unique cost patterns over its life cycle, predicting and scaling operating costs are all very important in business and financial planning".

In summary, overall, 85.7% of engineers rate the necessity of a specifically mathematical approach in their work as either "a little", "quite a lot" or "a very great deal". The degree a mathematical approach is necessary in engineering practice is greater than engineers' average *curriculum mathematics* usage. While some engineers present that management roles do not require a mathematical approach, it is also noted that 87% of engineers in management roles rate the necessity of a specifically mathematical approach in their work as "a little", "quite a lot" or "a very great deal", Figure A4-22. Affective factors are very evident in the engineers' reasons relating to the degree of necessity of a mathematical approach in their work. In particular task values (why should I do the task?), which are predictors of achievement behaviour, (Schunk et al. 2010) are evident. Negative task values include: work not requiring a mathematical approach; the time cost; and the availability of computer software. Positive task values include: cost savings resulting from mathematical accuracy; and data analysis which, with large numbers of variables and large population sizes, can only be done mathematically. Sociocultural influences are also evident in the engineers' responses. There is a strong sense that engineers need to convince their colleagues about the validity of mathematical solutions in that engineers' work "was open to public scrutiny" and that "objective" solutions are needed to "support their arguments".

The survey methodology doesn't give a deep picture of the necessity of taking a mathematical approach in engineering practice and the qualitative study, which is the second phase of this explanatory mixed methods approach, is required to give a more in-depth analysis.

5.5.2 Degree Engineers Seek a Mathematical Approach

Question: With regard to your work in the last 6 months, to what degree did you actively seek a mathematical approach?

Sample size: 365

Results: See results plots in Figures A4-23, A4-24 and A4-25 and Tables A4-15, A4-16, A4-17 and A4-18, Appendix 4, Volume 2.

Discussion:

Almost two thirds (63.3%) of engineers say that they actively seek a mathematical approach either “quite a lot” or “a very great deal”. A further 19.5% seek a mathematical approach “a little” and 9.3% of engineers say that they do not (“not at all”) seek a mathematical approach, Figure A4-23. The overall mean rating for the degree engineers actively sought a mathematical approach in their work is 3.62 Likert units, which is in the range “a little” to “quite a lot”, Figure A4-24.

With a p-value = 0.093 and due to a risk of a type 1 error, it cannot be asserted that the mean difference between the degree engineers seek a mathematical approach and the degree a specifically mathematical approach is necessary in their work is zero, Table A4-15. Using the “power and sample size” feature of Minitab, a sample size of 864 engineers would be required to verify if there is a difference between the degree engineers seek a mathematical approach and the degree a specifically mathematical approach is necessary in their work, Table A4-16.

Paired t-test analysis shows that there is a difference between the average degree engineers seek a mathematical approach and both engineers’ average *curriculum mathematics* usage, Table 5-17 and engineers’ average *thinking* usage, Table 5-18, as evidenced by p = 0.00 in both comparisons. The engineers rate the degree they actively seek a mathematical approach in their work considerably greater (by 0.73 to

1.05 Likert units) than their *curriculum mathematics* usage and less than their *thinking* usage (by 0.26 to 0.51 Likert units).

Some of the reasons given by the engineers as to why they didn't seek a mathematical approach in their work relate to management roles not requiring mathematics, common sense being more important in engineering, the availability of sufficient ready-made solutions and that "taking a mathematical approach may be risky and slow".

The reasons given by engineers for actively seeking a mathematical approach relate to using data to make decisions, using a logical framework for problem solving, confidence in mathematical solutions, greater understanding and comfort when taking a mathematical approach. Some engineers are of the view that "there is no other way" of solving engineering problems. For example, "mathematics was the only way to verify that the solutions were feasible" and "it was required by the client". There is a strong view that mathematics is "universally accepted and understood within the business" and that "mathematics is useful for explaining results to others". Mathematics is also "the quickest way to resolve complex problems".

According to the engineers, the majority of engineering decisions are based on data analysis and one engineer is of the view that "without data analysis everything is an anecdote". Many engineers demonstrate their confidence in mathematical solutions. Examples include: "maths provides certainty"; "a mathematical approach leaves little room for error"; "with maths product design is safe and will perform as required by product legislation"; "maths gives me confidence in my proposed solutions"; "it removes doubt and debate"; and "I use mathematics to satisfy myself and investors"; and "maths is needed when accuracy of work is crucial".

Many engineers appeared very comfortable using mathematics. Examples include: "I am comfortable with a mathematical approach"; "maths removes the subjective comment or indeed conflict"; "maths brought clarity to the solution being offered"; "maths helps me understand engineering problems"; "I am comfortable building a case mathematically"; "I felt comfortable using maths and mathematical evidence cannot be disputed"; "I always take an analytical approach when the result is subject to public scrutiny".

From the data gathered from the open question in the survey, engineers' reasons for seeking a mathematical approach are driven by affective variables. Negative task values such as the availability of sufficient ready-made solutions and "taking a mathematical approach may be risky and slow" are some reasons given by engineers for not actively seeking a mathematical approach. On the other hand, positive task values such as "the quickest way to resolve complex problems", "there is no other way", "maths is needed when accuracy of work is crucial" and "mathematics is useful for explaining results to others" are associated with engineers who actively seek a mathematical approach in their work.

In summary, 82.8% of engineers actively seek a mathematical approach either "a little", "quite a lot" or "a very great deal". Engineers rate the degree they actively seek a mathematical approach in their work considerably greater (by 0.73 to 1.05 Likert units) than their *curriculum mathematics* usage and less than their *thinking* usage (by 0.29 to 0.51 Likert units). This difference reinforces the importance of *thinking* usage in engineering practice. While some engineers present that engineers in management roles do not require a mathematical approach, 91.4% of engineers in management/ project management roles rate the degree they seek a mathematical approach as "a little", "quite a lot" or "a very great deal", Figure A4-25.

Positive affective experiences are very evident among the engineers who actively seek a mathematical approach. The engineers articulate the usefulness of mathematics in engineering practice and they particularly note the value of mathematics to the client, in meeting safety criteria, in supporting decisions and in satisfying the engineers themselves. Engineers' comfort with mathematics and mathematical solutions is particularly noticeable in the engineers' responses. Engineers' self-efficacy which is "people's judgements of their capabilities to organise and execute courses of action required to attain designated types of performances" (Bandura, 1986), are also apparent in the engineers' responses. For example one engineer demonstrates his confidence in mathematics by saying "I always take an analytical approach when the result is subject to public scrutiny". This comment also suggests a lack of confidence in alternative reasoning approaches and a lack of self-confidence in representing other forms of reasoning to an audience.

The engineers demonstrate a strong need to stand over their solutions and to convince their colleagues about the correctness of their solutions and mathematics provides engineers with this security.

5.5.3 Degree Engineers Enjoy Using Mathematics

Question: With regard to your work in the last 6 months, to what degree did you enjoy using mathematics?

Sample size: 365

Results: See results plots in Figures A4-26 and A4-27 and Tables A4-19, A4-20 and A4-21, Appendix 4, Volume 2.

Discussion:

Almost three quarters (74.0%) of engineers presented that they enjoy using mathematics in their work either "quite a lot" or "a very great deal". A further 15.9% rate the degree they enjoy using mathematics in their work as "a little" and 3.8% of engineers say that they do not ("not at all") enjoy using mathematics in their work, Figure A4-26. The overall mean rating of the degree engineers enjoy using mathematics in their work is 3.89 Likert units which is in the range "a little" to "quite a lot", Figure A4-27.

Paired t-test analysis shows that there is a difference between the average degree engineers enjoy using mathematics in their work and both engineers' average *curriculum mathematics* usage, Table A4-19 and the degree engineers actively seek mathematical approach in their work, Table A4-20 as evidence by p-values = 0.000 in both cases. The engineers rate the degree they enjoy using mathematics in their work considerably greater (by 1.01 to 1.03 Likert units) than their *curriculum mathematics* usage and also greater (by 0.17 to 0.37 Likert units) than the degree they actively seek a mathematical approach in their work.

Paired t-test analysis also shows that there is a difference between engineers' enjoyment using mathematics in their work and their enjoyment of school mathematics, Table A4-21. Engineers' enjoyment of using mathematics in work is less (by 0.33 to 0.12 Likert units) than their enjoyment of school mathematics.

The main reason given by engineers who don't enjoy using mathematics in their work relates to their experience of mathematics in secondary school. Examples include: "poor teacher"; "it's my in built hatred of mathematics from secondary school"; and "I had no idea how school maths related to the real world".

The main reasons engineers give for their enjoyment of using mathematics relates to engineers' "satisfaction" and "sense of achievement when using mathematics to solve a problem". The majority of engineers note "the satisfaction of a result" and "the enjoyment of getting the mathematical result". One engineer says "I love the challenge in solving problems mathematically. Other engineers present that "it's very satisfying to express a real life phenomenon in mathematical terms" and "describing a real world problem mathematically, solving the math problem then implementing a technological solution is very satisfying".

For many engineers, mathematics "just seems natural" and it is "part of who" they are. Mathematics is "how engineers think" and engineers are "just hard wired that way". Engineers present a sense of confidence and familiarity when using mathematics. They are comfortable with "clear and concise answers". One engineer says that "to reduce apparently complex processes to a series of mathematical forms is a great feeling".

Engineers show a preference for "the solidity of numbers" over "report writing and answering emails". For example, engineers say: "maths has always been easier for me than English words"; "a number can say much more than a word"; "it is easier to communicate using mathematics than talking", "I prefer to present information in tables, graphs and trends than written words"; "I like a 100% right answer rather than the ambiguity of non-mathematical solutions"; "maths gives answers that I can take action on rather than discussions, hearsay, rumours"; "numbers will always give a more accurate assessment of a situation than discussions" and "there is a beauty and clarity to using numbers".

In summary 89.9% of the engineers enjoy using mathematics in their work. The average degree engineers enjoy using mathematics in their work is significantly greater than their average *curriculum mathematics* usage and a little greater than the degree engineers actively seek a mathematical approach in their work. Engineers' enjoyment of using mathematics in work is somewhat less than their enjoyment of school mathematics.

From the engineers' responses to the open question in the survey it is apparent that affective memories play a large role in engineers' enjoyment of mathematics in their work. Memories of school mathematics are the main reason engineers do not enjoy using mathematics in work. One engineer describes this as his "in built hatred of mathematics from secondary school".

For the engineers who enjoy using mathematics in work, there is a sense that mathematics is "part of who" they are. Their mathematics education and usage of mathematics in work has "hard wired" them to think mathematically. Engineers' positive affective memories include: their "satisfaction"; "sense of achievement when using mathematics to solve a problem"; reducing "apparently complex processes to a series of mathematical forms is a great feeling"; mathematics "just seems natural"; and "there is a beauty and clarity to using numbers". Engineers' self-efficacy and confidence also contributes to their enjoyment of mathematics particularly when compared to non-mathematical activities. For example, engineers maintain that they are much happier working with "numbers" compared to "words".

5.5.4 Degree Engineers Feel Confident Dealing with Mathematics

Question: With regard to your work in the last 6 months, to what degree did you feel confident dealing with mathematics?

Sample size: 365

Results: See results plots in Figures A4-28 and A4-29 and Tables A4-22, A4-23, A4-24, A4-25, A4-26, A4-27 and A4-28, Appendix 4, Volume 2.

Discussion:

Over 80% (80.6%) of the engineers say that they feel confident dealing with mathematics in their work either "quite a lot" or "a very great deal". A further 14.2% of engineers rate their confidence using mathematics in their work as "a little" and 2.5% of engineers say that they are "not at all" confident using mathematics in their work, Figure A4-28. The overall mean rating of the degree engineers feel confident dealing with mathematics in their work is 4.03 Likert units which is just above the point "quite a lot" on the 5 point Likert scale, Figure A4-29.

Paired t-test analysis shows that there is are differences between the average degree engineers feel confident dealing with mathematics in their work and (i) engineers' average overall *curriculum mathematics* usage ratings, (ii) the average degree engineers actively seek a mathematical approach and (iii) the average degree engineers enjoy using mathematics in their work. The engineers rate the degree they feel confident dealing with mathematics in their work: considerably greater (by 1.16 to 1.43 Likert units) than their overall *curriculum mathematics* usage, Table A4-22; greater (by 0.31 to 0.52 Likert units) than the degree they actively seek a mathematical approach in their work, Table A4-25; and also greater (by 0.07 to 0.23 Likert units) than the degree they enjoy using mathematics in work, Table A4-26.

With a p-value = 0.874, it cannot be asserted whether or not there is a difference between the average degree engineers feel confident dealing with mathematics in work and their average *thinking* usage (in the 6 months previous to the survey), Table A4-23. A sample size of 1,309 is required to verify this, Table A4-24.

Similarly, with a p-value = 0.109, it cannot be asserted that there is a difference between the average degree engineers feel confident dealing with mathematics in work and their enjoyment of school mathematics Table A4-27. A sample size of 1,278 is required to verify this, Table A4-28.

While overall engineers are very confident dealing with mathematics, the engineers maintain that mathematics education and "practice" are key factors in engineers' confidence dealing with mathematics in work. Engineers attribute "poor grounding" and "lack of usage" as reasons for low confidence while a "good basis from school and university and practice in industry" is consistent among engineers who demonstrated high confidence. As well as "poor grounding in early schooling", for some engineers the "mathematics learned at college was very theoretical" and they don't "know how to convert this to the real world". Some engineers are concerned about their "lack of mathematics usage" in their work and one engineer said "I would be concerned about how much I have retained over the years".

There is a sense that some engineers are aware "of the risk involved using maths in work" and that using mathematics in work is "too time consuming".

Many engineers, while confident working within their "limits", are not confident using mathematics outside their "comfort zone". For example, some engineers say: "I regard myself as numerate and logical in approach however lacking in in-depth experience of mathematics"; "I feel confident in using the tools that I use frequently but a little slow to tackle the more difficult techniques"; "I have only tackled problems mathematically if I felt at least reasonably confident of finding a solution"; and "I feel confident in the capabilities I have and aware of my limitations".

There is a sense that for many engineers, including engineers with reasonably high confidence and high *curriculum mathematics* usage, they "would like to have a higher level of maths". Some engineers respond to this feeling by avoiding mathematics in their work. For example, one engineer says "I was comfortable with basic analytical thought but not with more advanced mathematical concepts and I prefer to leave it to others with these skills". Other engineers respond by engaging in mathematics learning or up-skilling which is consistent with motivation theory in Chapter 3 whereby it is maintained that self-efficacy strongly influences the choices people make, the effort they expend and how long they persevere in the face of challenge (Schunk et al. 2010). Engineers, who engage in self-teaching when they encounter a mathematics challenge in work, say: "if more detailed application was required at times, it was possible to refer to text books"; "I am satisfied I know where the information is should I need to draw on it"; "I still have to revise and brush up on certain aspects of maths"; and "I know where to look for clarification in areas where I am rusty".

Many engineers with high confidence in using mathematics also show high confidence in mathematics solutions and in the "logical and objective nature of maths". These engineers note the need to "check if solution is correct" and they are also of the view that "there is no reason for ambiguity in maths, there is only a right or wrong answer".

The majority of engineers with high confidence attribute their confidence to "a good grounding in mathematics", their comments include: "a good (enthusiastic) teacher is always a good start"; "I was taught in a way that made me sure of what I was doing"; "maths has been an in-depth part of my education"; "good training in school (and college) means I never doubt my mathematical ability"; "good understanding after completing education at degree level"; "engineering training brings with it a confidence in using mathematics"; and "the level of maths required in work is a lot less than I coped with at university".

The high confidence engineers have positive affective memories of school mathematics and high mathematical self-efficacy. Examples of what they say include: "I was in the habit of getting 100% in maths and maths-based exams in school and college"; "I never saw maths as a difficult subject from the start of schooling right through 3rd level education"; "I have always been quite good at maths"; "I have a self-belief and track record of producing 'right' answers"; "maths has worked for me in the past and I expect maths to get the job done for me in the future"; and "not too many mistakes, touch wood".

In summary, 84.8% of the engineers say that they feel confident dealing with mathematics in their work to the degree of either "a little", "quite a lot" or "a very great deal. The engineers rate the degree they feel confident dealing with mathematics in their work considerably greater than their overall *curriculum mathematics* usage. There is also a gap (0.31 to 0.52 Likert units) between the degree engineers feel confident dealing with mathematics in their work and the degree they actively seek a mathematical approach in their work.

From the engineers' responses to the open question in the survey, it is evident that the engineers' "grounding" in mathematics and subsequent usage are two major factors in their confidence to use mathematics in work. Negative task value factors such as questioning the value of "theoretical" mathematics in "the real world", "the risk involved" and the "time" required all contribute to engineers' low confidence using mathematics in work.

One interesting aspect of the engineers' views about their confidence using mathematics in work is the idea that many engineers are confident working within their "limits" and not outside their "comfort level". This is very similar to both Csikszentmihalyi's theory of flow and Vygotsky's theory of the zone of proximal development presented in Chapter 3. Csikszentmihalyi's theory posits that the teacher should keep the ratio between the learner's skills and the challenge within a range called the flow channel so that the learner experiences neither boredom nor anxiety (Csíkszentmihályi 1992). Vygotsky defines the zone of proximal development as "the distance between the actual development level as determined by independent problem-solving and the level of potential development as determined by problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky 1978). Together these theories present that there is both an optimum cognitive level and an optimum affective level for presenting learning challenges to students. Following on from both these theories and the survey data, it appears that individual engineers also use mathematics within their own optimum level which they call their "comfort level".

For many engineers high mathematical self-efficacy begins in school when they learn to check their answers and when they were also "in the habit of getting 100% in maths and maths-based exams". In addition to developing a "good understanding" of mathematics, the engineers identify confidence as an important learning outcome of mathematics education. It is interpreted here that the value of mathematics education for practising engineers is their confidence to use mathematics after school and university or that without the confidence to use mathematics, the value of engineers' mathematics education in engineering practice is greatly reduced. This interpretation is supported by the engineers views that low and high confidence with workplace mathematics stems from poor and good "grounding" respectively of school mathematics and that when engineers encounter new mathematical problems in work, the low confidence mathematics engineers prefer "to leave it to others with these skills" while high confidence mathematics engineers opt to "revise and brush up" on the required mathematics.

While it cannot be asserted statistically that there is a correlation between the average degree engineers feel confident dealing with mathematics in work and their enjoyment of school mathematics, any such relationship will be further investigated in the qualitative phase of this research.

5.5.5 Degree Engineers have a Negative Experience when Using Mathematics

Question: With regard to your work in the last 6 months, to what degree did you have a negative experience when using mathematics?

Sample size: 365

Results: See results plots in Figures A4-30, A4-31 and A4-32, Appendix 4, volume 2.

Discussion:

Just 3.9% of the engineers say that they had a negative experience when using mathematics either "quite a lot" or "a very great deal". A further 16.2% say that had a negative experience when using mathematics "a little" and 77.8% of engineers say that had a negative experience when using mathematics either "not at all" or "very little", Figure A4-30.

The overall mean rating for the degree engineers had a negative experience when using mathematics is 1.76 Likert units which is in the range "not at all" to "very little", Figure A4-31.

The majority of engineers, due to confidence in their mathematical ability and mathematical solutions, say that they did not have any negative experience when using mathematics in the previous six months. Some of the reasons given by the minority of engineers who had a negative experience when using mathematics include: making mistakes, difficult problems, time requirements and communicating mathematics.

It is apparent from the data that engineers don't like making mistakes and that while "checks and balances usually pick up the errors", for some engineers the "quirkiness of computational tools" and their "lack of understanding" and "over reliance of computer analysis" generate errors.

While difficult problems create negative feelings for some engineers, engineers generally relish such challenges. For example, one engineer says "even when a particular problem was very difficult or indeed impossible to solve with the mathematics, the effort was a very positive experience". The engineers' motivation to persist with a difficult problem is noticeable. This is consistent with expectancy-value research in Chapter 3 which substantiates that students with positive self-perceptions of their competence and positive expectancies of success are more likely to perform better, learn more and engage in an adaptive manner on academic tasks by exerting more effort, persisting longer and demonstrating more cognitive engagement (Schunk et al. 2010).

Some engineers note that time is often an issue when solving problems mathematically. Comments about time include: "there are just some areas that I would like to have a better understanding and knowledge which would allow me to make faster decisions"; "occasionally I have spent a long time trying to shoehorn something into mathematical language and failed, which was frustrating"; and "the time allocated to solving the problem did not justify the level of reading required to be up to speed with the mathematical approach".

The greatest reason attributed by the engineers to negative experiences using mathematics relates to communicating mathematics. Examples of this include: "being strictly logical and clinically mathematical on its own usually causes problems when dealing with people"; "I only had negative experiences when dealing with complex mathematics which was not understood by others"; "I would sometimes find that having used maths or statistics to analyse figures or to come to a conclusion would put non-numerically minded colleagues off or confuse them"; "sometimes when I explain basic maths to non-numerical people they turn off"; "wasting my time trying to convince people why things are important" and "sometimes, it can be difficult to influence business decisions, based on complex analysis, just because there are many others who don't have a maths background".

In summary, 77.8% of engineers say they had a negative experience when using mathematics to the degree of either "not at all" or "very little". The engineers cite difficulties communicating mathematics as a major cause of engineers' negative experiences when using mathematics. Their views include: using mathematics "put non-numerically minded colleagues off"; wasting "time trying to convince people why things are important" and "it can be difficult to influence business decisions". This has some resemblance to learning environments whereby student peer networks strongly influence students' academic motivation. Students often select their peer group on the basis of some similarity in values, attitudes or beliefs and students in networks tend to become similar over time which can lead to more or less engagement in school activities. In Chapter 3 it is reported that the desire for peer approval effects students choice of goals (Schunk et al. 2010). When students progress from engineering education to engineering practice, they move from a world of mathematically competent people to a more diverse environment where mathematics is not as obvious and where there is less time to engage in mathematical analysis. According to motivation theory, such changes in new graduate engineers' sociocultural influences are likely to impact their motivation. It may be that new engineers miss the peer approval associated with doing well in mathematics exams when they use mathematics in engineering practice. Engineers' difficulty communicating mathematics to their colleagues is compounded by the data in section 5.5.3 where engineers present that mathematics is "part of who" they are and that their preferred method of communication is through mathematics. Ironically engineers' task value of mathematics is less in engineering practice compared to engineering education and consequently this could generate negative affective memories. In the engineering education literature in Chapter 2 there is a view that social issues such as communications and team work contribute significantly to the gap between engineering education and engineering practice (Tang and Trevelyan 2009). The engineers' views here suggest that changes in affective influences on graduate engineers when they move from engineering education to engineering practice is also a factor. According to the survey data, the sociocultural influences in engineering practice are considerable given that engineers, who can deal with difficult mathematical problems by expending greater effort, are not as well able to deal with negative experiences they encounter due to their colleagues' lack of mathematics understanding. Overall the engineers demonstrated high affective engagement with mathematics in their work, Figure A4-32.

5.6 SCHOOL MATHEMATICS

5.6.1 Engineers' Enjoyment of School Mathematics

Question: Did you enjoy mathematics in secondary school?

Sample size: 365

Results: See results plots in Figures A4-33 and A4-34, Appendix 4, Volume 2.

Discussion:

80% of the engineers surveyed enjoyed mathematics in school either "quite a lot" or "a very great deal". A further 15.3% of engineers enjoyed school mathematics "a little". Only 4.7% of engineers liked school mathematics either "not at all" or "very little", Figure A4-33. The overall mean value of engineers' enjoyment of school mathematics is 4.11 Likert units which is greater than "quite a lot" on the 5 point Likert scale, Figure A4-34.

Engineers' enjoyment of school mathematics is further investigated in the next section and in the qualitative phase.

5.6.2 Factors Within and Outside of School that Contributed to Engineers' Interest in and Learning of Mathematics

Question: What events, experiences, aptitudes or other factors within and outside of school contributed to your interest in and learning of mathematics?

Sample size: 365

5.6.2.1 Within Primary School

Results: See results plot in Figures 5-2 and Figure A4-35, Appendix 4, Volume 2.

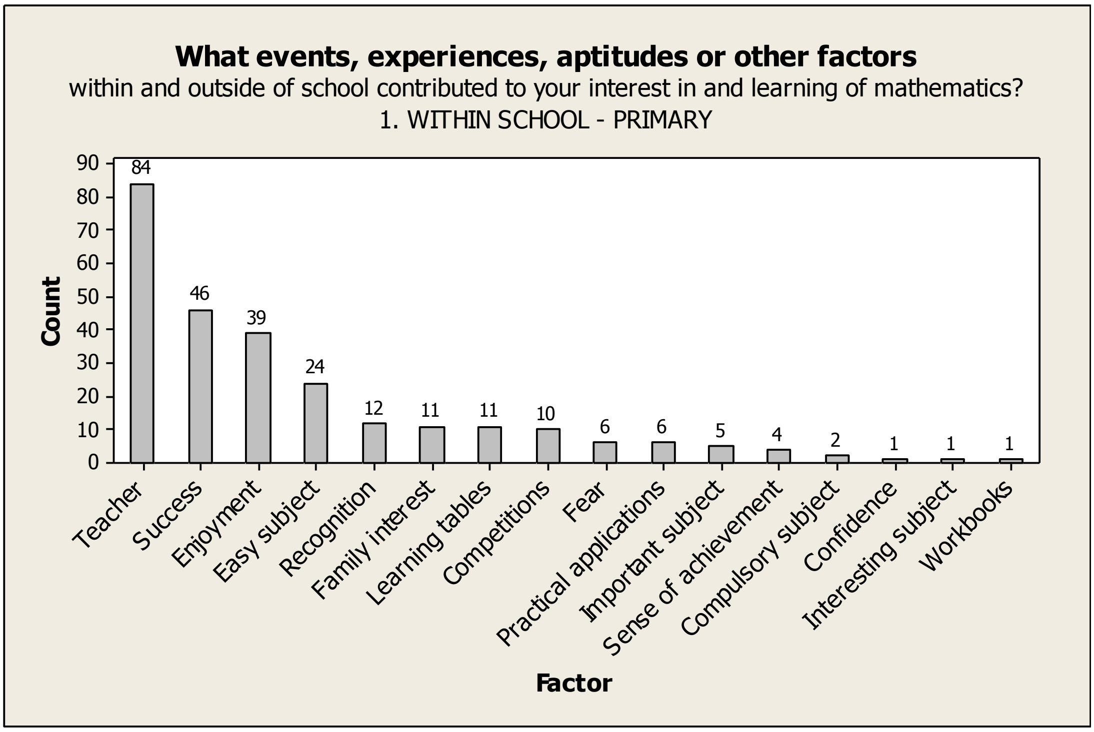

Figure 5-2: Factors within primary school contributing to mathematics learning.

Discussion:

The top three factors contributing to engineers' interest in and learning of mathematics in primary school are teacher, success and enjoyment. According to the engineers' response to the open question, teacher is the main factor contributing to their mathematics education in primary school. Apart from teacher, many of the factors contributing to engineers' primary school mathematics learning are affective variables e.g. success (self-efficacy), easy subject (views); enjoyment (value) and recognition (value), Figures 5-2 and A4-35, Appendix 4. These factors are further investigated in the qualitative phase.

5.6.2.2 Within Secondary School - Years 1 & 2

Result: See results plot in Figure 5-3 and Figure A4-36, Appendix 4, Volume 2.

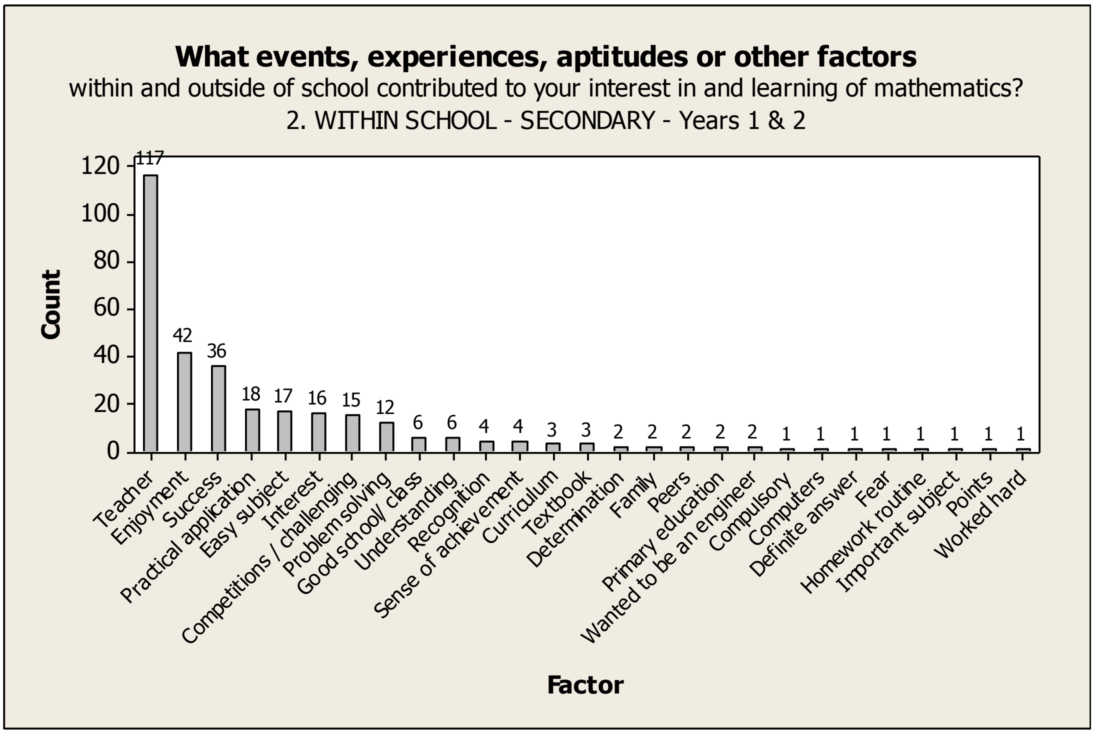

Figure 5-3: Factors within secondary school (years 1 & 2) contributing to mathematics learning.

Discussion:

Similar to primary school, the top three factors contributing to the engineers' interest in and learning mathematics in secondary school years 1 and 2 are teacher, enjoyment and success. Compared to the other factors, teacher is by far the most significant factor contributing to the engineers' interest in and learning of mathematics. The main change between primary school and secondary school years 1 and 2 is the greater influence of task value variables such as practical applications, interest and problem solving in secondary school years 1 and 2, Figures 5-3 and A4-36, Appendix 4.

5.6.2.3 Within Secondary School - Junior Certificate

Results: See results plot in Figure 5-4 and Figure A4-37, Appendix 4, Volume 2.

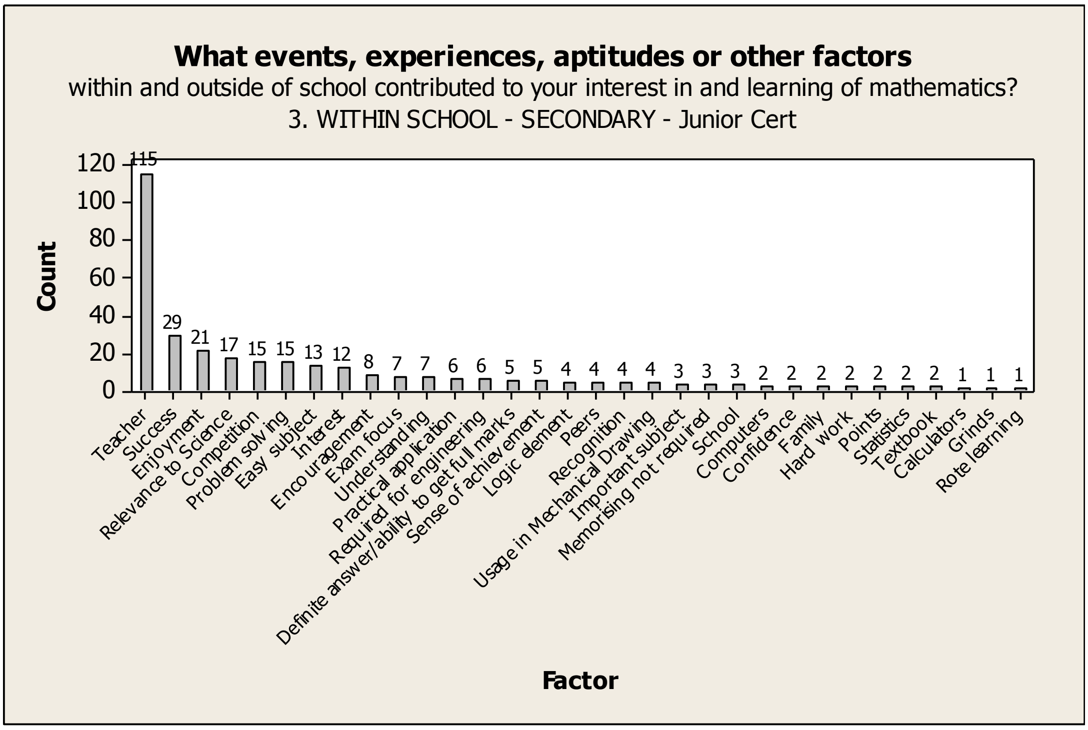

Figure 5-4: Factors within secondary school (Junior Certificate) contributing to mathematics learning.

Discussion:

The top three factors contributing to the engineers' interest in and learning mathematics in secondary school Junior Certificate years are the same three factors as in the earlier school years: teacher; success; and enjoyment. Again teacher is considerably ahead of the other factors impacting engineers' mathematics learning. It is noted that "relevance to science" and "required for engineering" emerge in secondary school Junior Certificate years, Figures 5-4 and A4-37, Appendix 4.

5.6.2.4 Within Secondary School – Leaving Certificate

Results: See results plot in Figure 5-5 and Figure A4-38, Appendix 4, Volume 2.

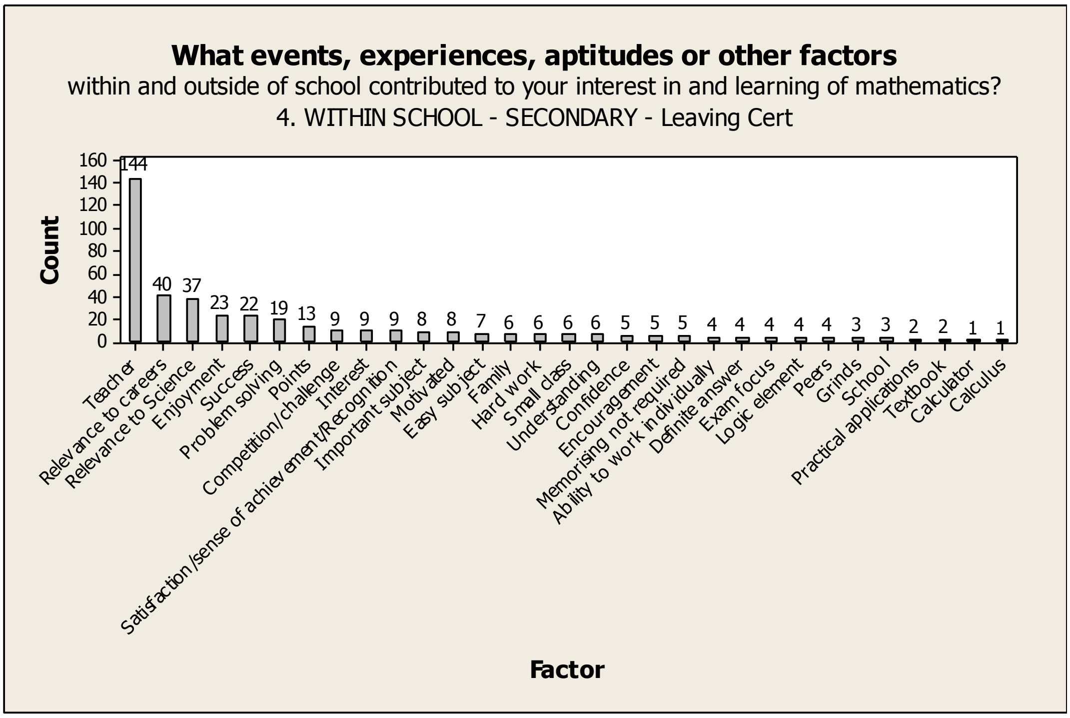

Figure 5-5: Factors within secondary school (Leaving Certificate) contributing to mathematics learning.

Discussion:

While many factors contributed to the engineers' mathematics learning in their Leaving Certificate years, teacher is by far the greatest factor. In Leaving Certificate years value variables, including relevance to careers and relevance to science are in second and third place respectively. As in the earlier school years, enjoyment (value) and success (self-efficacy) are strong factors. Points[19], an important value variable for current students, is in seventh place after problem solving, Figures 5-5 and A4-38, Appendix 4.

[19] Points [CAO Points]: Points are awarded to students based on their achievements in the Leaving Certificate examination. The maximum number of points is 600 (up to 2011). Students applying for third level education courses apply to the CAO and students, who meet the minimum points required for a course for which they have applied, are offered places. When the demand for a particular course exceeds the number of available places, places are offered to those students with the highest score in the CAO points system.

The variation of factors within school contributing to mathematics learning with engineers' progression through school is illustrated in Figure 5-6. The plot illustrates that teacher, compared to other factors, is a major influence on mathematics learning and is of increasing influence as students progress from primary school through to Leaving Certificate.

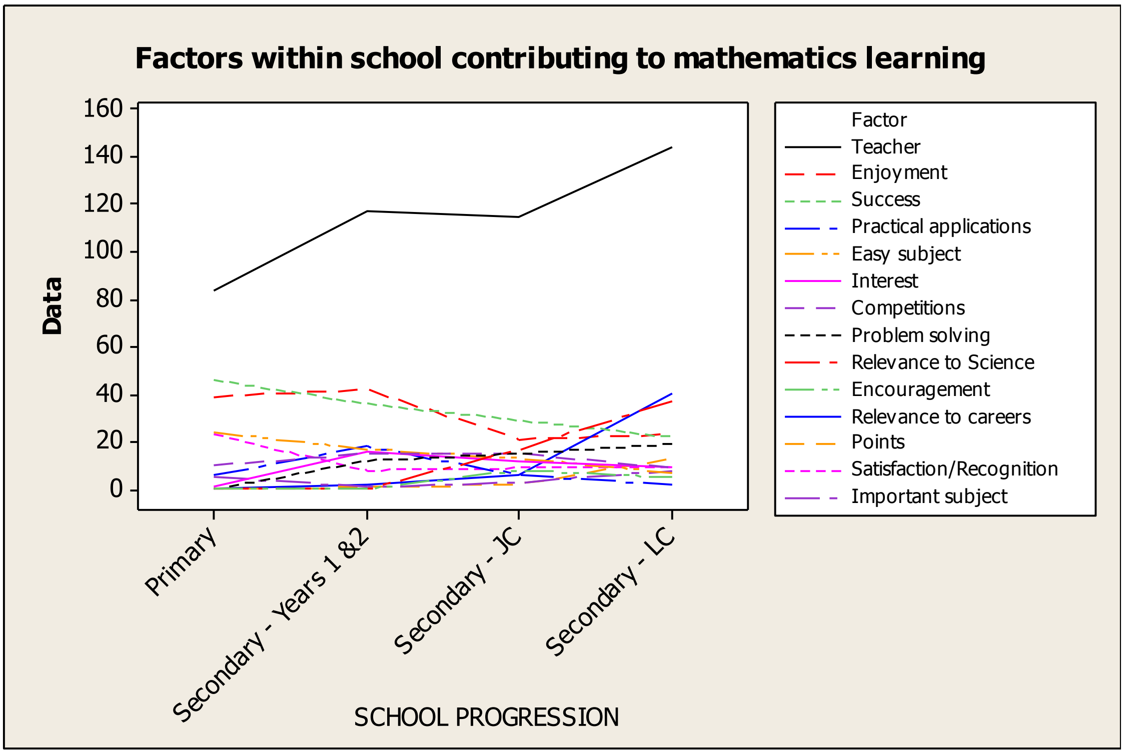

Figure 5-6: Variation of factors within school contributing to mathematics learning with school progression.

While open questions in the survey allow engineers to present factors that contributed to their interest in and learning of mathematics, the data does not explain why engineers present these variables and this is further investigated in the qualitative phase.

5.6.2.5 Outside Primary School

Results: See results plot in Figure 5-7 and Figure A4-39, Appendix 4, Volume 2.

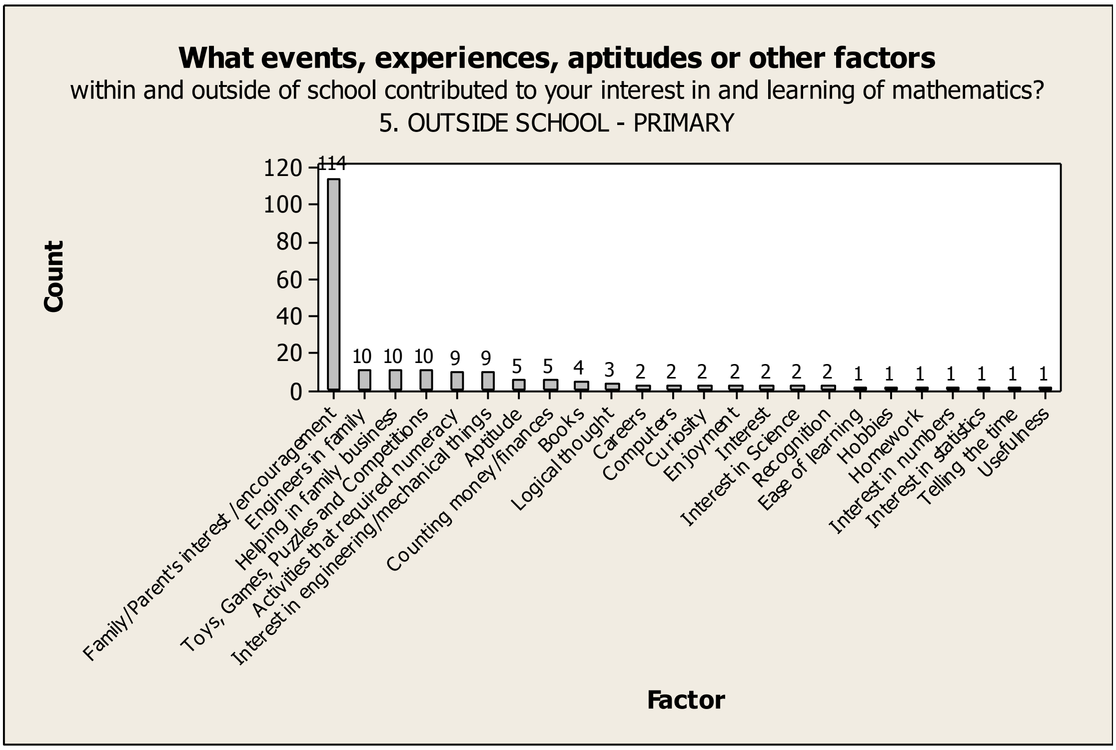

Figure 5-7: Factors outside primary school contributing to mathematics learning.

Discussion:

The data shows that family and parents were a very strong outside-of-school influence of the engineers' mathematics learning in their primary school years, Figures 5-7 and A4-39, Appendix 4.

5.6.2.6 Outside Secondary School - Years 1 & 2

Results: See results plot in Figures 5-8 and Figure A4-40, Appendix 4, Volume 2.

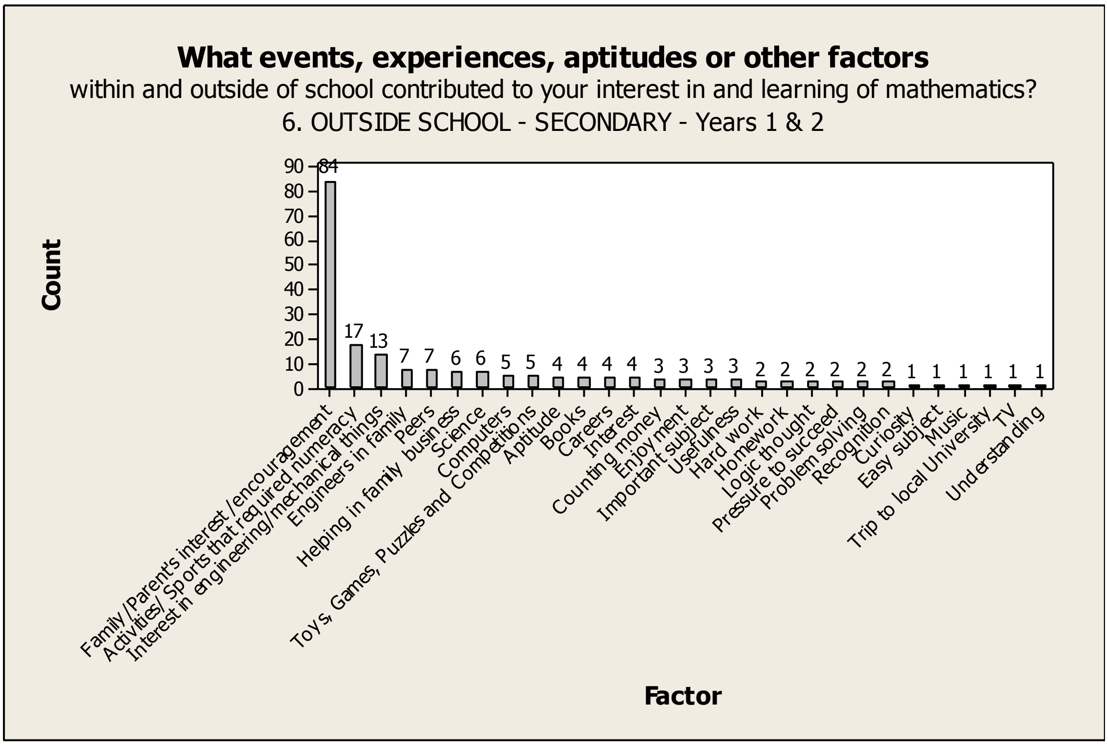

Figure 5-8: Factors outside secondary school (years 1 & 2) contributing to mathematics learning.

Discussion:

When the engineers moved from primary school into secondary school, family and parents remained a strong, but slightly reduced, influence on students' mathematics learning. In secondary school years 1 and 2 students' interests in activities requiring numeracy and their interest in engineering/ mechanical things emerged as small influencers. The engineers were also influenced by engineers in their families and by their peers, Figures 5-8 and A4-40, Appendix 4.

5.6.2.7 Outside Secondary School - Junior Certificate

Results: See results plot in Figures 5-9 and Figure A4-41, Appendix 4, Volume 2.

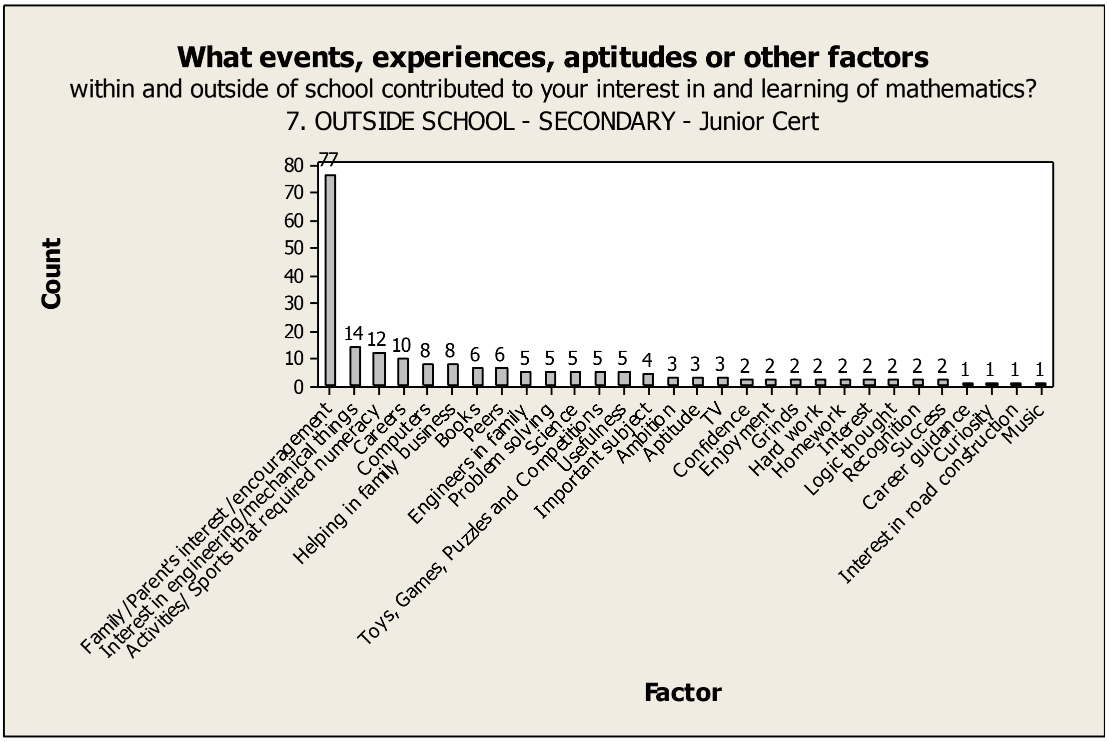

Figure 5-9: Factors outside secondary school (Junior Certificate) contributing to mathematics learning.

Discussion:

The main influencers on the engineers' mathematics learning outside of school in secondary Junior Certificate years include: family and parents; interest in engineering/ mechanical things; and activities that require numeracy. It is noticeable that careers emerge as an influence at this stage of the engineers' development, Figures 5-9 and A4-41, Appendix 4.

While careers is a factor contributing to engineers' interest in and learning of mathematics outside of school at Junior Certificate, it is noticed that careers is not apparent in the factors within secondary school (Junior Certificate) years, Figures 5-4 and A4-37, Appendix 4. This suggests that outside of school factors associated with mathematics learning are a greater influence on career choice compared to within school factors at Junior Certificate stage. In Chapter 2 it is reported that choosing a career is an evolutionary process; in the tentative period (typically aged 11 to 17 years) career choices are based on personal criteria: interests; abilities; and values.

Adolescents consider the things they enjoy or are interested in doing, their abilities and talents, salary, satisfaction specific occupations offer, work schedule and other value-related facets. (Ginzberg et al. 1951). Junior Certificate students are typically aged 15 years. Given that only 45% of Junior Certificate students take higher level mathematics in Ireland and thus by age 15 years the engineering pipeline has 55% leakage, it may be that potential engineers would benefit from career guidance or an appreciation of the task value of higher level mathematics prior to Junior Certificate years.

5.6.2.8 Outside Secondary School - Leaving Certificate

Results: See results plot in Figures 5-10 and Figure A4-42, Appendix 4.

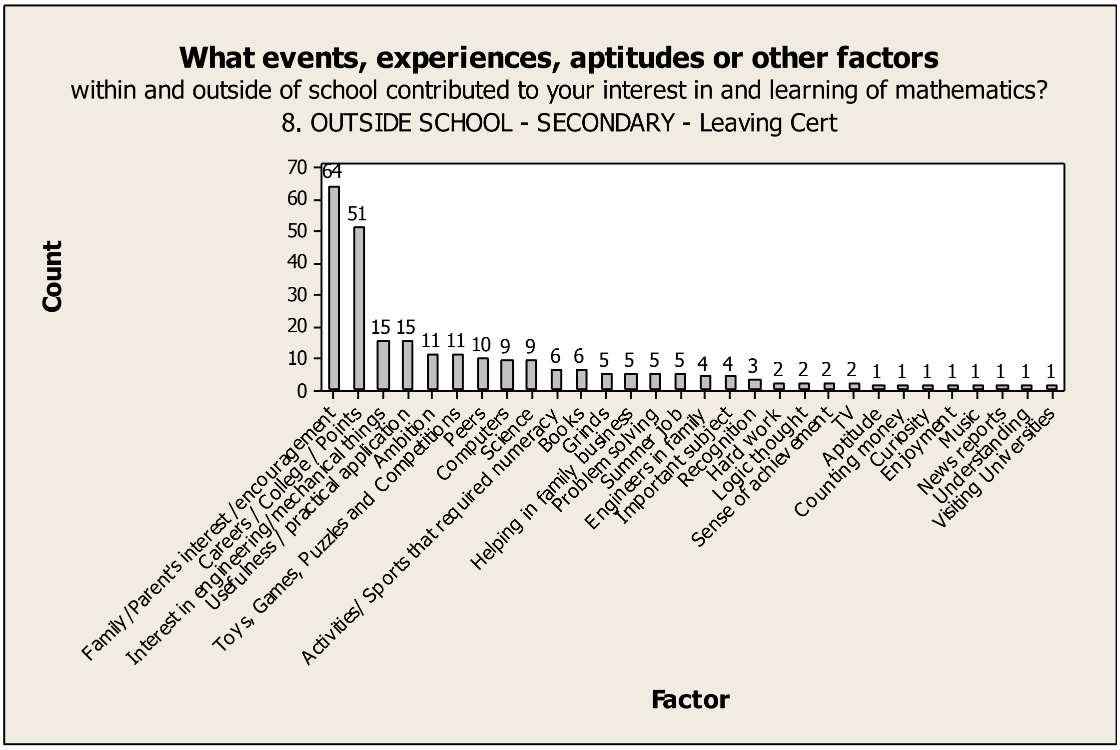

Figure 5-10: Factors outside secondary school (Leaving Certificate) contributing to mathematics learning.

Discussion:

In secondary Leaving Certificate years family and parents continue to be the biggest influence on students' mathematics interest and learning. The big change at this stage of the engineers' development is that careers/ college and points have moved up to second place. The influence of affective variables including: family (sociocultural influences); careers (task value); interest in engineering (task value); usefulness (task value); ambition (motivational belief); toys and games (task value); and peers (sociocultural influences) are evident at this stage of the engineers' development, Figures 5-10 and A4-42, Appendix 4.

The variation of factors outside school contributing to mathematics learning with engineers' progression through school is illustrated in Figure 5-11. Family and parents is a major influence on mathematics learning and is of decreasing influence as students progress from primary school through to Leaving Certificate. Similarly theory posits that parental involvement declines during adolescents (Schunk et al. 2010). After Junior Certificate careers is of increasing influence on mathematics learning.

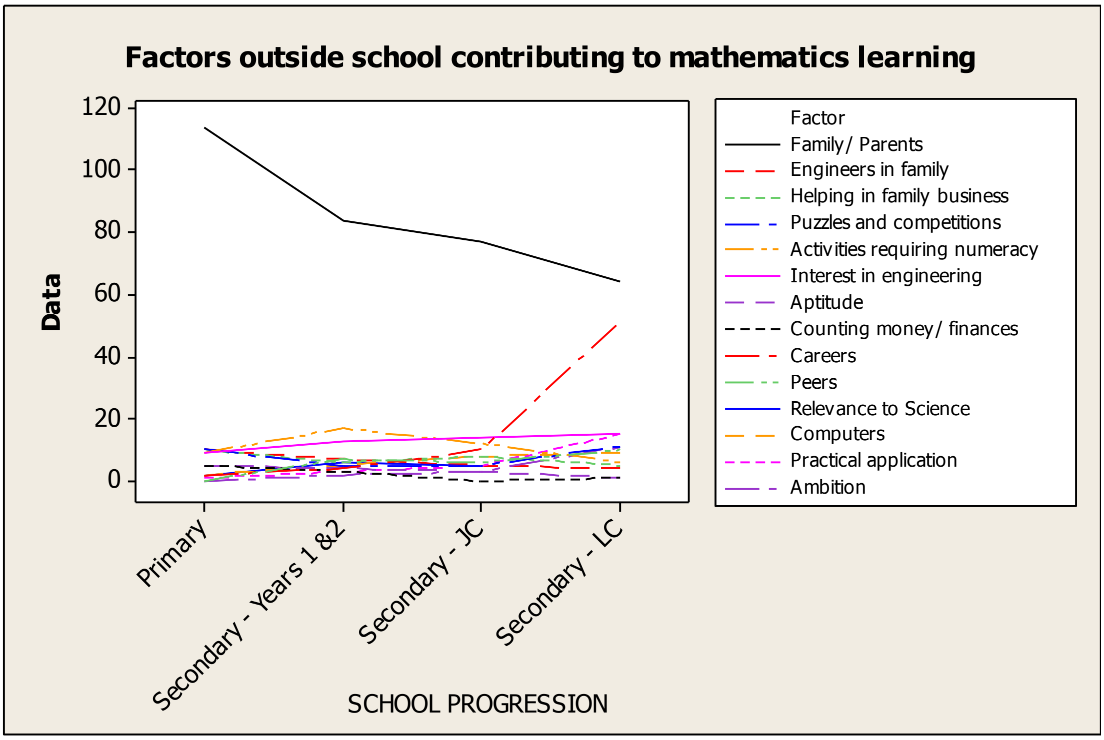

Figure 5-11: Variation of factors outside school contributing to mathematics learning with school progression.

5.7 IMPACT OF FEELINGS ABOUT MATHEMATICS ON CHOICE OF ENGINEERING CAREER

Question: To what degree did your feelings about mathematics impact your choice of engineering as a career?

Sample size: 364

Results: See results plots in Figures 5-12 and Figures A4-43 and A4-44 and Table A4-29, Appendix 4, Volume 2.

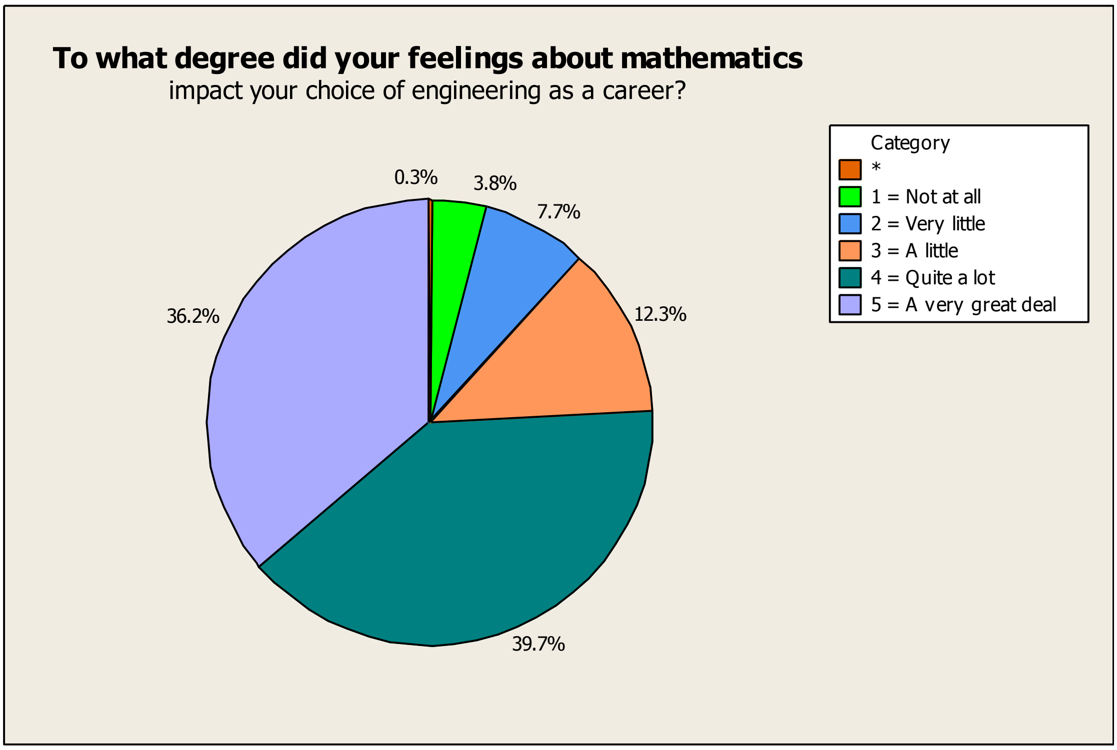

Figure 5-12: Degree that feelings about mathematics impacted engineers' career choice.

Discussion:

Three quarters (75.9%) of engineers say that their feelings about mathematics impacted their choice of engineering as a career either "quite a lot" or "a very great deal". A further 12.3% of engineers say that their feelings about mathematics impacted the choice of engineering career "a little". It is just 4.1% of engineers whose feelings about mathematics impacted their choice of engineering as a career "very little" or "not at all", Figures 5-12 and Figures A4-43, Appendix 4.

The mean value of the degree engineers' feelings about mathematics impacted their choice of engineering as a career is 3.97 Likert units, which is just under the "quite a lot" level on the Likert scale, Figure A4-44, Appendix 4.

Paired t-test analysis shows that there is a difference between the average degree engineers' feelings about mathematics impacted their choice of engineering as a career and the average degree engineers enjoyed school mathematics. The engineers rate the average degree their feelings about mathematics impact their choice of engineering as a career less (by 0.24 to 0.04 Likert units) than their average enjoyment of school mathematics, Table A4-29, Appendix 4.

The relationship between engineers' experiences with school mathematics and their choice of engineering as a career is further investigated in the qualitative phase.

5.8 HOW TO IMPROVE YOUNG PEOPLE'S AFFECTIVE ENGAGEMENT WITH MATHEMATICS

Question: Only a minority of students sit higher level Leaving Certificate mathematics and many of those subsequently choose not to stay with numerate studies. How, in your view, could young people's affective engagement (e.g. enjoyment) with mathematics be improved?

Sample size: 364

Results: See results plot in Figures 5-13 and Figure A4-45, Appendix 4, Volume 2.

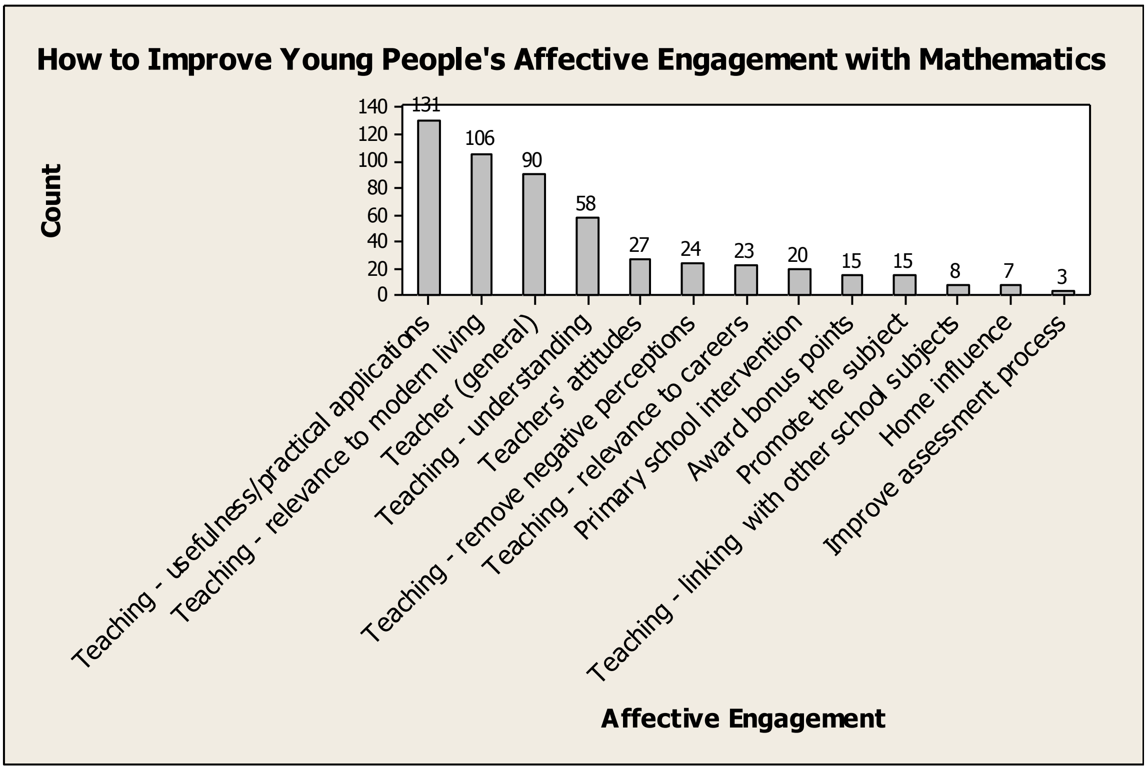

Figure 5-13: How to improve young people's affective engagement with mathematics.

Discussion:

When asked, in an open question, how young people's affective engagement with mathematics could be improved, the majority of engineers' responses relate to teachers and teaching, Figures 5-13 and A4-45, Appendix 4. Following categorisation of the engineers' responses, the four most popular views on how to improve young people's affective engagement with mathematics are:

i. Teaching - usefulness/ practical applications (24.86%);
ii. Teaching - relevance to modern living (20.11%);
iii. Teacher - general (17.08%); and
iv. Teaching - understanding (11.06%),

- Some examples of engineers' responses include:

- Teaching - usefulness/practical applications (24.86%): "show students worked examples of usefulness and applicability to real life situations"; "I enjoyed maths more at college because I could see its uses in other disciplines"; "I feel that pupils wonder why do I need to know this"; "school work should involve activities that mean something to the students and not just be a series of problem solutions that they neither understand nor see a use for"; "I disliked pure maths, applied maths was very interesting"; "calculus seems useless until you see it used in fluids and thermodynamics, statistics likewise is used extensively in both engineering and finance disciplines but from memory seemed quite obtuse in secondary school"; and "look at industry and design, figure out what maths is used and develop a curriculum around these topics".
- Teaching - relevance to modern living (20.11%): "make the curriculum more relevant to modern living"; "engage with the recent achievements that maths has produced"; "maths education needs to be more sociable, associate it with visual arts, new communications, etc."; "young students must see and experience where maths fits into their own everyday lives"; "mathematics is generally taught as an abstract subject and not really identified with the practicalities of modern life or everyday experience"; and "make maths more relevant to modern society".
- Teacher (general) (17.08%): "teaching is the biggest issue facing maths"; "I put my affective engagement with maths mainly down to the teacher"; "To me, it is all down to the teacher"; "If you don't like the teacher you won't like maths"; "I had an excellent maths teacher, he was approachable, and I guess made maths as fun as it could possibly be"; "a good teacher is paramount to the success and engagement of the student"; "mathematics is a difficult subject for the vast majority of people and teachers must have the skills, enthusiasm and ability necessary to teach the subject"; "engineers and persons with high mathematical achievements must be encouraged to look at teaching second level maths"; employ teachers that enjoy maths and who can teach"; "the teacher needs to have a sound grasp of maths, and a genuine interest in the subject, in order to fully impart the theory of maths to students and to give students the chance to learn maths from someone who is confident in their knowledge of the subject";

"some excellent maths teachers developed in me a love of maths"; "my excellent teacher"; and "much of the problem sadly lies with teachers and teaching methods and particularly those teaching maths without a major in maths at University".

- Teaching - understanding (11.06%): "the biggest difficulty with maths is the ability of students to visualize the concept"; "a strong reason for students not enjoying maths is that they don't understand it"; "I am of the strong view that considerably greater effort needs to go into maths at primary level so that pupils going into second level understand the basics"; "some students can be very intimidated by higher maths, teachers must make it easier to understand and thus encourage students to take higher maths"; "better instruction in the classroom by people who can relate the subject matter to reality, and speak in a language that can be understood by students"; "my leaving certificate maths teacher (arts student) did not understand what she was teaching"; "less emphasis on mechanical routines and formula based solutions and more emphasis on understanding and logical arguments"; "get teachers who actually understand maths"; "I dropped out of honours maths in my leaving cert year due to a lack of understanding"; "maths is a subject where students quickly get left behind when they do not understand the principals"; "we had a very interesting teacher who took time to get us to understand the reasons for approaching problems in a particular way rather than force us to learn by rote"; and " some brilliant mathematicians I have known have been very poor teachers, frequently unable to understand why a student could not grasp the concept being taught".
- Teachers' attitudes (5.12%): "teachers' interest in maths and their attitude to it decides a student's interest and attitude"; "in my experience the personality of mathematics teachers has always been quite dour and boring and especially those who dress accordingly in knitted cardigans or bow ties"; "my maths teacher would have preferred that we all did pass maths, she continuously tried to persuade us that honours maths was too difficult"; "need a motivated and enthusiastic teacher"; "teachers with a genuine love of the maths"; "teachers who pass their enthusiasm onto the students"; "fear of maths stems from teachers attitudes"; and "the key in my view is having an enthusiastic teacher at second level that brings the subject alive and brings students along with him/her".
- Teaching – remove negative perceptions (4.55%): "there is a perception amongst young people (and accepted by teachers) that higher level maths is difficult, other higher level subjects are considered 'easier' and so pass maths is often used as a 7^{th} subject"; "teacher should work to remove the stigma about the difficulty of higher level maths"; "teachers present higher level maths as very time consuming and that is a big 'turn-off' for students in leaving certificate"; "many students are intimidated by the perceived difficulty of higher maths, teachers should dispel this myth"; "only teachers can remove the 'fear factor'"; "there appears to be a disproportionate amount of fear among secondary level students about the difficulty of maths"; and "teachers have done little to change the negative image of maths".
- Teaching - relevance to careers (4.36%): "I believe that work and career exposure showing the massive opportunities for mathematically inclined individuals would surely encourage a higher participation rate"; "the correlation between "hard

sums" and elevated opportunity in business and life, is well recognised in industry and commerce, but is inadequately communicated to high school students"; "if teachers showed the link between maths and jobs"; and "teachers should modernise their teaching to provide an appreciation for students on the usage of maths in the working environment".

- Primary school intervention (3.79%): "the problem with mathematics starts with our primary school system"; "I see from my own children that much more emphasis is placed on reading, writing and art compared to maths in primary school"; "I developed my interest for higher level maths in the primary school"; "I got a good start in primary school"; "instill a greater interest in maths from a very early stage by making it fun to do in primary school"; "improve maths education at primary level"; and "if a primary school child dismisses maths (or rather, themselves as able mathematicians) it's difficult to re-engage them".

In summary 92% of the engineers' views about how to improve young people's affective engagement with mathematics relate to teacher or teaching, Figures 5-13 and A4-45, Appendix 4. This is consistent with motivation theory which posits that teachers are a huge influence on students' motivation. In particular teachers' decisions about what activities students engage in are deemed to affect motivation (Schunk et al. 2010). In this study the engineers' views are that teachers should teach mathematics content that illustrates: the usefulness of mathematics; the relevance of mathematics to modern living; mathematics that is used in various careers; and mathematics that has links with other school subjects. All of the content proposed by the engineers has a high task value.

Many engineers are of the view that because mathematics is a difficult subject, "teaching is the biggest issue facing maths". They say that "teachers must have the skills, enthusiasm and ability necessary to teach the subject" and that "much of the problem sadly lies with" unqualified teachers. Engineers also draw attention to the influence of teachers' own attitudes about mathematics on students and they are of the view that it is teachers' responsibility to correct the "stigma about the difficulty of higher level maths" and the "fear factor" associated with mathematics. This is consistent with motivation theory in Chapter 2 whereby an important type of teacher expectation is teacher self-efficacy or teachers' beliefs about their capabilities to help students learn. It is maintained that efficacious teachers are more likely to plan challenging activities, persist in helping students learn and overcome difficulties, and facilitate motivation and achievement in their students. Research literature suggests that constructivist teaching (theory contending that individuals construct much of what they learn and understand through individual and social activity), discussed in Chapters 2 and 3, changes the focus from controlling and managing student learning to encouraging student learning and development (Schunk et al. 2010; Vygotsky 1978). While the majority of engineers' views relate to affective variables, the engineers also present that "a strong reason for students not enjoying maths is that they don't understand it" and they advocate that mathematics teaching should place "more emphasis on understanding". This view is similar to Vygotsky's theory of social constructivism in Chapter 2 whereby understanding is accomplished when teachers present appropriate challenges for learners to engage in and make sense of concepts rather than students just passively receiving facts and skills. Vygotsky's zone of proximal development posits that there is a difference between what learners could achieve by themselves and what they could do with assistance from a skilled person. (Vygotsky 1978).

5.9 ENGINEERS' ADDITIONAL COMMENTS

Question: Would you like to make any additional comments?

Sample size: 171

Results: See results plot in Figure 5-14 and Figure A4-46, Appendix 4, volume 2.

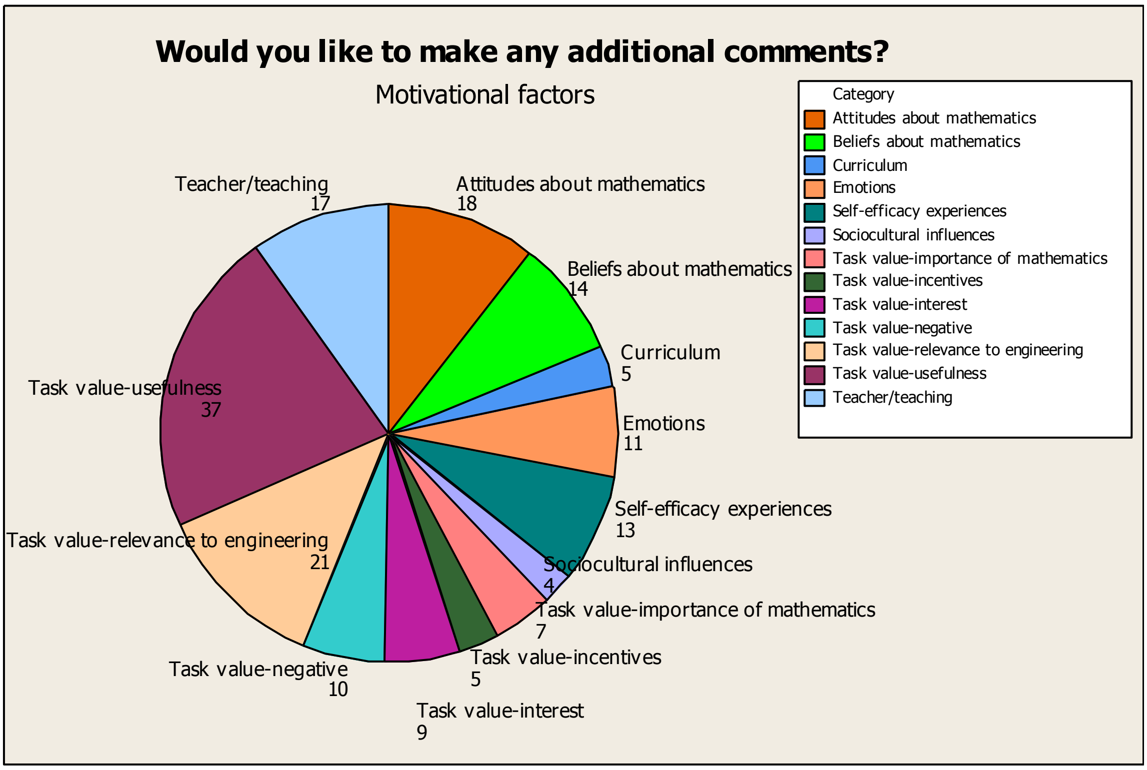

Figure 5-14: Engineers' additional comments.

Discussion:

A review of engineers' additional voluntary comments in the survey shows that the majority of comments relate to the affective domain, teaching and curriculum, Figures 5-14 and A4-46, Appendix 4. More than half (52%) the engineers' comments relate to task value (why should I do mathematics). Here engineers note benefits of mathematics education and how an awareness of these benefits encourages students in their mathematics learning. Engineers maintain that mathematics education is useful in engineering, finance, general management, in the home and for Ireland. On top of mathematics used in engineering practice, structured thinking and logical decision making are further benefits of a mathematics education. While engineers mostly present positive task values, there is a lesser view that practising engineers do not use the level of mathematics learned in engineering education. Interest, incentives, self-efficacy and positive beliefs about mathematics, as presented by the engineers, are all necessary motivators in mathematics education. The engineers note that sociocultural influences, both positive and negative, from families, teachers and peers significantly impact mathematics learning. In particular engineers express a strong view about the necessity for teachers' love and understanding of mathematics. An interpretation of the engineers' overall comments are that they associate mathematics and mathematics learning with values, attitudes, beliefs, self-efficacy, emotions and sociocultural influences, so much so that mathematics could be regarded as a highly "affective subject". A sample of the engineers comments include:

- Task value - usefulness/practical applications:

  > "it's important that teachers are able to explain where a branch of maths would ultimately be used"; "in every period of my career the structured thinking that mathematics teaches served me well"; "maths instilled in me a train of thought that allows me to analyse situations thoroughly"; "ability in mathematics demonstrates capability of rational thought that universities and employers consider essential in a wide range of jobs"; "knowing maths at engineering level makes finance very easy"; "my maths education encourages me to think, it is a great benefit in general management"; "I strongly believe that a sound basis in mathematics is essential for all aspects of life, regardless of professions, monthly household budgets, tax returns, ability to save, risk analysis, decision making etc. If more people had stronger skills in the area, I believe that social / economic problems would reduce".

- Task value - relevance to engineering:

  > "maths is fundamental to engineering"; "I could not envisage working in engineering without a good grounding and interest in mathematics"; "statistics, risk theory, logic and similar are all necessary in engineering"; "a thorough knowledge of maths is vital to ensure correct safe engineering designs are actual carried out with due diligence"; "maths gave me that practical, logical approach on which engineering and project management rely on"; "there is hardly a day that goes by that I don't use my secondary school maths. When it gets to the third level maths, I use them only occasionally, and as for pure

maths, there are a few of my engineering colleagues who use them, but not very often"; and "maths may not be obviously used by engineers at all times but a mathematical ability is necessary to making crucial decisions".

- Task value – interest:

"I believe that interest inspires mathematical ability and visa-versa"; "students should be encouraged from a very young age to take an interest in maths"; "it is important that maths is taught in an interesting way to keep young people interested"; "my son struggling with higher maths until we employed a grind teacher who was able to explain the subject and make it more interesting"; " I was an average pupil in secondary school however it wasn't until university that maths interested me and then I excelled in maths"; " with modern internet facilities and computer resources there are ample opportunities for students to be taught maths in a way that interests them".

- Task value-importance of mathematics:

"I cannot over emphasise the importance of higher mathematics"; "there is a need for an environment where maths is valued"; "maths is important in a society like Ireland where there are many difficulties": "in Ireland there is a misguided acceptance in society that mathematics is not important"; and "studying mathematics was my best investment".

- Task value- incentives:

"incentives, such as higher points, for maths would, in my view, bring a greater number of students back to studying maths again"; "humour, practical participation and competition with intrinsic and extrinsic rewards are the key ways to improve mathematics"; "some students need to visually see the problem, solution, benefits and rewards of mathematics"; and "while maths is quite enjoyable, there is need to link excellence in maths to possible rewards in life".

- Task value-negative:

"I really have done little with the higher maths I studied in college since I left a big consultants practice where I did a lot of design work"; " In an engineering career a very high level maths is only required by the few who go into computer modelling and research"; "advanced maths such as third order integration and Laplace transforms etc. are of little benefit to 99% of engineers"; "drop the requirement for honours maths, it is not necessary, for engineering" and "engineering is not so much about mathematics; it's about communication and creative thinking" ; "since graduating I have not used any of the maths taught in college, nor could I remember any of it".

- Attitudes about mathematics:

"I found maths boring and difficult"; "maths was time-consuming at school"; "for me attitude is the biggest single factor affecting students' maths achievements"; "the impression that people had of higher maths being difficult impacted on my enjoyment and performance at leaving certificate"; "my classmates' attitude towards me was very negative when I performed well at maths in secondary school"; and "I got away with murder in school because I was good at maths".

- Beliefs about mathematics:

  "If you have not developed a logically thinking brain/aptitude for mathematics by Junior Cert level it is already too late"; "there is a belief that it is cool to be poor at maths"; "when I was in school I was told that computer programming was all about maths, in fact it isn't"; "the perception that mathematics is overly difficult and time consuming at leaving cert is leading many schools and students to drop higher level mathematics in favour of subjects considered to be easier"; "my 12 year old son got the perception from school that math is hard and you have to be really smart to do well in it".

- Self- efficacy experiences:

  "I struggled with maths in primary school and I believed I was not good at maths"; "students often and wrongly lose confidence in their maths abilities in secondary school due to lack of primary school basics"; "the active involvement in practical application of engineering contributed to my increasing confidence in using mathematics as a tool"; "schooling gave me the knowledge that mathematical tools exist and the confidence to go try apply them"; "I am not comfortable with statistics beyond the very basic level"; "I would enjoy using more maths in my work however I have lost the ability over the years"; and "I believe that I have a natural ability to understand mathematics".

- Emotions:

  "there is a fundamental flaw in Irish education (beginning even in primary school), where students are allowed to develop a fear or discomfort with maths"; "with maths there is always the fear of mistakes"; "too many teachers impart a fear or dislike of maths"; "biggest impact for kids developing a love of maths is a good teacher"; "I believe the teaching approach has a very significant impact on students feelings surrounding maths"; and "I love the beauty of numbers".

- Curriculum:

  "the Junior and Leaving Cert curricula are frighteningly broad"; "the leaving cert maths course has been embarrassingly dumbed down"; "do not remove calculus from the higher level mathematics course"; "an appropriate syllabus approach needs to be developed at secondary and third level that reflects the realities of our needs”; “the level of mathematics studied at college was ridiculous, it was 100% theoretical and had no connection with real life”.

- Sociocultural influences:

  “If you don't have a good teacher or you can't ask for help at home, you probably will find it more difficult to succeed in maths”; “it was a friend that helped me restore my love of maths”; “it is important that children have the necessary support to do home projects on maths and related topics”; “I was influenced by my family who have been designers, builders, engineers and teachers of various types, for the past five generations”; and “a good standard of maths was almost a rule in my family”.

- Teacher/teaching:

  “maths needs to be taught by persons who fully understand the subject and have a significant qualification and training in maths”; “teaching by rote doesn’t work with maths”; “it is critical that those teaching maths have a love for it, even at primary level”; “maths teaching at primary level is critical and an aptitude for teaching maths should be developed in teacher training”; “it's particularly important to have exceptional teachers for maths as it is viewed as the most boring subject by many”: “I think a lot of maths teachers in secondary school are bad and don't fully understand maths themselves” and “the standard of maths in schools will improve only if the quality and interest of the teachers improves”.

5.10 GENERALISATION OF SURVEY FINDINGS

In statistical analysis sample data is used to make generalisations about populations, assuming the sample is representative of the population from which it comes, as discussed in section 4.2. Statistical estimation is based on the fact that "sample means taken from any population are normally distributed if the samples are big enough ... there is a 95% probability that the sample mean lies within 1.96 standard errors of the population mean … we can use the sample mean to construct a confidence interval that contains the unknown population mean with 95% probability" (Reilly, 2006).

The response rate in this study was noted to be broadly representative of the population of Chartered Engineers in Ireland across engineering disciplines, gender, industry and geography. The sample size of 365 chartered engineers is satisfactory for precision to within 0.15 units (on a Likert scale with five outcomes) and 95% confidence, i.e. 95% probability that the findings from the survey questionnaire represent the population of Chartered Engineers in Ireland, as calculated in section 4.2. Based on the assumption that the sample is random, it is concluded with 95% probability that the survey findings herein are representative of the population of Chartered Engineers in Ireland.

5.11 SUMMARY OF SURVEY FINDINGS

The two main research questions in this study are:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

In response to the question regarding the role of mathematics in engineering practice, there is evidence in the survey data to conclude that:

(i) Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation.

Engineers demonstrate high affective engagement with mathematics and they rate the following engagement variables: the necessity of a mathematical approach in their work; the degree they actively seek a mathematical approach; their enjoyment of mathematics; and their confidence using mathematics all high while they also rate the degree they had a negative experience using mathematics as very low. Mathematics is "part of who" engineers are, they "love the challenge in solving problems mathematically" and they prefer to communicate using mathematics rather than words.

Task value factors are a big influence on engineers' engagement with mathematics. Confidence in mathematical solutions, necessity of mathematics in complex situations and in large data analysis, the need to understand software solutions, the value of objective solutions in decision making, the quickest way and accuracy of solutions all increase engineers' engagement with mathematics. However the "quirkiness of computational tools" and their "lack of understanding" and "over reliance of computer analysis" sometimes generate errors. In engineering practice, engineers' time is often limited and thus the task value of engaging in lengthy problem solving reduces when students become engineers. While engineers use mathematics to discover objective solutions to support their decision making, their colleagues' lack of understanding of mathematics can make their mathematical solutions redundant.

(ii) While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, mathematical *thinking* has a greater relevance to engineers' work compared to *curriculum* mathematics.

Engineers' usage of *curriculum mathematics* is 2.73 Likert units based on a score of 5 for 75 domain-level-usage combinations of *curriculum mathematics* from Junior Certificate ordinary up to level 8. 64.4% of engineers use higher level Leaving Certificate mathematics either "a little", "quite a lot" or "a very great deal" in their work.

Engineers' *thinking* usage (4.02 Likert units) is between 1.15 and 1.43 Likert units higher than their *curriculum mathematics* usage. The modes of *thinking*, that influence engineers' work performance are: problem solving strategies (26.4%), logical thinking (26.2%); critical analysis (7.2%); modelling (7.2%); decision making (6.3%); accuracy/ confirmation of solution (4.8%); precision/ use of rigour (4.6%); organisational skills (4.6%); reasoning (3.6%); communication/ teamwork/ making arguments (3.2%); confidence/ motivation (3.1%); numeracy (2.2%); and use of mathematical tools (0.7%).

(iii) Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and role. Their *mathematical thinking* usage is independent of engineering discipline and engineering role.

Analysis shows that the interaction of engineering discipline and role has an effect on engineers' mean *curriculum mathematics* usage. Engineers' *thinking* usage is independent of engineering discipline and engineering role.

In response to the second research question; whether there is a relationship between students' experiences with school mathematics and their choice of engineering as a career, there is evidence in the survey data to conclude that:

(iv) Engineers' feelings about mathematics are a major influence on their choice of engineering as a career.

Engineers present mathematics as a highly "affective subject" where engagement is driven by motivational beliefs. Three quarters (75.9%) of engineers say that their feelings about mathematics impacted their choice of engineering as a career in the range “quite a lot” or “a very great deal”.

(v) Teachers, affective factors and sociocultural influences are the main contributors to engineers’ interest in and learning of mathematics

Teachers and affective factors are the main contributors to engineers’ interest in and learning of mathematics. Affective factors such as success (self-efficacy), enjoyment (value), practical applications (value), interest (value), problem solving (metacognitive activity), relevance to science (value) and relevance to careers (value) are all ahead of points (value) as contributors to engineers’ interest in and learning of mathematics within school. Outside of school, sociocultural experiences are the main influences on engineers’ interest in and learning of mathematics.

Engineers maintain that teachers are the key to improving young people’s affective engagement with mathematics. In particular teachers should communicate the value of mathematics by teaching content that illustrates the task value of mathematics. Teachers’ own beliefs about mathematics are responsible for the general “stigma about the difficulty of higher level maths” and teachers should place “more emphasis on understanding” mathematics.

While engineers are of the view that confidence dealing with mathematics develops in school where engineers learn to check their answers and where they are “in the habit of getting 100% in maths and maths-based exams”, it cannot be asserted statistically that there is a difference between the degree engineers feel confident dealing with mathematics in work and their enjoyment of school mathematics and this requires further investigation.

5.12 DISCUSSION OF SURVEY FINDINGS

Compared to the minority (16%) of Leaving Certificate mathematics students who take the higher level Leaving Certificate mathematics paper, 84% of the engineers in this survey have higher level Leaving Certificate mathematics and 80% of the total sample say they enjoyed school mathematics “quite a lot” and “a great deal”.

Engineers consider mathematics as a highly “affective subject” where motivational beliefs such as affective memories (previous emotional experiences with mathematics), goals, task value (why should I do mathematics?) and expectancy (am I able to do mathematics?) influence choice, persistence, quantity of effort, cognitive engagement and actual performance. A significant finding in this study is that positive feelings about mathematics are a strong influence on choice of engineering careers. This finding is particularly interesting as it is reported in Chapter 2, that students’ difficulty with higher-level school mathematics is considered to be a major contributor to the declining number of entrants to engineering degree courses (Bowen et al. 2007; King 2008; Prieto et al. 2009). Also (in Chapter 2) a significant shift away from engineering careers was observed as students progressed through second level school in Ireland (Lynch and Walsh 2010). Given the declining interest in engineering career choice, the strong influence of engineers’ feelings about mathematics on the choice of engineering as a career found in this study suggests that school mathematics education should aim to improve students’ emotional experiences with mathematics. This finding has some similarity with a study where a majority of high achievers in mathematics were interested in pursuing a mathematics related career (Leder 2008). It is also reported in Chapter 2 that self-efficacy is predictive of important indexes of career entry behaviour (Lent et al. 1986) and studies show that women’s lower mathematics self-efficacy compared to men’s perceptions of their capability to succeed in mathematics is a major influence on career choice (Correll 2001; Løken et al. 2010; Zeldin and Pajares 2000). The task value of mathematics is also a factor in engineering career choice. It is observed in this study that careers is an influence on students’ interest in and learning of mathematics within school in Leaving Certificate but not in Junior Certificate or earlier. Given that higher level Leaving Certificate mathematics is a requirement for entry into level 8 engineering education courses and that only 45% of Junior Certificate students take the higher level course, the provision of career guidance at an early stage of secondary school conveying the career value of higher level mathematics would likely assist students' task value and take-up of higher level mathematics.

Despite the widespread view that higher level mathematics competence is critical to a technology economy and necessary for engineering practice, almost a third of engineers agree that they could perform satisfactorily in their current job without higher level Leaving Certificate mathematics. However while engineers also present, what initially appears as a low (2.73 Likert units; "very little" - "a little") value of *curriculum mathematics* usage in their work, considering that the engineers' usage ratings relates to the entire spectrum of *curriculum mathematics* education from Junior Certificate up to level 8 degree mathematics including usage types ranging from reproducing to mathematising, a score of 2.73 out of 5 for overall mean usage is interpreted as a high score. Consistent with this is the finding that almost two thirds of engineers (64.4%) use higher level Leaving Certificate mathematics; 57.3% of engineers use engineering mathematics; and 41.4% of engineers use B.A./ B.Sc. mathematics in their work. This is an important finding given that "there is a belief among some practising engineers that the mathematics they learned in college is not applicable to their daily work" (Cardella 2007).

In addition to *curriculum mathematics* usage in engineering practice, engineers show significantly higher mathematics *thinking* usage compared to *curriculum mathematics* usage in work. This finding is consistent with Ernest's view in Chapter 2 that mathematics comprises explicit knowledge and "know how" that comes from the experience of working with mathematics which he describes as personal knowledge of mathematics (Ernest 2011). The strongest modes of *thinking*, resulting from engineers' mathematics education that influence their work performance are: problem solving strategies; logical thinking; critical analysis; modelling; decision making; accuracy/confirmation of solution; precision/ use of rigour; organisational skills and reasoning. These are important findings as it is suggested in Chapter 2 that "the use of mathematics within the job of an engineer is not necessarily self-evident to an undergraduate student, and hence it is not easy for students to make a connection between what they are learning at university and what they will be doing after graduation” (Wood et al., 2011).

In this study, it has been found that engineers enjoyed school mathematics at a very high level. Teachers, affective factors and sociocultural experiences are the main contributors to engineers’ interest in and learning of mathematics. Affective factors such as success, enjoyment, practical applications, interest, problem solving, relevance to science and relevance to careers and CAO points are contributors to engineers’ interest in and learning of mathematics within school. Outside of school, sociocultural experiences are the main influences on engineers’ interest in and learning of mathematics. This is consistent with: affective theory in Chapter 3 (Schunk et al. 2010) with the view that societal beliefs influence children’s learning of mathematics in Chapter 3 (Schoenfeld 1992); with the view that “knowledge is usually learned in social contexts” in Chapter 2 (Ernest 2011); and with the findings of a study of high achievers in mathematics where being good at mathematics and the ability to get 100% marks in tests are the main reasons for students’ enjoyment of mathematics in Chapter 2 (Leder 2008).

There is no overestimating the role of teachers in engineers’ mathematics education. This study found that teachers overshadow all other factors that contribute to engineers’ interest in and learning of mathematics and they are of increasing influence as students progress from primary school through to secondary school and Leaving Certificate. Engineers say that high mathematical self-efficacy develops in school. Memories of school mathematics are also the main reason engineers do not enjoy using mathematics in work. For example, one engineer demonstrates his “in built hatred of mathematics from secondary school”. According to the engineers who participated in this study, teachers should communicate the value of mathematics by teaching content that illustrates the task value of mathematics. One reason for students not enjoying maths is that they don’t understand it and engineers advocate that mathematics teaching should place “more emphasis on understanding”. They also maintain that teachers have a responsibility to correct the “fear factor” and general “stigma about the difficulty of higher level maths”. The engineers’ views are consistent with research literature in Chapter 2 where it is maintained that "students' understanding of mathematics, their ability to use it to solve problems and their confidence in and disposition toward mathematics are all shaped by the teaching they encounter in school" (National Council of Teachers of Mathematics 2000).

A requirement of engineers' mathematics education, that is apparent from the survey data, is their confidence to subsequently use mathematics in modern engineering practice. It is observed that high confidence mathematics engineers readily "revise and brush up" on the required mathematics and that some engineers' mathematics confidence is often constrained within certain "limits" or they sometimes avoid mathematics in their work. Engineers' say that high mathematical self-efficacy developed in school where they learned to check their answers and where they enjoyed "the habit of getting 100% in maths and maths-based exams". Similarly in work engineers demonstrate high affective engagement with mathematics; they "love the challenge in solving problems mathematically", they enjoy "the satisfaction of a result" and "to reduce apparently complex processes to a series of mathematical forms is a great feeling". Task value factors are a big influence on engineers' engagement with mathematics. For example, confidence in mathematical solutions, necessity of mathematics in complex situations and in large data analysis, the need to understand software solutions, the value of objective solutions in decision making, the quickest way and the accuracy of solutions all increase engineers' engagement with mathematics. The availability of computer solutions, the "risky and slow" nature of mathematics and colleagues' discomfort with mathematics reduce engagement.

One interesting finding in this study is that the main source of engineers' negative experiences using mathematics relates to communicating mathematics. This finding is aligned with the observation noted in Chapter 2 that communicating mathematics is often neglected in school mathematics education (National Council of Teachers of Mathematics 2000) and in undergraduate education (Wood 2010). It is observed in this study that engineers value "objective" solutions provided by mathematics and "a 100% right answer rather than the ambiguity of non-mathematical solutions" gives engineers confidence in their "proposed solutions". While engineers are comfortable with objective solutions and they rely on objective solutions to support their decision making, their colleagues' lack of understanding of mathematics makes their mathematical solutions redundant. This study also shows that engineers progress from an education environment where mathematics and "objective knowledge" are highly valued to a working environment where mathematics is less valued and thus graduate engineers encounter an affective hurdle. This hurdle comprises two elements: confidence to use mathematics after school and university; and an ability to communicate mathematics to non-mathematics people. Consequences of this hurdle could be that engineers' mathematics usage is compromised and/or undervalued. An implication of this finding is that engineers' mathematics education should address this hurdle and better prepare engineers for engineering practice. It is anticipated that the inclusion of practical applications and the relevance of mathematics to modern living in mathematics education, as suggested by the engineers, would benefit engineers' mathematics usage in situations where "a 100% right answer" may not always be the best practical solution. Furthermore it is suggested that engaging in active or social learning environments, in both school and university, where students are required to present and defend their mathematical solutions to both their peers and their teachers, would develop students' mathematics communications skills and would also enhance their mathematics thinking and confidence. Similarly, according to Vygotsky, in Chapter 2, learning environments should involve interaction with experts; discussions between teacher and students and amongst students themselves enhance students' mathematical thinking and communication (Vygotsky 1978). There is also a view in Chapter 2 that even though we live in a technological society, that "engineering departments possess a vast knowledge that is not readily available to school teachers" (Heywood 2005). The findings here have implications for mathematics teacher training given the strong influence teachers have on students' mathematics learning and also the influence of students' feelings about mathematics on engineering career choice.

The significantly higher mathematics *thinking* usage compared to *curriculum mathematics* usage in engineering practice has implications for both mathematics education and engineering education given that students are taught *curriculum mathematics*. Also in work situations, unlike education, engineers' time is often limited and the task value of engaging in lengthy problem solving reduces when students become engineers. A corresponding view in the research literature is that the way experts engage in mathematical practices differ from school mathematics (Ernest 2011; Schoenfeld 1992). This study highlights the importance of problem solving in engineering practice and this is relevant to mathematics teaching in Ireland given the emphasis on problem solving in the new Project Maths Leaving Certificate curriculum. It is also observed in this study that problem solving contributes to interest in and learning of mathematics in secondary Leaving Certificate years. Another major difference between mathematics taught in school and mathematics used in engineering practice is the use of computer solutions. In this study, while it is observed that the availability of sufficient ready-made solutions reduces the degree engineers actively seek a mathematical approach engineers also say that the "quirkiness of computational tools" and their "lack of understanding" and "over reliance of computer analysis" sometimes generate errors.

It is concluded that both the cognitive and affective domains of mathematics education are relevant to engineering practice. Almost two thirds of engineers use higher level Leaving Certificate mathematical knowledge in their work and engineers say their confidence to use mathematics is formed by their school experiences with mathematics. Feelings about mathematics are a major influence on engineering career choice. While affective factors and sociocultural influences contribute to students' interest in mathematics, teacher is the main influence on students' mathematics learning. There is a need to better match the type of mathematics used in engineering practice with that taught in schools and universities. Teaching practical applications and the relevance of mathematics, teaching mathematics communication skills, teaching mathematics thinking modes (problem solving strategies; logical thinking; critical analysis; modelling; decision making; accuracy/ confirmation of solution; precision/use of rigour; organisational skills and reasoning), teachers' own beliefs about mathematics, students' emotional experiences with mathematics and students' value of higher level Leaving Certificate mathematics are identified as essential components in the mathematics education of engineers. Given that students' relationships with mathematics develop in school and their feelings about mathematics are a major influence on engineering career choice, these findings have implications for mathematics teacher training.

While the survey contains some qualitative open questions, some aspects of the survey data are not substantial and in some areas the "why" questions are not sufficiently answered. The subsequent qualitative phase provides for a deeper insight into the research questions and the survey findings.

In conclusion there are five main survey findings:

1. Engineers' feelings about mathematics are a major influence on their choice of engineering as a career.
2. Teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics.
3. While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, *mathematical thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*.
4. Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and role. Their *mathematical thinking* usage is independent of engineering discipline and engineering role.
5. Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation.

CHAPTER 6: INTERVIEW METHODOLOGY & DATA ANALYSIS

6.1 INTRODUCTION

Following the collection and analysis of survey data in this two-phase sequential explanatory mixed methods research study, semi-structured interviews are employed to further investigate: (i) the role of mathematics in engineering practice and (ii) the relationship between students' experiences with school mathematics and their choice of engineering as a career. Its purpose is to explicate and expand on the survey findings. This chapter presents the methodology used for the collection and analysis of interview data and is organised as follows:

Page number

6.2 SELECTION OF INTERVIEW PARTICIPANTS

Purposeful sampling is the dominant sampling strategy in qualitative research whereby information-rich cases are selected to study the research questions in depth (Patton 2002). In the qualitative phase of this study, a purposeful sampling strategy is used to select a diversity of interview participants. A diversity of participants is considered necessary in this phase particularly as it is not possible to verify the randomness of the survey participants or to determine if engineers who have strong opinions about the research topics were over represented in the initial quantitative phase. A maximum variation sampling strategy provides a balanced approach to investigating the research questions and the survey findings in this study. It captures the central themes or principal outcomes that cut across a great deal of participants. Unlike quantitative research, the objective of qualitative research is not to seek generalisability or prediction, instead the focus is on understanding human experience (Crotty 1998). Hence, generally qualitative studies do not involve large and statistically representative sample sizes. There are no minimum sample size requirements, and Collis & Hussey (2009) assert that it is possible "to gain rich and detailed insights of the complexity of social phenomena ... with a sample of one" (Collis and Hussey 2009). While maximum variation sampling can yield detailed descriptions of each participant, for small samples a great deal of heterogeneity can be a problem because individual cases are so different from each other. Patton (1990) says that the maximum variation sampling strategy turns this weakness into a strength because "any common patterns that emerge from large variation are of particular interest and value in capturing the core experiences and central, shared aspects or impacts of a program" (Patton 2002).

Given the sequential nature of the qualitative phase of this study, interview participants were chosen from the sample of engineers who participated in the survey. This allowed for diverse engineering types to be identified from the pool of engineers whose background, educational, work and mathematics usage information were already available. A further advantage of this sampling strategy was that the qualitative phase could build on the outcomes of the quantitative phase of the study and participants could explain why they responded to the survey questions in a particular way thus also enhancing the validity of the overall study. A diversity of participants also contributed to discovering new and objective knowledge as this reduced the possibility that any such knowledge would be biased towards or against any category of engineers or indeed the researchers' own biases, if any.

It is well established that the majority of engineers and engineering students worldwide are male. 7.4% of Chartered Engineers registered with Engineers Ireland are women (Engineers Ireland 2011) and 10.7% of the survey participants are female.

In order to provide an adequate perspective of women's engineering career decisions and their mathematics experiences, the proportion of female interviewees was raised to 25%.

Given that one main aim of the survey phase was to discover new and objective knowledge about professional engineers' mathematics usage in engineering practice generally, interviewees comprised of low, mid and high mean *curriculum mathematics* users in their work as measured in the survey analysis. A diversity of engineering disciplines and roles, a diversity of employers, a diversity of urban and rural backgrounds, a diversity of Leaving Certificate mathematics levels (higher and ordinary levels) and a diversity of engineering education routes (direct entry into level 8 degree courses and progression from level 6 diploma to level 8 degree courses) was also included. It is noted that Chartered Engineers, by requirement, have many years' experience in engineering practice and they are a rich source of information regarding professional engineering practice. However at an early stage in the interview process it became apparent to the researcher that a diversity of engineers' ages was an important factor in the context of the research questions given that interviewees suggested that engineers' roles evolve over their career lifetime. The sample size was increased to accommodate this. In order to capture the broad picture of engineering practice, 25% of the interviewees were specifically selected to be no older than early 30s (or having sat their Leaving Certificate exam no earlier than 1997.) On the other hand, one interviewee was retired and his lifetime perspective of engineering practice was considered relevant to the research questions. A final sample size of twenty engineers gave sufficient variation to the study without overcrowding it with detailed descriptions of too many participants whereby emergent themes and new knowledge would be less visible.

Based on the factors above, the interview participants were selected from the pool of Chartered Engineers who completed and returned the survey questionnaire. Initial contact with the participants was made by email from the researcher (Appendix 5, Volume 2 of this thesis) and further arrangements regarding the interviews were made by telephone. All but two of the initial twenty interview participants selected by the researcher were available for interview and these engineers were substituted with another two Chartered Engineers with similar profiles. A profile of the twenty Chartered Engineers who participated in the interview phase of the study is presented in Table 6-1.

Name	Company Sector	Gender	Engineering Discipline	Engineering Role	Mathematics Usage	Maths Level	LC Year
A	Pharmaceutical	M	Chemical	Design/ Development	1.28	H	1990
B	Telecommunications	M	Electronic / Electrical	Technology Service Sales Manager	1.52	H	1984
C	Project Engineering	M	Mechanical	Design/ Development	1.76	O	1985
D	Project Engineering	M	Mechanical	Project Management	1.88	H	1966
E	Project Engineering	F	Civil	Design/ Development	2.04	H	1997
F	Energy distribution	M	Mechanical	Project Management	2.08	H	1985
G	Electricity distribution	M	Electronic/ Electrical	Commercial	2.09	H	1994
H	Project Engineering	F	Civil, Rail, Water	Design/ Development, Resident Eng.	2.33	H	1997
J	University	M	Biomedical	Education, Research	2.67	A-level	1971
K	IT consultancy	M	Electronic/ Electrical	Information Technology Consultancy	2.68	H	1995
L	Project Engineering	M	Electronic/ Electrical	Design/ Development	2.90	H	1997
M	Consumer electronics	M	Manufacturing / Production	Design/ Development	2.91	H	1991
N	Local authority	M	Civil	Maintenance	3.34	O	1981
O	Software	M	Software	Design/ Development	3.51	H	1979
P	Retired	M	Electronic/ Electrical	General Management	3.53	H	1963
Q	Medical Devices	F	Medical Devices	Design/ Development	3.54	H	2003
R	Local authority	F	Civil	Design/ Development	3.60	H	1980
S	University	M	Electronic/ Electrical	Education	3.84	H	1980
T	Electricity	F	Electronic/ Electrical	Design/ Development	4.17	H	2002
U	Telecommunications	M	Electronic/ Electrical	Design/ Development	4.23	H	1984

Table 6-1: Interview Participants.

Of the twenty engineers, there are five female engineers and fifteen male engineers working in a variety of roles and disciplines and one engineer is recently retired. The overall study is confined to engineers working in Ireland; ten of the twenty engineers work in Dublin, seven engineers work in Cork, two engineers work in Kildare and one engineer is retired. Engineers are assigned alphabetic pseudo names in order of increasing *curriculum mathematics* usage as determined in survey data analysis in Chapter 5. Of the twenty engineers, engineer A has the lowest *curriculum mathematics* usage and engineer U has the highest *curriculum mathematics* usage in their work. It is noted that both C and M have ordinary level Leaving Certificate mathematics, J has A-level mathematics and the other engineers all have higher level Leaving Certificate mathematics. The engineers' Leaving Certificate year (LC year) gives an indication of the engineers' ages whereby students usually sit the Leaving Certificate at age eighteen years. At the time of conducting the interviews engineers are estimated to range in age from twenty seven to sixty six years.

6.3 INTERVIEW DESIGN

The interview design is based on the research questions and the survey findings.

The main research questions are:

1. What is the role of mathematics in engineering practice?
    a) How can mathematics usage in engineering practice be measured?
    b) How do engineers use mathematics in their work?
    c) What motivates engineers to engage, or not, with mathematics?

2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?
    1. To what degree do students' feelings about mathematics influence engineering career choice?
    2. What factors in mathematics education influence students' affective engagement with mathematics?

The survey findings are:

1. Engineers' feelings about mathematics are a major influence on their choice of engineering as a career.
2. Teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics.
3. While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, *mathematical thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*.
4. Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and engineering role. Their *mathematical thinking* usage is independent of engineering discipline and engineering role.
5. Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation.

6.3.1 Interview Protocol

An interview protocol was compiled to assist the semi-structured interview process. This was a list of questions and predetermined inquiry areas that the researcher wants to explore during each interview and it helps to make interviewing multiple participants more systematic. The main objectives of the interviews were to capture the engineers' personal experiences in relation to the research questions and to give a more in-depth exploration of the survey findings. The interview design was organised according to the two main research questions:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

The interviews were limited to two hours maximum and each interview question had a corresponding time limit. Occasionally, after some interviews were complete, the interview protocol was revised. A copy of the final version of the interview protocol is included in Appendix 6, Volume 2 of this thesis.

6.3.1.1 Role of Mathematics in Engineering Practice

The main interview questions relate to the following:

1. What is interviewees' need for higher level Leaving Certificate mathematics in their work? Why don't 32% of engineers who participated in the survey need higher level Leaving Certificate mathematics in their work?
2. What is interviewees' *curriculum mathematics* usage? Why is engineers' overall average *curriculum mathematics* usage, as measured in the survey, in the range "very little" to "a little" (2.735 Likert units)?
3. What is the impact of engineering discipline and role on engineers' *curriculum mathematics* usage? What other factors influence mathematics usage in engineering practice?
4. How do interviewees' rate their *thinking* usage over the course of their careers? What modes of thinking are relevant to their work?
5. Why do engineers, who participated in the survey, rate their *thinking* usage as "quite a lot" and significantly greater than their *curriculum mathematics* usage?
6. What is interviewees' *engaging* usage? Why is engineers' engagement with mathematics in the range "quite a lot" to "a very great deal"?

6.3.1.2 Relationship between Students' Experiences with School Mathematics and their Choice of Engineering Careers

The interview questions relate to:

1. Did the interviewees enjoy school mathematics? Why did 80% of engineers who participated in the survey enjoy school mathematics at the levels of "quite a lot" and "a great deal"?
2. What are interviewees' views about improving young people's affective engagement with mathematics? Why do engineers who participated in the

survey consider usefulness/ practical applications and examples, relevance to modern living, teacher/training and understanding important factors in young people's affective engagement with mathematics?

3. What factors within and outside of school contributed to interviewees' interest in and learning of mathematics? Why are "teacher", "success" and "enjoyment" so important for engineers' school mathematics learning?
4. What are interviewees' school mathematics experiences? What influence had teachers on interest in and learning mathematics? Who are "good" and "bad" mathematics teachers?
5. What are interviewees' feelings about mathematics and learning mathematics? What was the impact of affective factors and sociocultural influences on interviewees' mathematics learning?
6. What was the impact of interviewees' feelings about mathematics on their choice of engineering careers? Why did 75.9% of engineers surveyed say that their feelings about mathematics were a major influence on their decision to choose engineering careers?

6.4 CONDUCTING THE INTERVIEWS

The purpose of the interviews was to capture the engineers' personal stories, to elicit their direct experiences of mathematics learning and usage and their feelings about mathematics in the context of engineering career choice and to explain the survey findings.

Seventeen of the twenty interviews were conducted in the engineers' workplaces, two interviews were conducted in the university where the researcher is a post graduate student and one interview was conducted in the engineer's home. To help put the participants at ease, to build rapport with the interviewees and in accordance with ethical guidelines, each interview opened with a brief description of the study where the researcher discussed the purpose of the research, the format of the interviews, analysis of the data and the proposed publication of any findings. The interviewees were assured of anonymity and were asked to confirm their consent to the audio recording of their interviews. Prior to embarking on the main interview questions, participants were asked about their work and their educational background. The interviews followed the general structure of the interview protocol and interviewees were allowed to present additional concepts that were relevant to the research questions. While the main focus was on the interviewees' own experiences, the interviewees were also questioned in relation to the survey findings. Probing questions were used extensively to extract deeper information from the interviewees. In order to reduce research bias, the researcher avoided leading questions and refrained from commenting on the interviewees' responses. The researcher regularly sought clarification and confirmation that the interviewee's views were interpreted correctly.

Overall the interviews were conducted in a friendly and casual manner. It is noteworthy that the interviewees appeared equally comfortable with discussing factual, positivistic aspects of their mathematics usage as with describing the affective influences such their emotional experiences of school mathematics in school and work. They were open in acknowledging the contribution of various people and factors to their education and careers and they showed no hesitation in criticising other people and factors.

6.5 INTERVIEW DATA ANALYSIS

Qualitative data analysis is an inductive process in that the researcher converts the detailed data into a coherent patterned picture. The main purpose of data analysis is to generate new knowledge or theory that is intellectually rigorous.

Interview data can occupy hundreds of pages of interview transcripts and analysis can be done manually or by computer. Manually sorting and organising interview transcripts are labour-intensive activities and computer analysis is convenient for analysing large data bases. However Johnny Saldaña (2011) recommends doing the analysis manually and this gives the researcher more control over and ownership of the work compared to using software. He adds that “only the human observer can be alert to divergences and subtleties that may prove to be more important than the data produced by any predetermined categories of observation or any instrument" (Saldaña 2011).

Given the subjective nature of interpretivism there is not a single correct way of analysing qualitative data. King and Horrocks (2010) say that the "researcher's subjectivity shapes the research process" and that it is highly unlikely that two different researchers using the same methodology would produce the same findings in qualitative studies. Instead the advantage of qualitative research is the richness and context of the data and hence methodical rigour is essential with analysing the data (King and Horrocks 2010). Miles and Huberman (1994) describe qualitative analysis as "a form of analysis that sharpens, sorts, focuses, discards and reorganises the data in such a way that final conclusions can be drawn and verified" (Miles and Huberman 1994). Bogdan and Biklen (1997) define qualitative data analysis as "working with data, organizing it, breaking it into manageable units, synthesizing it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others" (Bogdan and Biklen 1997). Saldaña (2011) describes qualitative analysis as "the search for patterns in data and for ideas that help explain why those patterns are there in the first place" (Saldaña 2011).

Qualitative analysis generally involves coding the data and identifying themes of interest that emerge from the data. Saldaña (2011) describes coding as the "transitional process between data collection and more extensive data analysis … it is the initial step toward an even more rigorous and evocative analysis and interpretation … coding is not just labelling, it is linking, it is a method that enables similarly coded data to be organised and grouped into categories". Saldaña adds that coding is not an exact science instead it is an interpretative process and it is the researcher's "judgement call" (Saldaña 2011).

King and Horrocks (2010) present a three stage process of thematic analysis that includes descriptive coding, interpretative coding and overarching themes. The descriptive phase is about identifying and labelling parts of the transcript data that are likely to be helpful in addressing the research questions. The interpretative phase is about grouping together descriptive codes that share some common meanings to create interpretative codes. The third stage is about identifying overarching themes that characterise key concepts. King and Horrocks say that as the researcher's thinking about the coding process develops there is a need to redefine codes and to go back over coded transcripts and reapply the new codes. Saldaña also says that coding is a cyclical process and that "subsequent cycles further manage, filter, highlight, and focus the salient features of the qualitative data record for generating categories, themes, and concepts, grasping meaning, and/ or building theory". His model of qualitative analysis is one where clusters of coded data are grouped into categories and he says that when "major categories are compared with each other and consolidated in various ways, you begin to transcend the reality of your data and progress toward the thematic, conceptual and theoretical" (King and Horrocks 2010).

Themes are derived from patterns within the data such as topics, meanings and feelings. King and Horrocks define themes as "recurrent and distinctive features of participants' accounts, characterising particular perceptions and/ or experiences, which the researcher sees as relevant to the research question" (King and Horrocks 2010). Saldaña says that "a theme captures and unifies the nature or basis of the experience into a meaningful whole … the analytic goals are to winnow down the number of themes to explore in a report and to develop an "overarching theme from the data corpus, or an "integrative theme" that weaves various themes together into a coherent narrative"(Saldaña 2011).

In this study a manual approach to data analysis was chosen because it allows the author to have greater control over the data analysis compared to doing the analysis using computer software. The first stage of analysing the interview data was to transcribe the audio taped interview conversations into a word document. The entire interviews were transcribed verbatim and this allowed the researcher to become familiar with the participants and their stories and it also preserved the integrity and meaning of the participants' views for subsequent data analysis. While it was tempting to correct mispronunciations and bad grammar, the purpose of transcription was not to produce a corrected version of what the interviewees said but an accurate account. However when presenting quotes to support analysis in a research paper or dissertation, it is appropriate to carry out minor tidying up without distorting the meaning in order to aid comprehension (King and Horrocks 2010).

In order to protect the identity of the interviewees, pseudonyms were used instead of the actual participant names. The twenty interviewees were identified alphabetically and in accordance with increasing mean overall *curriculum mathematics* usage as determined in the statistical analysis of their survey data, table 6-1.

While the goal of qualitative data analysis is to produce a consolidated picture of the research data, King and Horrocks (2010) maintain that a challenge in qualitative analysis is about getting the right balance between within-case and cross-case analysis (King and Horrocks 2010). Within-case analysis is about individual experiences and cross-case is about analysis of the group of participants as a whole. While the main focus of this study is about cross-case analysis, however given the diversity of engineering disciplines, roles and work in this study, attention to individual cases is also warranted. A further advantage of the dual approach of employing both within-case and cross-case analysis compared to just cross-case analysis is that the researcher becomes more familiar with the qualitative data and there is greater confidence about the overall quality of the data analysis.

6.5.1 Engineers' Stories

During the interview transcription process, it became apparent that the interview data comprised of engineers' stories about their background, their education experiences, their career decisions and their work in engineering practice. Aspects of some engineers' stories were strikingly similar to other participants' stories. A number of broad patterns of common themes, relating to the research questions, were immediately apparent across the interview data. These include: the impact of the engineers' background and family on their education and career choice, engineers' decisions to study engineering, the nature of engineers' work, engineers' Leaving Certificate mathematics experiences, engineers' experiences of mathematics in engineering education, engineers' use of mathematics in their current job, engineers' views about what engineering is, engineers' views about the engineering profession and engineers' general views about mathematics.

The diversity of interview participants in the context of their engineering disciplines, roles and work warranted an initial within-case analysis of the interview data whereby individual engineers' interview data were analysed separately. In this analysis each participant's interview data was individually studied with a view to identifying the participants' views and experiences of topics relating to the research questions. This resulted in a summary of each participant's interview data under the following broad headings:

- Gender
- Background
- Family
- Leaving Certificate mathematics level
- Education
- Decision to study engineering
- Current work
- Chartered Engineer
- Leaving Certificate mathematics
- Engineering mathematics
- Use of mathematics in current job
- What is engineering?
- Views on engineering
- Views on mathematics

The engineers' stories are included in Appendix 7 in Volume 2 of this thesis.

6.5.2 Coding the Data

The initial within-case analysis, as well as documenting the engineers' stories about their education and careers allowed the researcher to become familiar with the data and to contemplate how a consolidated picture of the research data could be produced.

The first step in the cross-case analysis was open-coding which sought to identify sections of the transcript data that were likely to be helpful in addressing the research questions. After reading each transcript, passages of text deemed relevant to the research questions were highlighted. Subsequently this data was placed in the left hand column of a table containing four columns. Notes as to the interest in and/ or the relevance of the data to the research questions were placed in the corresponding second column of the table. This process was repeated for each of the twenty interview transcripts. The entire transcripts were then re-read and the highlighted text was either discarded or added to and the remaining data (text) was assigned a descriptive code which was noted in the third column of the table. The descriptive codes used were short self-explanatory phrases used to label the highlighted text. New codes were assigned as required. The process was repeated for all transcripts. As the coding process developed there was a need to go back to earlier transcripts and modify the codes. In the first cycle of coding, 107 descriptive codes were identified and these are included in Appendix 8 in Volume 2 of this thesis.

6.5.3 Identification of Themes

The next stage of data analysis involved reducing the number of codes by grouping together sections of interview transcripts corresponding to the 107 descriptive codes that shared some common meaning or pattern. This resulted in nineteen interpretative codes which were included in the fourth column of the table. The entire interview transcripts were then reread and recoded according to the nineteen interpretative codes. As the process developed there was a need to redefine and reapply the interpretative codes while keeping the research questions in mind. Extracts from the interview transcripts relating to each of these codes were studied with a view to developing overarching themes that characterised key concepts of the analysis. When the process was completed, ten themes, four of which had sub-themes emerged from the data. King and Horrocks (2010) define themes as “recurrent and distinctive features of participants’ accounts, characterising particular perceptions and/ or experiences, which the researcher sees as relevant to the research question” (King and Horrocks 2010). The 107 descriptive codes and the ten themes are included in Appendix 8, Volume 2 of this thesis. The ten emerging themes are:

Theme 1: School mathematics

1. Subject
2. Teaching

Theme 2: Motivation to engage with mathematics

1. Family
2. School
3. College/ university
4. Engineering practice
5. Outside of engineering
6. How to improve young people’s affective engagement with mathematics

Theme 3: Factors influencing engineering career choice

1. Engineering career choice influences
2. The engineering profession
3. Modern young people’s career choices

Theme 4: Engineering practice, roles and activities

1. Engineering practice
2. Roles and activities
3. Use of resources

Theme 5: Career development paths in engineering practice

Theme 6: Engineering practice, *curriculum* mathematics usage

Theme 7: Engineering practice, mathematics *thinking* usage

Theme 8: Engineering practice, communicating mathematics

Theme 9: Engineering practice, *engaging* with mathematics

Theme 10: Relevance of engineering education to engineering practice

6.6 SUMMARY

Qualitative data concerning the two main research questions and the survey findings was collected from a diversity of twenty Chartered Engineers using semi-structured interviews. A manual data analysis process was employed to interpret the data from both a within-case and a cross-case perspective. Resulting from the analyses are (i) the personal stories of twenty Chartered Engineers concerning their use of mathematics in work and their relationship with school mathematics and (ii) ten emergent themes relating to the research questions. The interview findings are presented in Chapter 7.

CHAPTER 7: INTERVIEW FINDINGS

7.1 INTRODUCTION

The interviews formed the second phase of the sequential explanatory strategy mixed methods design employed in this study. Analysis of the interview data generated the career stories of twenty Chartered Engineers (Appendix 7, Volume 2) and identified ten emerging themes (Chapter 6) relating to the main research questions:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

This chapter presents the interview findings and is organised as follows:

Page number

7.2 EMERGING THEMES

In this section the interview findings are presented according to ten themes identified from an analysis of the interview data in Chapter 6. The findings are the results of interviews conducted with twenty Chartered Engineers, who are identified by alphabetic pseudo names in increasing order of *curriculum mathematics* usage as determined in the survey analysis in Chapter 5. The sample of twenty engineers interviewed represent a diversity of gender, industry type, engineering discipline, engineering role, *curriculum mathematics* usage (based on a score of 5), Leaving Certificate mathematics standard (higher, H or ordinary, O levels) and year of Leaving Certificate (LC), Table 7-1. The ten emerging themes, identified in the interview data analysis (Chapter 6), are:

Theme 1: School mathematics

1. Subject
2. Teaching

Theme 2: Motivation to engage with mathematics

1. Family
2. School
3. College/ university
4. Engineering practice
5. Outside of engineering
6. How to improve young people's affective engagement with mathematics

Theme 3: Factors influencing engineering career choice

1. Engineering career choice influences
2. The engineering profession
3. Modern young people's career choices

Theme 4: Engineering practice, roles and activities

1. Engineering practice
2. Roles and activities
3. Use of resources

Theme 5: Career Development Paths in Engineering Practice

Theme 6: Engineering practice, *curriculum mathematics* usage

Theme 7: Engineering practice, *thinking* usage

Theme 8: Engineering practice, communicating mathematics

Theme 9: Engineering practice, *engaging* with mathematics

Theme 10: Relevance of engineering education to engineering practice

Name	**Gender**	**Company Sector**	**Engineering Discipline**	**Engineering Role**	**Mathematics Usage**	**LC Mathematics Level**	**LC Year**
A	Male	Pharmaceutical	Chemical	Design / Development	1.28	H	1990
B	Male	Telecommunications	Electronic / Electrical	Technology Service Sales Manager	1.52	H	1984
C	Male	Project Engineering	Mechanical	Design / Development	1.76	O	1985
D	Male	Project Engineering	Mechanical	Project Management	1.88	H	1966
E	Female	Project Engineering	Civil	Design / Development	2.04	H	1997
F	Male	Energy distribution	Mechanical	Project Management	2.08	H	1985
G	Male	Electricity distribution	Electronic/ Electrical	Commercial	2.09	H	1994
H	Female	Project Engineering	Civil, Rail, Water	Design / Development, Resident Eng.	2.33	H	1997
J	Male	University	Biomedical	Education, Research	2.67	A-level	1971
K	Male	IT consultancy	Electronic/ Electrical	Information Technology Consultancy	2.68	H	1995
L	Male	Project Engineering	Electronic/ Electrical	Design / Development	2.90	H	1997
M	Male	Consumer electronics	Manufacturing / Production	Design / Development	2.91	H	1991
N	Male	Local authority	Civil	Maintenance	3.34	O	1981
O	Male	Software	Software	Design / Development	3.51	H	1979
P	Male	Retired	Electronic/ Electrical	General Management	3.53	H	1963
Q	Female	Medical Devices	Medical Devices	Design / Development	3.54	H	2003
R	Female	Local authority	Civil	Design / Development	3.60	H	1980
S	Male	University	Electronic/ Electrical	Education	3.84	H	1980
T	Female	Electricity	Electronic/ Electrical	Design / Development	4.17	H	2002
U	Male	Telecommunications	Electronic/ Electrical	Design / Development	4.23	H	1984

Table 7-1: Profile of interviewees.

7.2.1 Theme 1: School Mathematics

The findings concerning the engineers' views of their school mathematics are presented in this section. Theme 1 is presented as follows:

Page number

7.2.1.1 Mathematics is different compared to other school subjects

All engineers are of the view that mathematics is different from the majority of other school subjects. There is a view that mathematics is different because it "looks different" to many other school subjects. Mathematics looks different because compared to the interesting stories in many other subjects, mathematics consists of formulae and symbols. Mathematics learning requires understanding the concepts while learning many other subjects is about retaining information. Engineers find it easier to learn mathematics by developing understanding compared to memorising as in other subjects. However without understanding students can "fall behind" very quickly. The processes of learning mathematics and problem solving require a lot of practice and hence mathematics learning is time consuming. A major difference between mathematics and other subjects is that mathematics focuses on getting the right answer and other subjects lean towards subjective analysis. Engineers like having "a right answer" because it removes the subjectivity from exam grades. Mathematics has an extra dimension compared to other subjects; mathematics learning involves application in different contexts or situations.

7.2.1.1-1 Mathematics looks like formulae and symbols, not interesting stories

Some engineers say that because of the numerical nature of mathematics, the subject looks different to many other school subjects (D, F, H, N, Q, and R). Examples of engineers' views include: "because it is so numerical mathematics is different to other school subjects" (H); while most subjects have interesting stories school mathematics "is numbers" (Q); "you've got a good story" in history while mathematics is about "breaking everything down into bite sized bits" (R); mathematics comprises "hard figures" compared to "the softer stuff" (F); and mathematics comprises "formulae and symbols" and "looks different" to other subjects (N).

7.2.1.1-2 Mathematics learning requires understanding, not information retention

The interview data shows that the process of learning mathematics is different to learning other school subjects. Engineers say that mathematics learning is a "process" of problem solving and/ or application of mathematics and that "understanding" is an essential part of learning unlike other school subjects where learning is about "information retention" and "regurgitation"(A, B, C, D, E, F, G, H, J, K, L, M, N, O, Q, R, S, T, and U). The majority of engineers describe mathematics as a "process" or an activity: mathematics is "a very well defined process coming to a well-defined solution"(A); mathematics is a "logical process" (E); mathematics is the "process" of solving problems and students have to "figure it out for themselves" (Q); mathematics is a "process of understanding" (K); learning mathematics is "trying to work it out and get the solution" (M); mathematics is breaking everything down into bite sized bits" (R) and mathematics learning involves "getting on top of various concepts"(J) and mathematics is different from other subjects because "maths has an application" (T).

While the engineers say that learning many school subjects is about knowledge retention, there is no overestimating the engineers' views on the value of understanding in mathematics learning. J asserts that the key to mathematics learning is "finding that you are able to do it" and this "unique skill doesn't come up much in any of the other subjects". Memory is not important in mathematics because understanding the concepts enables students to go back to first principles and work out formulae (J, K, and T). For example, T used to “prove the theorems in an exam rather than learn them off” and she is of the view that even though “rote learning” is effective in most other subjects, people who attempt to learn mathematics “by regurgitation struggle to understand mathematics”. With mathematics there is a need to understand certain concepts from first principles and other subjects don’t have “the same depth” (T). Learning without understanding doesn’t work well for higher level Laving Certificate mathematics (L). “You really truly have to understand it [mathematics] and not just learn it” and understanding is like “an individual concept “where every person takes responsibility” for their own understanding (K). “Understanding is essential to mathematics learning and you can see it in students’ face when they grasp a mathematics concept” (Q). “Rote learning” does not work for mathematics and because mathematics contains “abstract concepts” and “vague ways of quantifying things”, it requires a higher level of understanding than many other subjects (S). G describes mathematics as “a building block type” of subject where learning is “based on building on the fundamentals” and without a good “foundation” students “won’t get a grasp” of a particular concept. Similarly C is of the view that each mathematics topic is related to the previous topic and that an understanding of each topic is necessary prior to moving on to the next topic. When students “get stuck” in higher level mathematics, they can “fall behind” very quickly (H).

7.2.1.1-3 Mathematics is about getting the right answer, not subjective analysis

The majority of engineers contrast the quantitative nature of mathematics with the qualitative nature of other subjects. They say that in mathematics the focus is on the right answer while other subjects are not as precise (A, B, C, D, E, F, G, H, K, L, M, P, Q, R, T, and U). With “maths there had to be the right answer” and the other subjects don’t have “right and wrong” answers (M). Mathematics is a “special” subject because it is “unique, it’s precise, there is a right answer” and because of the “precise nature” of mathematics “you can’t bluff it” (D). Compared to other subjects, mathematics is “a very well defined process coming to a well-defined solution” (A) and mathematics has a "wider opportunity" for success because of its "quantitative" nature" (B). In mathematics "you get your answer right or you get it wrong and you either get an A[20] or a D[21] grade" and while "you could wing the English paper" and "get a few marks, you couldn't do that in maths" (E). "English is so subjective" in that "no matter how much work" H put into it, her best grade ever was a "C1[22]". She feels that mathematics "is not so subjective" and that one's mathematics exam grade "is directly related to the effort you put into it". Q likes the fact that there was "a right answer" in school mathematics and that she could "do a sum in half a page and still get full marks … whereas in English you could write three pages of waffle" and not get full marks. Q says that by "checking the units … you always knew if you got the right equation" and that "checking the answer is something you do in maths". She says she "wanted to just write the answer that was all I wanted to write. I didn't want to write three pages of an answer".

7.2.1.1-4 Understanding mathematics is easier than memorising other subjects

While most subjects are perceived to have varying degrees of difficulty, mathematics is perceived as either difficult or easy. The majority of engineers interviewed have a view on the perceived difficulty of mathematics learning (A, B, C, D, E, G, H, J, K, L, M, N, O, Q, R, S, T, and U).

There is a view that because learning mathematics is more about understanding than memorising, it is easier than many other subjects (A, E, H, L, N, O, Q, T, and U). A was "much stronger in process" type learning, which he describes as "understanding" rather than "information retention". E "found it [mathematics] to be one of the easier subjects to do for homework" as it didn't involve learning. Mathematics was "easier" than "English, history, Irish and all other languages" because it "wasn't sitting down learning stuff off by rote" (H). Compared to subjects that required a lot of memorising "it was never a chore to do maths" (L). N says it "took me ages to get my mind around

---

20 Grade A: ≥85%
21 Grade D: ≥40%, <55%
22 Grade C1: ≥65%, <70%

it [mathematics]" and when he "saw the point of it … it clicked … it wasn't that difficult then". "Because maths is so much understanding based you don't forget it as easily" as the other subjects (O). T describes mathematics as "a risky subject" in that there is a "risk that you are not going to be able to work it out" and that "no matter what you do; you will always get a C[23] in English but you can get an A in maths". Compared to other subjects school mathematics "was more numbers and less learning (U).

7.2.1.1-5 Problem solving/ precise nature of mathematics is time consuming

Some engineers say that due to the "problem solving nature" or the "precise nature" of mathematics, they found the subject difficult or time consuming (A, B, C, G, and M). While Leaving Certificate mathematics is not "particularly difficult" it is "time consuming" because it is "a lot about practice" (A). Other subjects have a "wider opportunity" for success because of their "qualitative" nature and higher level Leaving Certificate mathematics is a "hard grind" compared to most other Leaving Certificate subjects (B)". Leaving Certificate mathematics is "hard" and "time consuming" because with mathematics "you had to think on the spot" while other subjects were about "regurgitating stuff" (G). Higher level Leaving Certificate mathematics is "a mixture of a hard subject and a huge amount of time" and most of M's study time was spent "trying to figure the stuff out and get the solution".

7.2.1.1-6 Mathematics is a diverse subject with an applications dimension

Some engineers are of the view that, because mathematics is a diverse subject, there are some parts of mathematics that are conceptually difficult to understand (D, K, J, N, R, S, T, and U). In secondary school, R says she believed that "it was only boys who had the ability to grasp most of higher level Leaving Certificate maths." "Mathematics is quite abstract in some ways" and for "some people there is genuinely an inability to appreciate abstract concepts" (K). T is of the view that functions and statistics are

[23] Grade C: ≥55%, <70%

“abstract” and both D and N never understood the “concept of a function”. Many teachers and students have particular difficulty understanding statistics (U) and “probability and statistics while useful, always seems to be one of these vague ways of quantifying things” (S). Statistics is “conceptually quite different” to the rest of mathematics and there are “a lot of people who are good at maths, who hate statistics … maybe because it often isn’t well taught, but certainly it requires a different mind-set” (J). “Some areas of maths need to be applied as opposed to just straight studied” and “to understand statistics and probability, it would need to have an application”(N). Students must engage in “transfer learning” whereby students “take what they learn and transfer it to a slightly different context or situation” and “learning happens when the student manages to make that little extra step” from the knowledge “they are comfortable with and that makes sense to them … to solve this new but related problem”(S).

7.2.1.1-7 Higher level Leaving Certificate mathematics is at a higher standard compared to other school subjects

There is a view that higher level Leaving Certificate mathematics is at a high standard relative to other subjects (C, D, H, M, and S). Even though C took ordinary level mathematics for his Leaving Certificate, he is of the view that higher level Leaving Certificate mathematics requires “so much more work” compared to other subjects. Leaving Certificate mathematics covers more “material” and is at a higher “standard” than other school subjects (D). Mathematics requires a higher level of understanding that many other subjects (S). H is of the view that there “is no comparison” between higher level and ordinary level Leaving Certificate mathematics. She asserts that when students “get stuck” in higher level mathematics, they can “fall behind” very quickly and are likely to change to ordinary level mathematics which is at a much simpler level. She is of the view that ordinary level “maths needs to be a bit more challenging”. Similarly M is of the view that ordinary level Leaving Certificate mathematics requires only “a fifth of the work” necessary in higher level Leaving Certificate mathematics.

Due to the higher level of understanding required in learning mathematics compared to other subjects and the application of mathematics in different contexts, there is a view that mathematics gets difficult quicker than other subjects (J, O). For example, J says that mathematics “gets difficult in a kind of a non-linear way, in an exponential way, that if you want to go to the next level of difficulty you are always looking at the upturning curve”.

7.2.1.2 Good mathematics teachers transform students’ mathematics learning and their enjoyment of the subject

Nineteen of the twenty engineers say they had good mathematics teachers at some stage throughout their school years. The engineers express a very strong view on the importance of good mathematics teaching. The ability to communicate mathematics is the predominant feature of good mathematics teaching. Many engineers also noted that their good mathematics teachers encouraged the students, challenged the students, knew mathematics and they were strict. When asked to describe the characteristics of good mathematics teachers generally, the engineers say that good mathematics teachers are “positive” about mathematics and they encourage the students to engage in the subject, they know mathematics, they are able to teach students with differing abilities and learning styles, they show students the relevance of mathematics in the real world, they are attractive to students and they are organised and disciplined.

There is no overestimating the engineers’ views of the impact of good mathematics teaching on students’ performance (A, B, C, D, E, G, H, J, K, M, N, O, P, Q, R, S, T, and U). Many engineers say that good mathematics teachers transformed their mathematics learning and their enjoyment of the subject and many others note the importance of a good grounding in mathematics. In addition to mathematics learning, the engineers’ positive feelings about mathematics are a very significant outcome of having had good mathematics teachers. “A good maths teacher will create more successful students, who will then probably enjoy it more … they will get more recognition and the fear will lessen” (O). When N changed from a “very poor” mathematics teacher to a good one, he “saw the point of it [mathematics] … it clicked” and it “wasn’t that difficult then”. O’s new mathematics teacher “transformed” him “from being someone who didn’t like maths or didn’t care about it to someone who loved it” and he “went from being this average student to being someone who was in the top five in the school”. O asserts that “the power that a teacher has to capture some child’s imagination and make them think they like something is immense and so it just drives everything else”. He says that his own “fabulous” mathematics teacher “woke up” the mathematics in him and “it’s frightening in a way to think that if I never had him I might have never liked maths and that my life would have been different”. Similarly when R switched from a public school to a grind school[24], she says her new mathematics teacher “was a revelation in that he “totally revitalised her feelings of what maths was about” and she began to think that “maths were easy”. S “was a real problem student in primary school” but when he got a new teacher, he says “the change that happened as a result was amazing”. According to A, “Leaving Cert students are hugely influenced by individual teachers”. C’s “maths teacher had probably a big influence” on his “enjoyment of maths”. E maintains that teaching is the “number one” factor in mathematics education. Compared to her twin sister, who struggled with ordinary level mathematics and whose mathematics teacher “didn’t have a clue what she was doing”, E “loved secondary school maths” because her teacher “knew the maths and she was able to teach it well”. “Good teachers were probably the biggest single thing within school” that impacted U’s mathematics education. Mathematics “teaching is very important from an early age” and many young children who “don’t get a chance to learn the basics” have “very negative experiences of maths” (G). With “a good grounding” in mathematics K was “ahead on the maths when compared to your [his] peers in secondary school”. H notes that, in school, students depend on the teacher and she attributes her own enjoyment of school mathematics to “very good teachers”. J asserts that teachers have “an enormous effect in all subject areas” and when “you get on with a particular teacher things just work for you”. M “relied” more on the teacher as the “maths got tougher”. According to P, “bright kids without good teachers only achieve part of their

[24] In Ireland, grinds are private tuition, grind schools are private secondary schools that provide students with intensive coaching in preparation for Junior Certificate and Leaving Certificate exams.

potential". Only twelve girls in Q's class of one hundred and fifty took the higher Leaving Certificate mathematics exam because the mathematics teacher was "on a different wavelength" to most of the students.

T went so far as to self-teach Leaving Certificate mathematics because she was of the view that she would not realise her full potential in the mathematics class where "the pace was a bit slow". D says that due to his "very poor grounding" in mathematics, he was "afraid" of the subject right through university and work. B is concerned that his own son "has lost maths" because his teacher is "introverted and neurotic". Similarly O is concerned about his own daughter's feelings for mathematics whereby the previous year her teacher inspired her "to become someone who loved maths" and subsequently with a new teacher who "isn't great", O feels that his daughter's "maths may well fall off". Despite O's own love of mathematics there is nothing he can do to help. He says "it is all down to the teaching … it has to be the teacher".

7.2.1.2-1 Who are "good" mathematics teachers?

All of the engineers, with the exception of D, acknowledge that they had some very good mathematics teachers in school (A, B, C, E, F, G, H, J, K, L, M, N, O, P, Q, R, S, T, and U). A profile of the engineers' mathematics teachers is included in Volume 2 of this thesis in Appendix 9, Table A9-2.

7.2.1.2-1-1 The ability to communicate mathematics is the predominant feature of good mathematics teaching

From the engineers' experiences of their "good" mathematics teachers, it is noted that the ability to communicate mathematics is the predominant feature of good mathematics teaching (A, B, C, F, J, M, N, O, Q, R, S, T, and U). The importance of communication in mathematics teaching is illustrated by many engineers: A's teacher had an "ability to explain" mathematics; B's teacher also "explained maths well"; C's teacher didn't make mathematics "confusing" and "she waited for you to understand it before moving on"; one of F's good mathematics teachers would engage with individual students on particular areas of mathematics they were weak at; H's teacher was "never boring"; J and K's teachers made mathematics "interesting"; M's teacher "just connected with people through maths"; N's teacher "pitched maths at our level" and he "made sure that we understood something before moving on to the next topic"; O's "legendary" mathematics teacher "held our attention, he could tell a good story and he did tell a good story", his "history of maths" just captured O's imagination and he showed O "how the solutions were so wonderful and beautiful and just cool"; Q's teacher "sat down beside me [her] … and explained it"; R's grind school teacher "came in with a smile on his face and told us about the maths in everyday things we use" and he "explained the problems"; S's "famous" mathematics teacher made mathematics "interesting" and "he kept throwing, what to me [S] were interesting examples up on the board and then following them with interesting problems"; when T's primary school mathematics teacher was explaining maths problems, he would relate it back to practical examples" and U's school principal showed U that "presentation and showing how you got the right answer" was as important as the correct answer.

While D is the only engineer not to have experienced good mathematics teachers in school, some other engineers, in addition to having some good mathematics teachers, also encountered weak mathematics teachers. F's "bad" mathematics teacher was reluctant to take questions from the class; K's "poor" Leaving Certificate mathematics teacher "had no interest in answering questions". N, who took the ordinary level mathematics exam for his Leaving Certificate had "very poor" teachers in Junior Certificate. R's Leaving Certificate mathematics teacher is described as "manic depressive". However D stands out for having the worst mathematics teacher. He says his Leaving Certificate mathematics teacher was "plain ordinary bad" because the teacher "just could not explain the consequence" of any mathematics topic".

7.2.1.2-1-2 Good mathematics teachers are "positive" about mathematics and teaching

Good mathematics teachers are "positive" about mathematics and they encourage students to engage in the subject (B, C, D, F, G, H, J, M, N, O, P, Q, S, and T). The engineers' descriptions of good mathematics teachers include: "mathematics needs to be illustrated in a very positive way"; higher level Leaving Certificate mathematics requires a teacher that is "enthusiastic to the point where he can foster interest and enthusiasm for the subject with a broad profile of students within the classroom"; "someone to explain it [mathematics] to them [students] or motivate them"; teachers who are "very interested in actually the maths and very interested in teaching"; someone who gives kids "the message" that being good at maths "opens up a huge number of careers"; someone who presents mathematics in a way that "is fun and it's easy because you don't have to remember it"; someone who encourages students "to see that actually if they just remember one thing, then they can derive all of these other things"; someone who "challenges both good and bad students to do better"; teachers who "encourage" students; teachers "who love maths"; teachers who "can inspire"; a teacher who encourages "weak" students; teacher who "challenges" brighter students; teachers who are "enthusiastic" about mathematics; a teacher who "reinforces the point that everyone can do it"; teachers who "pass on an appreciation" of mathematics; a teacher who is "nice or funny" and teachers who "encouraged people to do a little bit of work on their own". H presents that "bad" teachers" have poor attitudes and they often label specific parts of course as "too hard" and they do not teach the entire syllabus. D asserts that "teachers' attitudes submerge all other issues" in mathematics education.

7.2.1.2-1-3 Good mathematics teachers know mathematics

Teachers need to know their subject and be comfortable with mathematics (A, B, C, H, L, M, O, and T). They need to be "confident in their ability" to "field any kind of a question" presented by students in mathematics classes. There is a strong view that mathematics teachers should have a mathematics qualification. For example, O says that it is well known that “the teachers who have maths qualifications” are assigned to the Leaving Certificate students and that first and second year students in secondary schools get the majority of the “unqualified” teachers. O says that young students, instead of getting the teacher “who loves maths … knows all about it … can answer any questions that is thrown at them … can inspire”, get the teacher “who is thinking I hope they don’t ask any questions because I don’t really know this thing”. C states that “honours maths[25] challenges a certain amount of teachers … if you have a damned good honours maths student, he is not just interested in the curriculum, he is pushing the boundaries and I think very few teachers are comfortable with that”.

7.2.1.2-1-4 Good mathematics teachers are able to teach a broad profile of students

Mathematics teachers need to be able to teach a “broad profile of students within the classroom” (B, G, K, N, P, Q, S, and T). A good mathematics teacher is someone who recognises the different paces of children picking up the “fundamentals” of mathematics (G) and someone who will encourage children and also ensure that the advanced children “don’t get bored”(K). Mathematics teachers have a responsibility to the students who “struggle to understand” and also to the students who “get bored when the poor students are driving the pace of the whole class” (T). Mathematics students “learn differently” … “you would nearly need to be a psychologist as well, you would need to be aware that you’re taking one approach explaining this and you need to be prepared to have some students get that and go off and do some work while you flip it around and explain it in a totally different way” (Q). “The single most important piece of information that a teacher can have about a student is their level of prior knowledge in the topic that you are trying to teach them” and mathematics teaching is about “building on” students’ “prior level of knowledge” (S).

[25] Honours maths: Higher level Junior Certificate or Leaving Certificate mathematics

7.2.1.2-1-5 Good mathematics teaching illustrates the relevance of mathematics

Good mathematics teaching involves showing students the relevance of mathematics in the real world (B, F, K, N, R, and T). “If I was teaching maths, I would relate it to everything around me. I would not do area without getting somebody to measure the floor. I would not do volumes without getting somebody to measure the volume of the room. I would not do liquids without bringing in a can of paint or a can of liquid or a bucket of water and that was never done when I was in school” (R).

7.2.1.2-1-6 Students are attracted to “interesting” teachers

Students are attracted to “interesting” teachers (C, F, J, O, and Q). Interesting teachers “pass on an appreciation” of mathematics and often they are “nice or funny” (Q). Good mathematics teachers present mathematics in a way that “is fun and it’s easy because you don’t have to remember it” (J).

7.2.1.2-1-7 Good mathematics teachers are organised and strict

Good mathematics teachers need to be organised and strict (A, H, and T). It is “important” to have a “rigorous and disciplined” mathematics teacher as mathematics “is much about precision” (A) and mathematics teachers “should be organised” when planning the mathematics lessons and they should teach the “entire syllabus” (T).

7.2.1.3 Discussion of theme 1

There are two main findings (F1.1 and F1.2) associated with theme 1, these are:

F1.1 Mathematics is different compared to other school subjects.

F1.2 “Good” mathematics teachers communicate mathematics well; they are positive about mathematics and teaching; they know mathematics; they are able to teach a broad profile of students; they illustrate the relevance of mathematics; they are interesting; and they are organised and strict.

7.2.1.3-1 F1.1: Mathematics is different compared to other school subjects

All twenty engineers are of the view that mathematics is different to the majority of other school subjects. Because of the numerical nature of mathematics, the subject looks different to many other school subjects and unlike many subjects, mathematics doesn't have an interesting story. Unlike other school subjects where learning is about "information retention" and "regurgitation", mathematics learning is a "process" of problem solving and/or application of mathematics and "understanding" is an essential part of learning. Mathematics learning is "based on building on the fundamentals" and an understanding of each topic is necessary prior to moving on to the next topic. Compared to most other subjects, mathematics learning has an extra dimension which is applications. One engineer calls this "transfer learning" whereby students "take what they learn and transfer it to a slightly different context or situation" and this "learning happens when the student manages to make that little extra step" from the knowledge "they are comfortable with and that makes sense to them … to solve this new but related problem". Because mathematics learning is about building understanding of concepts and situations, there is of the view that when students "get stuck" in mathematics, they can "fall behind" very quickly and are likely to change from higher level Leaving Certificate mathematics to ordinary level mathematics which is at a much simpler level.

A major difference between mathematics and other subjects is that mathematics focuses on getting the right answer whilst other subjects lean towards "subjective analysis". Because of the precise nature of mathematics "you can't bluff it" and one's mathematics exam grade "is directly related to the effort you put into it" and "no matter how much work" one puts into the "subjective" subjects one might not get "full marks". The right answer in mathematics enables students to objectively check their work prior to the teacher grading it.

Another difference between the subjects is the perceived difficulty of mathematics learning compared to other subjects. Nine of the twenty engineers say mathematics is easier than most other subjects because the "process" type learning of understanding and problem solving in mathematics is easier than memorising information and facts in other subjects. One engineer describes his mathematics learning as "more numbers and less learning" compared to the other subjects. The problem solving nature of mathematics is time consuming: mathematics learning is "a lot about practice; it is about "trying to figure the stuff out"; and unlike many other subjects students spend considerable amounts of their mathematics homework time "looking for a specific answer". Mathematics is a diverse subject. There are some parts of mathematics that are conceptually difficult to understand and cannot be directly learned. There is a view that *statistics and probability* "need to be applied as opposed to just straight studied".

The engineers' view that mathematics is different to the majority of other school subjects is consistent with views in research literature. For example, Smith (2004), in section 2.2.3, describes mathematics as "special" and he identifies what is widely known as the 'mathematics problem' where mathematics education "fails to meet the mathematical requirements of learners, fails to meet the needs and expectations of higher education and employers and fails to motivate and encourage sufficient numbers of young people to continue with the study of mathematics post-16" (Smith 2004). The view that mathematics comprises symbols and abstract ideas compared to interesting stories in many other subjects is supported by research literature in section 2.23 (Brown and Porter 1995; Nardi and Steward 2003; Skemp 1987).

The engineers' view that mathematics learning is "based on building on the fundamentals" is supported by Ridgway (2002) in section 2.2.3 of this thesis who describes mathematics as a "hierarchical subject" (Ridgway 2002) and by the NCTM's "Principles and Standards for School Mathematics" in section 2.3.2 where the need to learn mathematics with understanding by actively building new knowledge from existing knowledge is highlighted (National Council of Teachers of Mathematics 2000). Engineers are of the view that unlike most other subjects, rote learning mathematics does not work and without an understanding of concepts and situations, students “get stuck” and they “fall behind” very quickly. This is consistent with many views in Chapter 2 recommending a shift away from isolated facts and memorisation of procedures and a move towards conceptual understanding in mathematics learning (Chambers 2008; Jaworski 2002; Pietsch 2009; Schoenfeld 1994; Skemp 1987; Vygotsky 1978; Watson and Mason 2008). For example, Vygotsky’s theory of social constructivism, in section 2.3.2, maintains that understanding is critical in mathematics learning and according to his theory of students’ zone of proximal development, mathematics teachers should present students with the right level of challenge and assist them perform tasks just beyond their current level of understanding (Vygotsky 1978). Like the engineers in this study, Skemp (1987), in section 2.3.1, asserts that a student who attempts to learn by memorising suffers “distress” and ultimately “falls by the wayside” (Skemp 1987). It is noted that in Ireland mathematics teachers generally rank lower-order abilities (e.g. remembering formulae and procedures) more highly, and higher-order abilities (e.g. providing reasons to support conclusions, thinking creatively and using mathematics in the real world) less highly than do teachers in many other countries (Lyons et al. 2003).

Engineers maintain that a significant difference between mathematics and most other school subjects is the focus on the right answer in mathematics whilst other subjects lean towards subjective analysis. In section 2.2.3 it is maintained that the focus on the right answer creates “a fear of being seen to be wrong (Lyons et al., 2003) and it also creates a “hierarchy of students who either get good grades or who “sink to the bottom” of the class (Boaler 2006). Ernest (2011) is of the view that “stressing that every task has a unique, fixed and objectively right answer” can result in “mathephobia” (Ernest, 2011). Another view in the research literature in section 2.3.1 is that the ability to get 100% in mathematics tests is a strong reason for students’ enjoyment of mathematics (Leder, 2008). Leder’s view that students enjoy mathematics because of the clear cut answers involved (Leder, 2008) is similar to the engineers’ view that “no matter how much work” one puts into the “subjective” subjects one might not get “full marks. However it is maintained in section 2.3.2 that dialogical classrooms, where different perspectives are considered, create rich mathematics learning environments (Vygotsky, 1978, National Council of Teachers of Mathematics, 2000, Pietsch, 2009).

Compared to other subjects, engineers are of the view that mathematics is a diverse subject. Research literature supports the view that mathematics is a broad subject. In section 2.2.2, Schoenfeld (1992) says that mathematics is multidimensional and comprises five aspects of mathematical thinking (Schoenfeld 1992). According to Niss (2002), in section 2.2.1, mathematics has eight competencies, (Niss, 2003). In section 2.2.1 it is noted that international student assessments of mathematics proficiency are also multidimensional and are based on content, competencies and situations (Organisation for Economic Co-Operation and Development 2009) and on both "content domains" and "cognitive domains" (International Association for the Evaluation of Educational Achievement 2011).

While mathematics learning includes cognitive activities such as using and applying mathematical knowledge, there is also a metacognitive aspect. Metacognitive activities include "planning, controlling and monitoring progress, decision making, choosing strategies, checking answers and outcomes and so on" (Ernest, 2011). Furthermore as presented in section 3.3.2 there is no underestimating the significance of the affective domain in mathematics learning. From the interview data there is a sense that mathematics learning is more personal compared to other subjects. For example, engineers say that "every person takes responsibility" for their own [mathematics] understanding", mathematics students "learn differently" and "learning happens when the student manages to make that little extra step" from the knowledge "they are comfortable with and that makes sense to them … to solve this new but related problem". Furthermore school mathematics success is very visible whereby "you get your answer right or you get it wrong and you either get an A or a D grade". According to Ernest (2011) it may be that student feelings are stronger in mathematics than in other subjects because "in mathematics more than any other subject there is the possibility that they [learners] will experience absolute failure at the tasks they are given"(Ernest 2011).

Engineers say that compared to most other subjects mathematics learning has an extra dimension and that in addition to the knowledge base, mathematics is a "process" of problem solving and/or application that requires "transfer learning" whereby students "take what they learn and transfer it to a slightly different context or situation". Similarly Ernest (2011) in section 2.2.2 suggests that mathematics has an explicit dimension and a tacit dimension and that the process of "doing" mathematics differs from "textbook" problems. In section 2.2.2 Ernest notes the difficulty of transferring learning between contexts (Ernest 2011). Similarly in section 2.2.3 Schoenfeld notes that there is considerable difference between school mathematics and the way experts engage in mathematical practices (Schoenfeld 1992). According to Evans (2000), in section 2.2.1, doing mathematics includes processing, interpreting and communicating numerical, quantitative, spatial, statistical mathematical information in ways that are appropriate for a variety of contexts (Evans, 2000).

Another difference between the subjects is the perceived difficulty of mathematics learning compared to other subjects. However some engineers are of the view that due to the "process" type learning associated with understanding and the problem solving nature of mathematics, learning mathematics is easier than memorising information and facts in other subjects. Similarly Schoenfeld (1988) in section 2.3.2 says that with conceptual understanding mathematics makes more sense and it therefore easier to remember (Schoenfeld 1988). However the problem solving nature of mathematics is time consuming: mathematics learning is "a lot about practice; it is about "trying to figure the stuff out"; and students spend considerable amounts of their homework time "looking for a specific answer".

Given that all twenty engineers have at least a level 8 engineering qualification, they have all demonstrated proficiency in mathematics unlike the statistics presented in section 2.2.3 where only a 16% minority of all Leaving Certificate mathematics students take the higher level option (State Examinations Commission 2011b). In section 3.3.1 it is maintained that attributions which are perceived causes of outcomes are important influences on motivation and that ability and effort are the most frequently used attributions in mathematics learning. Effort is seen as controllable as the student is deemed responsible and ability is classified as uncontrollable and students lacking in ability can feel shame, embarrassment or humiliation which could lead to an avoidance of the subject (Schunk et al., 2010). Only one engineer in this study shows a concern about ability in school mathematics when at school she believed that "it was only boys who had the ability to grasp most of higher level Leaving Certificate maths". In section 3.3.2 Schoenfeld (1992) notes that parents in the U.S. are more likely than Japanese parents to believe that "innate ability" is a better predictor of children's mathematics success than is effort. Thus U.S. parents are less likely to encourage their children to work hard on mathematics. In contrast to the U.S., mathematics teachers in Japan and China allow more time for students to understand mathematics concepts and solve mathematics problems (Schoenfeld, 1992). In section 2.2.3 it is noted that even relatively successful students perceive that they perform poorly in mathematics and that there is a perception of "elitism" in mathematics where only a "clever core" of students are capable of learning mathematics (Hodgen et al. 2010; Nardi and Steward 2003). There is also evidence that students behave strategically by not choosing advanced mathematics because it is perceived to be more difficult compared to other subjects (Hodgen et al. 2010) The majority of engineers in this study were generally motivated to expend effort to "get the right answer". They say that Leaving Certificate mathematics is not "particularly difficult", and that one's mathematics exam grade "is directly related to the effort you put into it". One engineer asserts that the key to mathematics learning is "finding that you are able to do it" and this "unique skill doesn't come up much in any of the other subjects".

7.2.1.3-2 F1.2: "Good" mathematics teachers communicate mathematics well; they are positive about mathematics and teaching; they know mathematics; they are able to teach a broad profile of students; they illustrate the relevance of mathematics; they are interesting; and they are organised and strict

Teaching is the "number one" factor in mathematics education and good mathematics teachers "transform" students' mathematics learning and their enjoyment of the subject.

The ability to communicate mathematics is the predominant characteristic of good mathematics teachers. While one engineer's mathematics teacher was "excellent" because he "just connected with people through maths" the "plain ordinary bad" teacher "just could not explain the consequence" of any mathematics topic".

Good mathematics teachers are "positive" about mathematics and they are "enthusiastic to the point where [they] can foster interest and enthusiasm for the subject with a broad profile of students within the classroom". While good teachers encourage students to engage in mathematics "bad" teachers" have poor attitudes and they often label specific parts of course as "too hard" and they do not teach the entire syllabus.

Teachers need to know their subject and be "confident in their ability" to "field any kind of a question" presented by students in mathematics classes. Being able to teach a "broad profile of students" is important as mathematics teaching is about "building on" students' "prior level of knowledge" and understanding. Illustrating the relevance of mathematics in the real world, "interesting" teachers, being organised and strict are also characteristics of good mathematics teachers.

According to the engineers in this study teaching is the "number one" factor in mathematics education and good mathematics teachers transform students' mathematics learning and their enjoyment of the subject. This is supported by Bandura's social cognitive theory in section 3.3.1 whereby human learning is greatly expanded by the capacity to learn vicariously (Bandura 1986). In section 2.3.1 Skemp believes that mathematics is "very dependent on good teaching" (Skemp 1987) and in section 2.3.2 the NCTM maintain that "students' understanding of mathematics, their ability to use it to solve problems and their confidence in and disposition toward mathematics are all shaped by the teaching they encounter in school" (National Council of Teachers of Mathematics 2000).

Engineers identify the ability to communicate mathematics as the predominant characteristic of good mathematics teachers. While one engineer's mathematics teacher was "excellent" because he "just connected with people through maths" the "plain ordinary bad" teacher "just could not explain the consequence" of any mathematics topic”. In section 3.3.1 it is asserted that constructivist teaching (theory contending that individuals construct much of what they learn and understand through individual and social activity) changes the focus from controlling and managing student learning to encouraging student learning and development (Schunk et al., 2010). Skemp (1987), in section 2.3.2, says that “to know mathematics is one thing and to be able to teach it – to communicate it to those at a lower conceptual level – is quite another; and I believe it is the latter which is most lacking at the moment” (Skemp 1987). According to Vygotsky, in section 2.3.2, learning is fundamentally a social process whereby knowledge exists in a social context and it is initially shared with others instead of being represented solely in the mind of an individual. He says that the stimulus for learning comes from outside the individual and the individual’s construction of knowledge is secondary to the social context. Classroom discussion, dialogue and collaboration are critical components of social constructivist theory of mathematics learning. In section 2.3.1 Vygotsky’s theory of the zone of proximal development maintains that there is a difference between what learners could achieve by themselves and what they could do with the assistance from a skilled person such as a teacher. His theory suggests that learning environments should involve interaction with experts and that discussion between teacher and students and amongst students themselves enhance students’ mathematical thinking and communication. The role of teachers is to provide scaffolding [26] on which students construct their learning (Vygotsky 1978). Communication is one of the NCTM’s five Process Standards in section 2.3.2 and the NTCM says that teachers need to establish and nurture an environment conducive to learning mathematics that “encourages students to think, question, solve problems and discuss their ideas, strategies and solutions” (National Council of Teachers of Mathematics 2000). In section 2.2.2 Ernest (2011) asserts that knowledge is usually learned in a social context and that the transfer of learning between contexts often does not take place and that it is the social context that elicits the skills and knowledge from long term memory (Ernest 2011). In section 2.3.2 Pietsch maintains

[26] Scaffolding: is when a more skilled person imparts knowledge to a less skilled person through language and communication, Vygotsky, L. S. (1978). "Mind in Society: The Development of Higher Psychological Processes", in M. Cole, V. John-Steiner, S. Scribner, and E. Souberman, (eds.). Cambridge, MA Harvard University Press.

that developing specific mathematical forms of discourse that can be internalised by individual students is an important part of effective mathematics teaching (Pietsch 2009). In section 3.3.2 Pape, Bell and Yetkin (2003) say that learning occurs through social interaction with others and that mathematics inquiry, including "learning to reason statistically, to think algebraically, to visualise, to solve problems and to pose problems" is developed within classrooms that support reflective discourse (Pape et al. 2003). Teachers' role is to "establish the context for mathematical development" and to scaffold students' developing skills by presenting tasks that encourage students to value and enjoy mathematics and to articulate their thinking. By articulating their thinking over time, students learn to monitor their thinking and consequently they develop mathematical reasoning skills (Pape et al. 2003). However as noted in section 2.3.2, there is little evidence of group work, individualised work, whole class discussion or reflection in mathematics classrooms in Ireland (Lyons et al. 2003).

According to the engineers in this study, teachers need to know their subject and be "confident in their ability" to "field any kind of a question" presented by students in mathematics classes. Engineers are of the view that being able to teach a "broad profile of students" is important as mathematics teaching is about "building on" students' "prior level of knowledge" and understanding. In section 2.3.2 the NCTM say that for teachers to be effective, they "must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" and they "need to know the ideas with which students often have difficulty and ways to help bridge common misunderstandings". Because "students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know" (National Council of Teachers of Mathematics 2000). According to Pietsch (2009), in section 2.3.2, "mathematics teachers need to be comfortable with a wide range of mathematical abstractions, techniques, concepts, ideas and generalisations". They also "need to feel comfortable working with individuals, with people who are fundamentally unpredictable, beyond complete understanding, each person representing a unique exemplar of multiple overlapping abstractions" (Pietsch 2009). One reason advanced in section 2.3.2 to explain the decline in mathematical competencies of students in Ireland is untrained and under-qualified teachers of mathematics where it is estimated that only 20% of Leaving Certificate mathematics syllabus is taught by those with degrees in the subject and consequently "the problem-solving power and logical basis of mathematical manipulations is often lost and replaced by attempts by students to learn by rote and memorise numerous sets of complex rules" (Irish Academy of Engineering 2004).

According to the engineers in this study good mathematics teachers are "positive" about mathematics and they are "enthusiastic to the point" where they can foster interest and enthusiasm for the subject with a broad profile of students within the classroom. While good teachers encourage students to engage in mathematics "bad" teachers" have poor attitudes and they often label specific parts of course as "too hard" and they do not teach the entire syllabus. One engineer maintains that "the power that a teacher has to capture some child's imagination and make them think they like something is immense and so it just drives everything else" and when another engineer switched to a grind school, she says her new mathematics teacher "was a revelation in that he "totally revitalised her feelings of what maths was about" and she began to think that "maths were easy". In section 3.3.2 it is presented that there is a significant correlation between teachers' attitudes and student achievement in mathematics. For example, in his social cognitive theory, Bandura (1986) holds that teachers are role models and their attitudes, emotions, beliefs and values about mathematics impact their students' learning (Bandura 1986). In Koehler and Grouws' model of mathematics learning, it is asserted that students' mathematics learning is influenced by teachers' attitudes and beliefs about mathematics and teaching mathematics (Koehler and Grouws 1992). According to Lampert (1990), students acquire beliefs about mathematics through years of watching, listening and practicing mathematics in the classroom (Lampert, 1990). Yara (2009) found that students' positive attitude could be enhanced by teachers' enthusiasms, resourcefulness and behaviour, thorough knowledge of subject matter and by making the subject interesting (Yara, 2009). Ernest claims that classroom experiences are decisive in developing children's views of mathematics. He reports on a study where students often distinguish mathematical topics as “hard-easy” and “useful-not useful” and he suggests that “experiences in school mathematics form the basis for the conceptions, appreciation and images of mathematics constructed by learners, especially negative ones”. Ernest also says that many learners experience a “Dualistic” view where teachers give students a “myriad of unrelated routine mathematical tasks which involve application of memorised procedures and by stressing that every task has a unique, fixed and objectively right answer, coupled with disapproval and criticism of any failure to achieve this answer”, these teaching methods create images of mathematics as “cold, absolute, inhuman and rejecting” (Ernest 2004a; Ernest 2011). It is claimed that all young children like mathematics and that they do mathematics naturally but that as they become “socialized by school and society”, their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear and eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. Later, as parents, they pass this conviction on to their children. Some even become teachers and convey this attitude to their students” (National Research Council, 1989). Schoenfeld also suggests that teachers’ beliefs are formed by their own schooling experience and the same beliefs are apparent in successive generations of teachers, which he calls a “vicious pedagogical/ epistemological circle” (Schoenfeld, 1992). A study of second-level mathematics classroom practices in Ireland, noted in section 2.2.3, found that all students had “a fear of being seen to be ‘wrong’” and many suffered “mathematics anxiety” when teachers taught at a very fast pace and when teachers were critical of students who made errors (Lyons et al. 2003).

Engineers say that illustrating the relevance of mathematics in the real world, “interesting” teachers, being organised and strict are also characteristics of good mathematics teachers. In section 3.3.2 it is maintained that students’ perceptions of the importance, utility and interest in mathematics are strong predictors of their intentions to continue to take mathematics courses and that male and female adolescents differed in the relative value they attached to various subjects and that boys valued mathematics more than girls. Research has consistently shown a decrease in the mean level of self-perceptions of mathematics ability as children move into adolescents (Wigfield and Eccles 1992). It is also maintained, in section 3.3.2, that while students' value perceptions of mathematics, language, arts and sports declined in high school, mathematics declined most rapidly. Explanations for students' declining task value beliefs range from attributing poor performance to low ability, students becoming interested in social comparisons and the mismatch between the students' developmental needs and the organisation of the school. (Jacobs et al. 2002). In section 3.3.1 it is maintained that teachers are a huge influence on students' motivation. When teachers teach well-structured content, they engage in practices that are consistent with principles of contemporary cognitive learning which enhance motivation. Efficacious teachers are more likely to plan challenging activities, persist in helping students learn and overcome difficulties, and facilitate motivation and achievement in their students (Schunk et al., 2010).

7.2.2 Theme 2: Motivation to Engage with Mathematics

The findings concerning the engineers' motivation to engage with mathematics are included in this section. Theme 2 is presented as follows:

Page number

7.2.2.1 Family support and influence

In addition to helping with mathematics homework, engineers' families influenced their mathematics learning primarily by fostering an interest in mathematics related activities through "game playing" or by helping out in the family business. Dispelling negative views and engaging in discussions about mathematics with family members all motivate mathematics learning.

7.2.2.1-1 Family engagement in mathematics generates positive affective memories

Family influence, support and encouragement are evident in engineers' early motivation to engage with mathematics (A, B, G, H, J, K, L, M, O, Q, S, T, and U). Seven of the twenty engineers have engineers in their family (A, D, G, J, Q, T, and U) and it is noted that D is the only engineer whose father is an engineer and who didn't receive any particular support or encouragement in mathematics from his family. Engineers recall memories of engaging with mathematics with their family from a young age. Examples include: together with his father G worked out the "odds" of a particular horse winning a race; engaging in "mathematical type game playing" with his family

(J); “there was a value on money” in K’s family and as a child he learnt how to “count it and calculate change” when doing errands; when M was “five or six years old”, he had to count “hundreds of sheep” … he “had to count them three times … you couldn’t trust the first count, if the second one matched up you were okay but most likely it didn’t so then you had to count everything three times”; T played a game of recalling “car registration plate numbers” with her mother; and U worked in the family’s corner shop “taking the customers money and working out the change that had to be given to them” … he says he “had to be able to do sums quickly” in his “head in front of the customers”.

Without her father’s support, H would not have done so well in mathematics, she says her father’s approach to mathematics placed more emphasised on the “methodology” than on “the right answer” … he told her not to worry if she got the “wrong” answer that she would still get marks for her work. He also told her that mathematics “is not hard, it is a challenge” and it “opens up a lot of doors in different careers”. Q’s father corrected her negative views about mathematics when for her French homework she wrote “maths is hard, I can’t do it and stuff like that”. If she “hadn’t been pulled up on it” by her father, Q wonders if she would “have gone on to believe that”. In T’s family mathematics was “much more important than any other subject”; there was “a certain level of competition in the family about maths” and because her “older sister would have got an A so I wanted to get an A and then my brother wanted an A”. T says that during secondary school she would have “regularly discussed maths problems” with her older sister.

7.2.2.2 School mathematics

There are many factors that motivate students to engage with school mathematics. Motivational factors include: feelings; views/ beliefs, self-efficacy, value, peers and effort and engineers’ school mathematics. Motivational factors are included in Table A9-3, Appendix 9, Volume 2.

Engineers say that the feeling of success is the main contributor to enjoyment of school mathematics. There are stigmas and prejudices associated with school mathematics. Being good at mathematics causes social problems for students and in order to fit it with other students many students hide their mathematical ability in school. Confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. Engineers' value of school mathematics is mixed and examples include: mathematics is required for entry into engineering education; it is an interesting subject; it is an important subject; the recognition associated with success is enjoyable; and assisting other students with homework gives a sense of peer approval. The cost of engaging in school mathematics is the time required and not being able to "see the point" of it [mathematics] is a further cost. Engaging in social or group learning of mathematics with peers or role models contributes significantly to preparation for mathematics exams. In mathematics learning, motivation is important and results in mathematics exams are related to "the effort you put in".

7.2.2.2-1 Feelings of success contribute to students' enjoyment of school mathematics

Engineers enjoyed school mathematics quite a lot (A, B, C, E, G, H, J, K, L, M, O, P, Q, S, T, and U). The main reason many engineers give for rating their enjoyment of school mathematics so highly is that they were "good at mathematics" and the feeling of success that came with that (A, B, C, E, J, K, L, M, O, P, Q, S, T, and U). Examples of feelings of success include: "there is certain amount of fulfilment" in getting the correct answer while in subjects such as English, if you "think you have done a damn good job" you might only get "fifty per cent" (C); E "liked the challenge of it [mathematics]" and she "liked getting it right"; school mathematics was "instantly rewarding"(K); because L was "good at maths and enjoyed it, it became easy" and he was "automatically rewarded by the teachers" in primary school"; M enjoyed "solving problems and getting the right solution"; P "enjoyed" mathematics because it "came easy" to him; Q recalls the enjoyment of getting "the right answer and she says that getting the "wrong" answer "feels bad"; S loved mathematics in secondary school and he got a "great buzz" from "difficult homework"; as mathematics "got more difficult" O "started to enjoy it more" because he "found" he “was good at it” and “maths is so much easier” than other subjects because “you don’t forget” it when “you understand” it. O also expresses a “love” of mathematics because the subject is “beautiful”; T got a lot of “satisfaction” from the “reasoned ways of thinking out a mathematical problem” and getting the “right” answer and she “liked being good at maths”; J was “always attracted” to mathematics because he didn’t “have a particularly good memory” and for him the “fun” of mathematics was “to derive the solution on the spot” because “the deriver would always be successful and the learner might not be”; and H found mathematics “nicer study than the other subjects” as “it wasn’t just sitting down trying to remember loads of stuff” instead it was “doing” mathematics and “coming up with the answer”.

Of the engineers who don’t express a great enjoyment of school mathematics, poor teaching is a common factor (D, F, N, and R). D says he “was not a fan of maths”. He “found maths difficult”. He states that due to his “very poor grounding” in mathematics, he was “afraid” of the subject. D is of the view that “teachers’ attitudes” impact students’ learning of mathematics and he emphasises the need “for someone to explain it [mathematics] to them or motivate them”. While F was “drawn to physics, chemistry and maths” in school, he says that school mathematics “meant nothing and you couldn’t relate it really to everyday life”. He is of the view that school mathematics “should be more applied” and that students should be made aware of its usefulness. N’s feelings about school mathematics are mixed. While N enjoyed Leaving Certificate mathematics, he did not enjoy “Intermediate” [Junior] Certificate mathematics because he “had a very poor teacher” and he could not “relate” to the subject. N took the ordinary level Leaving Certificate option. Prior to switching to a grind school, R feared that she would not do well in Leaving Certificate mathematics. She says her new mathematics teacher in the grind school “totally revitalised her feelings of what maths was about”. R says “if I was teaching maths, I would relate it to everything around me”.

7.2.2.2-2 There are stigmas and prejudices associated with school mathematics

Some engineers say that being good at mathematics causes social problems for students and that in order to fit it with other students they had to hide their mathematical ability in school (H, J, K, L, O, P, Q, R, S, and T). H attended a particularly good all-girls school where a third of her class took the higher level Leaving Certificate mathematics exam and where higher level mathematics is “seen as quite prestigious”, a view which she believes “needs to be wiped out”. J is of the view that there is a “them and us culture” associated with mathematics. He says that mathematics “cut-off” happens at quite an early age when “people decide that they can’t do it” and “that the people who do it are somehow different from them”. Some of K’s secondary school class mates were “very much more isolated because of their abilities and skills in maths and they would have been pushed out of the groups”. K believes that many young people have a negative perception of those who are good at mathematics and students do “not to want to stand out by being good at something like maths”, instead they “want to fit in more” with their peers. In school L was branded as being good at mathematics and he was “angry” when he was “put “into a certain group who would be the geeks”. He therefore tried to hide “the guilty pleasure of enjoying maths” and he “let on that it took” extra time to answer a mathematics question. O would not be comfortable declaring his “love for mathematics” in school because he feared that he would have been branded the “school geek”. While P “would have been regarded as something of a phenomenon” in school due to his “innate mathematical ability”, he “tried not to alienate other students with mathematics”. According to Q, there is “kind of a stigma associated with school maths”. In Q’s school “maths was nerdy” and “the same twelve girls did honours maths, physics and chemistry”; if she was “the only one in the class” who got a particular mathematics question correct, the other students would look at her and she felt “like a closet nerd”. Q says she felt “alone in her enjoyment” of mathematics and she wouldn’t readily “come out” and say “maths is my favourite subject”. R is also of the view that there is a “stigma associated with being good at mathematics” and that “if you’re brainy you can’t be good at anything else … a swot can’t be an all-rounder”. She says that her own son and five other students who are very good at mathematics are excluded from the football team because of the stigma associated with mathematics. S attended an all-boys school where the boys were “too geeky” to work together and he felt like “such a sad individual” because unlike many other students he got a “buzz” from doing mathematics. According to T, there is a very negative view towards maths” and people who “were good at maths in school would have been deemed to be geeks” and because she was good at mathematics T adopted a strong personality and appearance so as “not to be branded a geek”.

7.2.2.2-3 Mathematics confidence is triggered by recognition of success

While engineers were confident in their mathematical ability in school (A, B, E, G, H, J, K, L, M, O, P, Q, R, S, T, and U), their mathematics confidence related to a recognition of success such as their latest test grades, getting top marks or being the best in the class. Examples of this include: A, who says he “was good at problem solving” and the “sense” of getting “the answer right” and knowing that he had “the right answer” was “very direct gratification”; if E got something wrong in a mathematics test, she would ask herself “why didn’t I get 100% when I could have”; The “instant recognition” K got from “getting the maths questions right” helped him to develop the “ability to stand up in class and answer the questions, from a very young age and be more correct than everybody else”; J asserts that the key to mathematics learning is “finding that you are able to do it” and this “unique skill doesn’t come up much in any of the other subjects”; M “got good results in primary school maths”, from Junior Certificate he got “strong results in exams that mattered” and while he “worked very hard at maths for the Leaving Certificate” if he “couldn’t get an A and be the best at it” he “would not be confident at it”; O “found” he was good at mathematics in secondary school; S became confident when he progressed to the top mathematics class; and T says “I got confidence in the fact that I was getting good results in mathematics and then I realised this is something that I could be good at”.

D shows very low confidence in school mathematics. He says that due to “bad” teaching, he developed an “inferiority complex about maths” and a “blockage” to learning mathematics in secondary school.

7.2.2.2-4 School mathematics has both value beliefs and cost beliefs

Engineers' views on their value (importance of doing well in mathematics) and cost (perceived negative aspects of engaging in mathematics) beliefs of school mathematics are mixed. For some engineers, mathematics was required for entry into engineering education, some engineers had an interest in the subject, some engineers say that they viewed mathematics as an important subject, some enjoyed the recognition associated with getting mathematics problems correct or assisting other students with homework and for some the effort did not justify the achievement or mathematics meant nothing.

7.2.2.2-4-1 Value of school mathematics is recognition of success

The main value of school mathematics is that some engineers found school mathematics rewarding and they enjoyed the recognition and gratification associated with being good at mathematics (E, G, K, J, L, O, P, S, and T). These include: getting mathematics right was "very rewarding" (E); there is something special about "being able to do something other people can't do" (G); K found mathematics "instantly rewarding" and he recalls at a very young age "getting the maths questions right" and "being rewarded for it and getting a gold star … maths has an instant answer and if you are correct you're great" (K); "being good at maths had almost as much of a cachet about it as being good at football … I got some brownie points for helping other students with their maths homework and they could see that the geeks had their uses" (J); I was in demand by students who needed help with their homework" (O); "maths was a pleasant intellectual exercise"(P); when doing mathematics, L got "satisfaction" and "pleasure" and "it was always relaxing" for him to do mathematics; when he was "the only one in the class who got it [difficult mathematics homework] right", the teacher would praise S putting him "on cloud nine"; and "you only need to get one good grade in secondary school mathematics and the teachers will leave you alone" (T).

7.2.2.2-4-2 Value of school mathematics is that mathematics is an interesting subject

For some engineers school mathematics had a value because they found the subject interesting (G, H, K, O, P, and E). In secondary school G attended "maths competitions" every Saturday morning where he learnt "a lot of number theory stuff" and "patterns" which he found "quite interesting". H's teacher was "never boring", she engaged the students. K's Junior Certificate mathematics teacher also made mathematics "interesting" by talking "about other elements where maths could be used". The "history of maths" just captured O's imagination and his teacher "held our attention, he could tell a good story and he did tell a good story". P was interested in mathematics; he says "you were almost for ever learning something new". S says that "maths was what I was interested in doing at the time, I just loved it".

7.2.2.2-4-3 Value of school mathematics is admission and persistence in engineering education

One value of higher level Leaving Certificate mathematics is admission and persistence in engineering education (A, B, D, K, and R). Examples of this value include: the ability to get "through engineering studies" (A); "entry into engineering education" (B). For D higher level Leaving Certificate mathematics was a "career requirement" and his interest in engineering as a career motivated him to continue with the higher level option; K knew "that maths always had a part to play in science and technology" and thus mathematics always had a "value" for him; and R says "I knew that I needed honours maths for engineering and I had to do it by hook or by crook in whatever way I could remember it to get a C in the honours exam".

7.2.2.2-4-4 Value of school mathematics is that mathematics is an important subject

Some engineers regarded mathematics as an important subject (E, K, L, and P). E believes that because mathematics was assigned such importance in her school she "put more work into maths rather than other subjects". K "always thought of it [mathematics] as an important subject. For L mathematics “was always important” and he “held it in high regard”. P was aware “that mathematics would have been an important subject” for him. T is of the view that “an honour in maths is actually worth something … it is worth more than an honour in Irish or English”.

7.2.2.2-4-5 Value of school mathematics is “points” earner

There is a view that school students currently value mathematics from a points [CAO points] perspective rather than from an applications perspective (K, N, and T).

7.2.2.2-4-6 Cost of school mathematics is time

On the negative side, a few engineers feel that the time required for higher level mathematics reduces its value (A, C, and M). There is little “value” in taking higher level mathematics in the Leaving Certificate if it consumes “almost all of the two years” (A). C’s secondary school “didn’t have the critical mass of students necessary to do higher level maths” and it would have “been too much” to do outside of school. Leaving Certificate mathematics took up more than half of M’s allocated three hours of study time and he believes that he risked passing his Leaving Certificate exam when he skipped the other subjects to do “an extra maths question”.

7.2.2.2-4-7 Cost of school mathematics is lack of relevance to everyday life

For some engineers mathematics had little applications value (F, N, and R). Leaving Certificate mathematics “meant nothing” to F because he “couldn’t relate it really to everyday life” and teachers did not explain “how it is actually used”. N says he couldn’t “relate” to school mathematics and he found many areas of mathematics “totally abstract”. N also says he “couldn’t see the point” of it and it “turned us off”. Similarly R says she “couldn’t relate the maths to anything”.

7.2.2.2-5 Peer learning contributes to mathematics success

Engaging in social or group learning of mathematics with peers or role models contributes significantly to preparation for the Leaving Certificate mathematics exam (B, D, J, K, M, O, Q, and U). The most "notable feature" of B's engagement with secondary school mathematics was his "peer group" of friends, who also "had proficiency in maths and were targeting an engineering qualification". Within the group there "was an interest in getting a common approach" in mathematics and they would "share perspectives" when presented with a difficult problem. B says the "comfort and positivity" of the group towards numerate subjects were "hugely important". D relied on some of his peers who "used to give tutorials" to the rest of the class in preparation for mathematics exams. J's group of friends were all good at mathematics and they collaborated over homework. They also "played football together because nobody else would play football" with them. One of K's school friends, who also became an engineer, was good at mathematics and they studied together. M recalls that in preparation for his Leaving Certificate he worked on mathematics problems with a group of four other boys. He says that "Leaving Certificate maths just turned into this challenge, this fun thing as I said a few of us doing the maths together". O's school friends also loved mathematics. He says that a core group of "six or seven friends formed" as a result of them attending extra maths classes after school and "that helped with the maths". Q sought out and "found a few female role models who had got As [A grades] in Leaving Certificate maths". She says that these "mentors or role models" helped motivate her and they helped her realise that she could get an A in Leaving Certificate mathematics. Q says she now wants her younger brother "to know that he too can get an A". U recalls, when in secondary school, that the school principal who was "a Presentation brother" and who "clearly had a love of mathematics" helped the mathematics teacher by giving a "free grind" to students every Saturday morning. U says that "everybody" in sixth year attended because "it was too good to miss".

7.2.2.2-6 Mathematics results are related to effort

There is a view that results in mathematics are related to “the effort you put in”. Engineers were motivated to do well in school mathematics (A, B, C, E, G, H, J, K, L, M, O, P, Q, R, S, and T). A says “I was determined to work out every maths problem that came my way”. B’s teachers “challenged” him with “maths problems and he “persisted” until he “worked out the answer”. C was competitive in primary school and he did the mathematics as fast as the teacher “could dole out the maths”. E was motivated to do well in Leaving Certificate mathematics and she “wouldn’t drop a difficult mathematics problem”, instead she would “wait and stick it out”. G’s mathematics teachers in school “challenged” him and he says he was “very motivated to do well in maths”. H was “very dogged on working stuff out” and when she did solve a mathematics problem she was “delighted” with herself. H believes that unlike other subjects, results in mathematics are directly related to “the effort you put in”; K “would persist and try and work it [difficult mathematics problem] out” and the sense of achievement when he solved a problem spurred him “to do more”. L “pushed” himself in mathematics because he would be “disgusted” if he “wasn’t at the top in mathematics” class. He says he “would have always gone ahead of the teacher in the maths curriculum”. M says mathematics is about “trying to work it out” and he says “I kept at it until I got the right answer”. O says his teacher gave him “a love of maths … he showed me how the solutions were so wonderful and beautiful and just cool … I wished I could think like that, I wanted to think like that … I wanted to find out all about the history of maths … I read every maths book I could find”. P also “tended to read ahead.” Q says that sometimes mathematics “required a lot of effort” and if “I didn’t get it I would go and move on and come back to it”. She was “diligent”, “methodical” and she would also “go back” over her work and she “filled in units” to verify that equations were correct. While R learnt mathematics “by slight rote and by default and by memory and everything else because of the poor teaching practices”, she says she was competitive in school and that she was determined to “find the solution” to mathematics problems. S says mathematics “was the subject that I put most into … it is said the more work you put into something the more you get out of it, so I used to do one hour of maths a day religiously, whether I needed too or not”.

He says he got “a certain buzz out of being able to get to a solution in a smaller number of lines … there was probably an element of arrogance in this as well … if it was really important you would go back and verifying every step”. He “liked problems where you could get your teeth into the problem and work out the exact answer … I was obsessed with quantifying things to the n^{th} degree”. T “worked harder at maths than probably any other subject” because she liked it. She says “I liked the challenge and I liked being good at maths, it was easy to be good at it and it just felt natural” and she adds that mathematics “is a personal thing and it is easier to work through it yourself”.

7.2.2.3 College mathematics

Engineers’ feelings about engineering mathematics are included in Table A9-4, Appendix 9, Volume 2.

The transition from school mathematics to engineering mathematics is difficult. There is a view that in university “lecturers don’t teach, they lecture”, “they tell you where the information is” and you “are very much left working it out for yourself”. Engineers say that mathematics is central to engineering education and engineering subjects are “based around maths”. Engineers express views that, in university, mathematics is “theoretical” mathematics, it is “applied” mathematics, it is the “discipline” of learning mathematics and it is the “level of mathematics required to become a professional engineer”.

7.2.2.3-1 Transition from school mathematics to engineering mathematics is difficult

There is widespread agreement among engineers that college (engineering) mathematics is difficult and that without higher level Leaving Certificate mathematics students would “struggle” with engineering mathematics (A, B, D, E, G, H, J, K, L, M, O, P, T, and U). In particular engineers say they struggled with the transition from school mathematics to engineering mathematics. D “endured the maths, to get through college" and his "blockage to learning mathematics" and his "lack of maths caught" him "all the way through college". D says that in engineering education "you're up against it" and a "good standard of maths" is necessary. While H always "found maths real easy to study", she says that there was "so much of it for the first two years" of college. J also found mathematics much more challenging at university than school, in university he says there were "a lot of us putting our heads together trying to get solutions". M "lost" his "love for maths as soon" as he went to college where mathematics "was hard, it was complicated … it was a completely different way of doing maths than the Leaving Certificate". M missed the "banter" of "the peer group that studied together" in school and he says that in college "the social element of the maths was gone". P who has an "innate" mathematical ability failed a mathematics exam in college because he wasn't "diligent". Q says that engineering "was tough … the hours were a lot longer than you know the arts block … it was just the workload" when compared to other courses. She had to "turn off" the students in her course "who weren't very studious" and "who were messing in class" and she "would not dare ask a question out loud in a lecture" but she attended tutorials where "the post-grads would come and just talk to you". She studied with her boyfriend up to third year when he took the civil engineering option and she chose mechanical engineering. R found her engineering course "very difficult" as she was in a class "with male colleagues who had done mechanical drawing, who had done applied maths, who had been told because they were male that they were probably better at maths than the girls". R says she had "never heard of applied maths" until she was in university. She says "everybody else had applied maths … they had a greater comprehension of what they were doing". R says she felt "completely disadvantaged" from the start of university and she also "found maths in university extremely hard".

Some engineers attribute the difficulty of engineering mathematics to the style of lecturing. D is of the view that mathematics teaching in universities is also "bad". He says that the "take it or leave it" approach to teaching mathematics in universities does not work. D believes that, in universities, there is a need for teachers who demonstrate a "willingness to teach rather than just throw" the mathematics at the students. H also notes that in university “lecturers don’t teach, they lecture”, “they tell you where the information is” and you “are very much left working it out for yourself”. She says that without confidence in her mathematics ability, engineering education would “intimidate” her. M also blames the lecturers, “it [university] was a completely different way of doing maths than the Leaving Certificate … in school the teacher interacted with a class of twenty five of us and there were two directions with the maths ... however in college when you are getting lectured on maths it’s one direction only … it was all just thrown to you”.

Two engineers say that their enjoyment of mathematics increased in university when they got to see applications of mathematics. C who took ordinary level mathematics for his Leaving Certificate, says that engineering mathematics was one of his best subjects “all through his degree” and he attributes his success in engineering mathematics to a lecturer who “could relate what he was doing to the practicalities of it” and who also “made it very interesting”. F, who says that Leaving Certificate mathematics “meant nothing” to him, got to like mathematics in college as he “could see it applied”.

7.2.2.3-2 Students have mixed views on value of engineering mathematics

Engineering mathematics has a number of different values. Some engineers say that mathematics was required for engineering subjects, others benefitted from the discipline and rigour of learning mathematics and for one engineer it was the level of mathematics required to become a professional engineer. However some engineers asked “what is the point of this and where are you ever going to use this” mathematics?

7.2.2.3-2-1 Value of engineering mathematics is for engineering subjects

Mathematics is “essential” in engineering education as many of the engineering subjects are “based around maths”(C). Many engineering subjects “are taught through mathematics” and students thus need a “very high standard of maths” to "grasp" subsequent engineering concepts (D). J describes his university mathematics as "applied maths" and "certainly not pure maths". For N "maths followed" the engineering problems. It wasn't until third year in university that R "started relating" to the mathematics in the course, she says "it kind of came to me … that's what 'dy/dx' means, finite element analysis was a huge eye opener, I thought it was marvellous … nobody ever related maths to me ever". T's interest in college mathematics was primarily in the applications of the subject. She says she is into the "practical use of maths … what can you use that for, why is that any good to you". U says the "civil students definitely gave me the impression they weren't that interested in maths, whereas the mechanical, the electrical and the electronic students were all very into maths".

7.2.2.3-2-2 Value of engineering mathematics is benefit from discipline and rigour

B and P note the value of the discipline of learning engineering mathematics. B says that people who pursue less numerate careers benefit from the discipline and rigour of learning engineering mathematics. P is of the view that engineering mathematics "serves as a platform on which one can undertake the kind of reasoning that is necessary when confronted with technical challenges or situational challenges of any sort". N says his engineering education gave him the "mentality to think."

7.2.2.3-2-3 Value of engineering mathematics is level required to become a professional engineer

Both B and J support the view that engineering mathematics should be at a high level. B says that engineering education should "aim the course for the top five or ten per cent of engineers that are going to bear that design responsibility". J maintains that the lecturer's job is to let the undergraduate engineering students out of university "with a level of maths that we think is appropriate for a professional engineer, a Chartered Engineer, which of course is a very high level of engineering".

7.2.2.3-2-4 Value of engineering mathematics is for purpose of passing an exam

For A the "value" of college mathematics was for the purpose of passing an exam at the end of the year. N is also of the view that engineering mathematics is "a means to an end". He says that apart from "report writing, problem solving and spatial awareness" the purpose of engineering mathematics is to pass the engineering exams. H wasn't aware of the value of engineering mathematics and the big question in her engineering class was "what is the point of this and where are you ever going to use this" mathematics? Similarly L says that his engineering mathematics "didn't relate to the other elements of the degree". It was "taught by the mathematics department and most of the other subjects were done through the engineering school". M's college mathematics was "very theoretical" and it was "difficult to apply and to internalise". Q's engineering mathematics was also very theoretical and she says there is "still a lot of maths" that she studied in college that she doesn't "know the application of".

7.2.2.4 Engineering practice

Engineers' feelings about mathematics in engineering practice are included in Table A9-5, Appendix 9, Volume 2. Engineers say their work requires many types of mathematics including: "differential equations"; "kind of maths and figures, particularly statistics required in my industry"; "maths in a great depth"; "problem-solving techniques"; "appreciation of mathematics"; "mathematical logic"; "discipline of maths"; "estimation of engineering solutions"; "having a feel for an answer" and "some checking by maths". Engineers' enjoyment of mathematics comes from their success when using mathematics and their confidence in mathematics and in mathematical solutions.

7.2.2.4-1 Mathematics education contributes to engineers' work skills

The majority of engineers note that the positive contribution of mathematics education to their work (F, G, H, K, J, L, M, O, P, Q, R, S, T, and U).

F is of the view that one "would need to have had higher level maths at some stage" to do his job. G says mathematics "is necessary" for his job and it makes his job "easier". He adds "if you have a fear of it or it turns you off it's just like not being able to use your driver in your golf bag, it's just going to handicap you". H says the need for higher level Leaving Certificate mathematics varies in her company and the engineers who do "modelling of drainage or water systems" need to know mathematics. Mathematics is "essential" to J's research work. He is of the view that while few engineers need "certain types of maths, applied maths and problem-solving techniques" in their work, there "are still quite a lot of engineers who couldn't do their jobs unless they can solve differential equations". He is also of the view that managers in engineering companies require an "appreciation of mathematics" and that if the managers never learnt the mathematics themselves they cannot properly manage engineering work. J asserts that "doing maths is just very good training for the brain and teaches you concepts, like abstraction which you know make you a better thinker in general". Mathematics is "valuable" in the ten per cent of K's work where he uses mathematics. He says he sees "circumstances where others in the company would be better" if they had mathematics and that "when they don't have that level of mathematical ability it restricts them in terms of analysing situations". L notes that while "in this day and age" engineers don't need to write down equations" to do their work, that engineers, because of their education and because they are "comfortable with maths and using maths", still approach their work with "a kind of mathematical logic". M presents that only "ten per cent of the engineers on site here would need some of the learning from higher level maths" and that "ninety per cent of us could do our roles without honours maths". M says he has "taken more from the discipline of maths than from the actual academic side of it" and that while "higher level Leaving Certificate maths isn't necessary" in his "day to day work", the "discipline that comes with it is a requirement". In his current role as a manager, O is of the view that while engineering managers generally wouldn't be using higher level Leaving Certificate mathematics "in their day to day jobs" that "they may need to understand certain parts of it". P asserts "that a good grasp of maths is essential to being a good engineer" and that "mathematics is an extremely useful tool … early on one learns how useful it is and simply continues to use it in one way or another as one progresses through one's career". P notes that "in engineering there is very seldom a unique solution, there is a balance between the amount of time you can spend on problem solving and the degree of certainty that you can have that the solution you've come up with is the ideal solution". Q maintains that there is "a need for mathematical engineers, because engineers need to be strong in maths to understand processes". She says "I do feel I am able to cope with things better because I have a grasp of the kind of maths and figures, particularly statistics required in my industry". R asserts that higher level Leaving Certificate mathematics is necessary for engineering practice because "in engineering you need to go into maths in a great depth". S is of the view that mathematics in general is "a real useful tool" in engineering. T couldn't do her job without higher level Leaving Certificate mathematics. She says "I just think you can do certain aspects of it [her job], but I don't think you understand the fundamental aspects unless you have a good grasp of maths". U says that he "simply wouldn't been able to do" his job without mathematics.

It is the five lowest *curriculum mathematics* users (A, B, C, D, and E) and also N who do not recognise the value of their mathematics education in their work. Of the engineers who do not express a need to use mathematics in their work, A, B, D and N consider engineering to be about solving practical problems rather than using mathematics. N says that in engineering practice "you don't have to be good at maths; you have to be good at problem solving". A, B and D do not consider estimation of engineering solutions to be mathematics. While A says his job does not require higher level mathematics, he is of the view that "having a feel for an answer or solution is more useful" than having an answer "correct to eight decimal places". Similarly B says that "so much of the value an engineer brings to his job and brings to society is to be able to do a reasonableness test to conceive a solution and within a good level of probability to be able to say yeah, that will meet the need, but then not being afraid to modify that and evolve that in subsequent observations or in practice". D is "much more confident" in his work about "having the principles right and conclusions right from a good understanding of the problem with some checking by maths rather than doing a big long calculation, coming up with the answer and saying bang, there's the answer".

While, in his work, C has "set up the computer" to do mathematical calculations and his job does not "require a huge level of maths" he says that "invariably something will come along where I need to do the maths". E would "prefer to use maths more" in her work."

7.2.2.4-2 Confidence in mathematical ability and solutions contributes to engineers' enjoyment of mathematics in work

The majority of engineers enjoy using mathematics in their work (B, C, E, F, G, H, J, K, L, N, O, P, Q, R, S, T, and U). Much of the engineers' enjoyment of mathematics comes from their success when using mathematics and their confidence in mathematics and in mathematical solutions. While the engineers generally enjoy using mathematics in work, some engineers are of the view that doing so is risky because they sometimes get something wrong and they get caught out in front of colleagues who may not have a similar respect for mathematics. There is also a degree of acknowledgement that mathematics is an individual activity unlike engineering practice where the focus is mostly on teamwork.

Mathematics instilled "great confidence" in B in terms of career progression. C recognises the security associated with a mathematical answer and he likens mathematics to "a safety valve" in his work. In her work, E likes "a maths way to do something", she likes getting an "exact solution" and she likes "to be able to prove that something is right with maths". E would "prefer to use maths more" in her work and she is more confident in her work when using mathematics compared to when she doesn't use mathematics. F likes "mathematical solutions" and he recalls "getting a bit of a kick out of doing spread sheet analysis". In his work, F uses "models" and "black box solutions" to do various computations such as gas flow rates and while he doesn't "have to develop the models" he needs "to know where they came from" if he is "to use them with confidence" and have "an appreciation of the limitations" of the models. Due to her "good grounding" in mathematics, H is confident using mathematics in work and she enjoys the mathematics in her work. G has “always found numbers to be the most interesting” part of his work. J “absolutely” enjoys using mathematics as he is “exercising a skill” that he has. He says that mathematics “coincides with the way” his “brain works” and consequently he “finds pleasure in doing” mathematics just like a “fit and strong” person would find it “a pleasure to lift heavy weights”. He often finds that he is “so engaged” when “solving a mathematical problem” that he would hardly notice when “hours and hours have gone by”. K says he is confident about using mathematics at work and he relishes the “challenge”. K is of the view that using mathematics is risky because “you have to stand on your own two feet” and “if you get it wrong it can look very bad”. He says he has to “double check” the mathematics before presenting his mathematical solution to his co-workers. N says that because he is good at mathematics, it is “always there at the back of my mind just because I enjoy doing it.” However he is of the view that there is an “isolation” associated with using mathematics as “maths is usually more of an individual activity than a team effort”. L enjoys “working with mathematics” and he has the “confidence” necessary to solve engineering problems he encounters in his work. O is confident enough in his own mathematics ability to know when he should use mathematics and he “would be very confident that maths will deliver a better way of doing something”. He says “if the maths works out … it’s a faster way of doing something”. In his work P says “there is a feel good factor” when “you’re faced with a problem which you can define mathematically”. In her work, Q is known to be “kind of good” at statistics and she enjoys when people are often referred to her for advice. Q recalls a time in work when she “just took a minute too long” to predict a mathematical solution and consequently she was “put down by a colleague”, who was “just a step ahead” of her. She said that she felt “just stupid” but determined “to be a bit more on the ball” from then on. When using mathematics R is “a lot more confident” than her work colleagues and she says that when “I achieve something that is kind of difficult then I will get bored at it”. S says that “to get the real buzz out of maths it has to be a real problem”. He recalls that, while working in a research and development role for Sony in Japan, he came up with “a minimal tweak” that made one of Sony’s products “comply with an international standard” which put him “on cloud nine for weeks”. S says that while he would like to formulate all his problems mathematically, he only feels confident using the mathematics he is “comfortable” with. He adds that he is “really good at only a few small branches of mathematics” and that he has experienced “a lot more negatives than positives” when using mathematics particularly “when you get the wrong answer … more so when you’ve identified a problem and you just can’t formulate it mathematically … if only I had the maths to formulate this problem and go and solve it”. T enjoys using mathematics in work because “it is clean … it is completely logical, … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it”. U enjoys using mathematics at work because he is confident in the mathematics he uses and he had “very few negative experiences” using mathematics, however once when he did get “something wrong” he “got caught out”. He says he mostly trusts mathematical software; “but not one hundred per cent and he adds “I just don’t have the time to check it all but I found that there are times where simply the software is wrong … but at least I know where it is wrong now”. U is of the view that “there is a certain respect for mathematics” in his company “but that seems to change as the management changes and I have seen that over the years where the CEO was an engineer there was a very large amount of respect for mathematics, whereas the current CEO currently is very much a marketing man and so definitely the emphasis is on sales and marketing and away from the maths right now”.

One engineer who does not express an enjoyment of using mathematics in work is A. He is of the view that his work doesn’t involve mathematics and he compares his work where “having a feel for an answer or solution is more useful” than mathematics which he sees as having an answer “correct to eight decimal places”. Due to “the very poor grounding” D “got in maths” he says he “was afraid of some of” the mathematics he encountered in engineering practice and he has “a nagging fear that” he has “got something wrong”. When he encounters a mathematics problem, he “refers” to his colleagues. While M enjoys his job, he doesn’t enjoy using mathematics in his work. He says he prefers the “buzz of working with people solving problems, working with teams and giving direction to teams” rather than “doing the maths which is working on your own.” M says that while he is confident “using Excel to run graphs, standard deviations, CPKs [process capability] that type of stuff” but “if you threw some honours maths type stuff in front of me now, I would probably fall off the chair". He is also of the view that there is a "risk" associated with using mathematics in work. He says "if you were doing or using some maths for your solutions … where nobody has done it before and you can't copy a template … you are putting yourself up, putting your neck on the line … you don't want to be the guy that puts something in place that goes wrong or is fundamentally flawed".

7.2.2.5 Outside of engineering

While this study is about mathematics in schools and engineering practice, many engineers' views go beyond engineering to society in general. Engineers' feelings about mathematics outside of engineering are presented in Table A9-6, Appendix 9, Volume 2**.** There is a view that mathematics has a "powerful" benefit outside of engineering but that society generally does not value mathematics sufficiently.

7.2.2.5-1 Low take up of higher level Leaving Certificate mathematics is accepted by society

Some engineers say that it is generally accepted by society that only a minority of students take higher level Leaving Certificate mathematics. The engineers maintain that there is a general belief in society that mathematics is difficult and there is a stigma associated with being good at mathematics (B, L, N, O, T, Q, and S). N is of the view that society does not value "maths … other subjects seem to take precedence over maths". B notes that "only 16% of any given school year is taking higher level maths" and that mathematics has been on "a sliding trend" for a number of generations. He says that "generations of students" have shown general disaffection with mathematics resulting from "a general dumbing down in society". O is concerned with "the number of adults that you meet who say that they hate maths, they are afraid of maths, maths is very hard and who would have negative experiences of maths at some level". He says that this attitude is accepted "because there are enough people to form a quorum who can feel not left out by being that way". O says this attitude is "wrong" and that if higher level Leaving Certificate mathematics was “made easier”, it would be “detrimental to more than just engineering”. T believes that many people have “a very negative” perception of mathematics and she adds that it is “perfectly acceptable to drop to pass maths whereas if you drop to pass English you will never be able to write a letter”. She is of the view that “society is at a loss” because of so few students taking higher level Leaving Certificate mathematics. L presents that “there are almost two types of people, the people who are good at maths and the people who aren’t good at maths”. He believes that because many teenagers have “so much choice” being good at mathematics is not as important now compared to when he was a teenager. S is of the view that there is a “general feeling that maths is important” but that “it’s not the be all and end all” for the majority of students. Q asserts that mathematics is one of those subjects where “people who can’t do mathematics call you a nerd and the people who are just amazing at it put you down”. She believes that students “in the middle are probably the ones who are probably going down into ordinary level, but would be capable of doing higher level” if they got support and encouragement to do so. Q says that mathematics makes many people “shut down” and many of these people “go on and they become parents and primary school teachers … and then their kids, are from a family who could never do maths … is a vicious circle”. Q believes strongly that “everyone can do maths”. She says “if someone says they can’t do something, they are never going to be able to do it … they just need the belief, they need a pace that suits them … everyone can do a certain level and I am not going to say that everyone is going to get PhDs in maths, but everyone can do maths”. She adds “I don’t think there should be 16% doing it, I think it should be 60% doing” higher level Leaving Certificate mathematics.

7.2.2.5-2 Mathematics has a “powerful” benefit outside of engineering and in society

There is a view that mathematics benefits many careers outside of engineering (A, B, J, O, P, and T). Some engineers note that mathematics has a “powerful” benefit outside of engineering and in society generally. Some applications of mathematics outside engineering range from counting money, statistically analysing social data, logical thinking in businesses, developing creativity and describing biological systems. "A grasp of statistics always useful" and society requires "numeracy" skills (A). For those outside the engineering profession, mathematics learning is not a "waste" (B). J presents that *thinking* skills developed from mathematics education are useful when solving "problems that are not essentially mathematical problems". He believes that "the ability to think logically" would be very useful in many jobs outside of engineering and mathematics. J's research work involves working with "teams of people who have degrees in biology or who have medical qualifications." He says it "is a big problem to work with these people because, not only do they not know any maths, they are scared of it, and their whole approach to solving a problem doesn't include maths". J says that "the biologists and the medical people" tend to "use maths after the fact" in that "they collect all their data and then they use statistics to try and see what they have got"; for example, when exploring the question "does smoking cause cancer"? J believes that using "maths at the beginning of the problem", for example in "systems biology" to determine "what is this drug doing to your bones", generates a more effective solution. J is also of the view that mathematics invites creativity. He says that while the first step in creativity is "to have the idea" he believes that if he "went back and studied or re-studied some of those parts of mathematics" he has stopped using, he "would probably start having different ideas as well", because he says "I have got a tool that I can use" to develop the ideas. O is of the view that "maths is a tool that enables you to really do powerful things in other disciplines". P is also of the view that mathematics is "an extremely useful tool" and that most people use mathematics "in one way or another" throughout their careers". T says that apart from engineering, "maths helps other occupations as well … the whole logical thinking training would help pretty much any occupation".

7.2.2.6 How to improve young people's affective engagement with mathematics

When asked how to improve young people's affective engagement with mathematics, engineers say that teachers and bonus CAO points are the key to more young people engaging in mathematics learning.

7.2.2.6-1 Teacher is "biggest influence" on students' relationships with mathematics

When asked how to improve young people's affective engagement with mathematics, all twenty engineers are unanimous in the view that "it's all down to the teaching" and that the "teacher is biggest influence" on students' relationships with mathematics. There is a view that primary school is where the "damage or good stuff" is done and that many primary school teachers "have no concept how any subject relates to anything" in the real world. There is a strong view that society is tolerant of "bad" mathematics teachers in Ireland in both primary and secondary schools. Many engineers are of the view that teachers' own attitudes to mathematics contribute to students' affective engagement with the subject and that the many "unqualified" mathematics teachers in the early years of secondary school are neither confident nor positive in their teaching of mathematics. Engineers say that teachers fail to communicate the value of mathematics and they also fail to demonstrate real world applications to students. Many teachers present mathematics as a "hard" subject in class and they opt for rote learning rather than understanding. Some engineers are critical about the mathematics assessment process. Engineers call for more student encouragement from mathematics teachers and for making mathematics more enjoyable for students.

"Teachers are the biggest influence on students' feelings for mathematics" (A). "Mathematics teaching is quite impoverished in Irish schools" and "there needs to be a root and branch reform of mathematics teaching" (K). J asserts that while teachers have "an enormous effect in all subject areas" the way mathematics is taught and assessed makes it more challenging than other subjects. P presents that "there is an absence of accountability on the part of teachers in the Irish school system" and that society is overly tolerant of "bad maths teachers". He states that "society needs to set certain expectations for kids coming out of school and teachers need to be accountable for the achievement by the kids of the expectation set by society". He criticises bad teachers and adds that the consequence of bad teaching is that students don't develop to their potential. P argues that mathematics education will not improve by bypassing the teachers and changing the kids. He says “you need to start by teaching the teachers”.

G is of the view that mathematics “teaching is very important from an early age” and he also calls for “specialist maths teachers” in primary schools. Such a specialist teacher is someone who “is interested in maths and children” and someone who recognises the different paces of children picking up the “fundamentals” of mathematics. K believes that primary school mathematics is “the foundation for everything else”. He says this is where the “damage or good stuff” is done. K suggests that “teamwork” where students “trash out” mathematics problems between them from a young age would benefit the “emotional side” of mathematics learning. When asked “how to improve young people’s affective engagement with mathematics”, R says she “firmly believes” that young people “need proper maths teachers” and she calls for a review of primary school teacher training whereby the “swots” of the Leaving Certificate who “have never lived” and who “have no concept how any subject relates to anything” are attracted to this “great job with great summer holidays”. T is also of the view that “the level of teaching in maths is very bad” in Ireland and that there is a need to go “back as far as primary school and how maths is taught there”.

According to B mathematics is “a bogey subject” loaded with “misunderstandings and misconceptions” that “teachers have failed to change”. H cites “teacher attitudes” as one key variable in how to improve young people’s affective engagement with mathematics. She says there is currently “no consequence for bad teaching” in Ireland; those teachers “are just left to teach badly”. She describes bad teachers as those who are “not interested” and “not able to take control of the class”. H also notes that many mathematics teachers’ attitudes are poor, they often label specific parts of course as “too hard” and they do not teach the entire syllabus. C is also concerned about teacher attitudes towards mathematics and he says that mathematics teachers need to be “comfortable” with mathematics and that mathematics classes need to be much smaller compared to other classes. D says that “mathematics teaching needs to be improved” and that “teachers’ attitudes are critical in mathematics education”. F is of the view that good teachers “should encourage students to stay with it [mathematics]" and with good teaching students would "grasp the maths, understand it and feel good about it rather than just learn it off by heart". He adds that a good mathematics teacher, "who interacts" with the class, makes a huge difference to students' attitudes". G calls for "personification of the teaching of maths" which he explains is "teacher support for individual students to promote confidence, understanding and importance of maths". L suggests that "confident teachers" would improve students' relationships with mathematics. M notes that mathematics teachers should be "qualified to teach mathematics" and they should present the subject "with confidence and positivity". O asserts that "unqualified teachers" are not "confident in maths" and that consequently students do not develop a "love for maths". Q says that "teacher and teaching is a big one", particularly "teachers' attitudes". She says "it's the whole feeling, if someone feels they can't do maths, they are just not going to do maths". She says that teacher and parents who themselves have low mathematics self-efficacies and classroom peers who either "put down" students who are weak at mathematics or call those who are good at mathematics "weird" contribute to students' poor confidence in their mathematical ability. T says that there is a perception that "maths is hard" and that many students "going into secondary school have already decided to do ordinary level mathematics for their Junior Certificate exam".

According to B, the challenge is to demonstrate the value of mathematics. He notes that despite young people's ability to engage with Facebook and Google, "there is a failure by teachers to communicate to young people that these modern tools only exists because of mathematics". B says that teachers should "open up the whole world of mathematics sitting behind" these modern tools, they should present the "linkage" with mathematics and they should "persuade young people of the relevance and important of maths". G says that "kids have to get the message" that being good at maths "opens up a huge number of careers". J believes that "in the earlier years" teachers "can emphasis more the applications of maths … say that this is why we are doing it, the place of maths in the world and make that part of the taught and examined subject". He suggests that "brain teaser type competitions" where students "are winning money" for thinking about a problem "in a logical way" would encourage more students to engage with mathematics. J also says that "the idea that maths is actually something that a lot of people will enjoy" might get children started with mathematics and if they discover that they are "good at it" they might enjoy it more and stick with it. K suggests that "making maths real" and illustrating how mathematics is "used in society" would improve students' understanding of mathematics. When asked how to improve young people's affective engagement with mathematics, L suggests greater "relevance to careers" and "applications and examples" in mathematics education. N is of the view that students "should see the value of it [mathematics] and its usefulness" and he calls for more applications in the teaching of mathematics. When asked how to improve young people's affective engagement with mathematics, O is of the view that good mathematics teaching is "about trying to make them [students] see maybe how good or how beautiful a subject it [mathematics] is or how important it is in life and how important it is across a range of other subjects". O adds that young people "don't see the usage of maths enough" and that young people need to acquire "that sense of how important maths is as a subject" and "that being good at maths might be quite helpful to them at some stage of their life". O argues "you have to pitch it at the applied level really". Q points out that "understanding" is essential in mathematics learning. She says that many young people ask "what is the point of maths" and that giving them "an understanding of the application of maths … would help". She says teachers need to "engage" with students and "make maths interesting so that students can have discussions in class or ask questions" and that teachers can tell "young people that they can do it … it is cool … it can be applied here, here and here … it is useful". S believes that students "would benefit from better enjoyment of maths by better teaching of it". He says that "relating maths to the real world is everything" and that "project based learning might be good training for people in grappling with problems which are bigger than maths itself and where only parts of the problem can be solved mathematically". T believes that mathematics teaching in secondary school would benefit from more practical applications and that a more rigorous assessment process would give students confidence in their "ability to do maths". U maintains that if teachers showed young people mathematics "in a way that is useful to them and in a way that they can understand" they would develop a greater interest in mathematics.

7.2.2.6-2 Decision to take higher level mathematics is driven by the students' perceptions of how time consuming and difficult it is and the corresponding reward in terms of points

There is a view that the decision to choose either ordinary level or higher level mathematics is driven by the students' perceptions of how time consuming and difficult each option is and the corresponding reward in terms of CAO points for the effort required to take higher level Leaving Certificate mathematics. A says that "the Leaving Certificate has become a machine for CAO points" and that higher level Leaving Certificate mathematics is "not an efficient way for a lot of people to get CAO points". Similarly B is of the view that mathematics has suffered as a result of the focus "on the points race rather than on a more holistic approach to education". Many engineers are of the view that awarding bonus CAO points for mathematics would encourage more students to take the higher level Leaving Certificate mathematics exam (C, D, E, F, G, H, J, M, P, R). C and D are both of the view that in Leaving Certificate mathematics the "reward should match the effort". E is of the view that "higher level maths probably takes up a fair bit of time" and that additional rewards "would encourage students to stay with it". While F is of the view that awarding "bonus points in maths" might "get more people doing honours maths" in the Leaving Certificate "to get more points", this "incentive will not necessarily steer people towards numerate careers". G believes that a doubling of Leaving Certificate points for mathematics would encourage more students to take the higher level exam. H also lists "bonus points" as a key variable in generating students' interest in mathematics. J suggests making "the other subjects harder" or giving "more points" for mathematics. M is of the view that if "you put half your study time into honours maths you should get bonus points for that subject". He is of the view that students get "easier" points in "pass maths, geography, history and home economics" than higher level mathematics. P is also of the view that "there should be some reward for them [students] in studying maths and the most obvious reward in an education system that is driven by points would be to give more points for higher level maths than for some other subjects that for one reason may be regarded as less important". He is also of the view that "if you can increase the number of kids who got higher level maths the number of kids who will have the confidence that they can successfully undertake an engineering programme will also increase". R is of the view that bonus points for mathematics would be an incentive for students to take the higher level exam.

While the majority of engineers are of the view that bonus points should be awarded for higher level Leaving Certificate mathematics none of the engineers say that higher level Leaving Certificate mathematics is too difficult. There is however a suggestion that some of the other subjects are too easy. Q is of the view that awarding bonus points for mathematics grades might encourage students to take "easier" subjects in their Leaving Certificate exam. T is of the view that making "the Leaving Certificate easier doesn't really work" and that bonus points would make "maths more of an elitist subject".

A summary of engineers' feelings about mathematics is presented in Table A9-7, Appendix 9, Volume 2.

7.2.2.7 Discussion of theme 2

There are two main findings associated with theme 2, these are:

F2.1 Teachers, task value (why should I do mathematics?), feelings of success and family, peer and societal influences are key motivators to engage in mathematics learning.

F2.2 Mathematics education contributes positively to engineer's work and confidence in mathematical ability and in mathematical solutions are the main motivators for engineers to use mathematics in their work.

7.2.2.7-1 F2.1: Teachers, task value (why should I do mathematics?), feelings of success and family, peer and societal influences are key motivators to engage in mathematics learning

In this study engineers present four main sources of motivation for both learning school mathematics and using mathematics in work:

- Task value (why should I do mathematics?)
- Feelings of success
- Sociocultural influences
- Teachers

7.2.7-1-1 Task value (why should I do mathematics?)

According to Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation in section 3.3.1, task value (why should I do the task?) is a predictor of achievement behaviour. In particular students' perceptions of the importance, utility and interest in mathematics are strong predictors of their intentions to continue to take mathematics courses. (Schunk et al. 2010; Wigfield and Eccles 2002). In this study it is apparent that engineers' task value of mathematics is a major source of motivation for both learning and using mathematics. Getting "the correct answer" is the key value of mathematics whereby engineers enjoy the recognition associated with success and consequently they are motivated to engage further with mathematics. The costs (perceived negative aspects) of engaging in mathematics are the time required to get "the correct answer" and the fear of getting the "wrong" answer. Families, teachers, work colleagues and society are all part of recognising mathematics success.

For some engineers the task value of mathematics was evident from a very young age when they enjoyed "mathematical type game playing" in the home. From counting sheep on the family farm to "working out the change", engineers engaged in real tasks where the "correct answer" was important. In school the main value of mathematics is the recognition of success where the "sense" of getting "the answer right” and knowing that the answer is correct is “very direct gratification”. Engineers say that the feeling of success is the main contributor to enjoyment of school mathematics. For example, one engineer says mathematics is “instantly rewarding” and he recalls at a very young age “getting the maths questions right” and “being rewarded for it” with “a gold star”. Other values of school mathematics are interest, requirement for entry into engineering education, important subject and a CAO points earner. A cost of doing higher level Leaving Certificate mathematics is the time requirement as evidenced by one engineer who says he risked passing his Leaving Certificate exam because mathematics consumed more than half his allocated homework time. Another cost is the lack of relevance of mathematics teaching to everyday life. The decision to choose either ordinary level or higher level mathematics is driven by the students’ perceptions of the cost of how time consuming and difficult each option is and the corresponding value in terms of CAO points for the effort required in taking higher level Leaving Certificate mathematics. This is supported in section 2.2.3 where a study of student participation in upper secondary mathematics education found evidence of students behaving strategically by not choosing mathematics, particularly advanced mathematics, because it is perceived as being more difficult than other subjects or one in which it is harder to achieve higher grades (Hodgen et al. 2010).

Engineers say that the transition from school mathematics to engineering mathematics is difficult. It is similarly noted in the literature review, in section 2.5, that engineering students are generally challenged by more complex mathematics delivered at a faster rate than what students experience in school (Irish Academy of Engineering 2004; Manseur et al. 2010a). The engineers’ value of engineering mathematics includes: an understanding of engineering subjects that are based on mathematics; benefits of discipline and rigour associated with learning mathematics; and the level of mathematics required to pass exams and become a professional engineer. The cost of engineering mathematics is related to its usefulness whereby engineers ask “what is the point of this [engineering mathematics] and where are you ever going to use this”?

The value of mathematics is very evident in engineering practice where engineers use varying degrees of mathematics in a variety of ways. Examples of the value of engineering mathematics are apparent in its many uses: “an appreciation of mathematics ”; “problem-solving techniques”; “mathematical logic”; “discipline of maths”; “estimation of engineering solutions”; “having a feel for an answer”; “checking by maths”; “useful tool”; “a safety valve”; “it’s a faster way of doing something”; and being “able to cope with things better because I have a grasp of the kind of maths and figures, particularly statistics required in my industry”. Mathematics also has an affective value, for example: one engineer “finds pleasure in doing” mathematics; engineers are “comfortable with maths and using maths”; engineers show “confidence in mathematical solutions”; “to get the real buzz out of maths it has to be a real problem”; and “it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it”. The cost of doing mathematics in engineering practice includes: time; risk of being “wrong”; and colleagues’ lack of respect for mathematics. One engineer notes that “in engineering there is very seldom a unique solution, there is a balance between the amount of time you can spend on problem solving and the degree of certainty that you can have that the solution you’ve come up with is the ideal solution”. Engineers say that mathematics is “risky” and one engineer, when using mathematics, has “a nagging fear” that he has “got something wrong and when he encounters a mathematics problem, he “refers” to his colleagues. There is also a view that “maths is usually more of an individual activity than a team effort” whereby “you have to stand on your own two feet” and “if you get it wrong it can look very bad”. Even when engineers get the mathematics correct, their colleagues may not have “respect for mathematics” and one engineer says his “current CEO currently is very much a marketing man and so definitely the emphasis is on sales and marketing and away from the maths right now”. Engineers are also of the view that the value of mathematics to society is not fully realised because society itself does not value mathematics sufficiently and it is generally accepted by society that only a minority of students take higher level Leaving Certificate mathematics.

7.2.2.7-1-2 Feelings of success

In section 3.3.1 it is presented that goal setting is a key motivational process and learners with a goal and a sense of self-efficacy for attaining engage in activities they believe will lead to attainment (Schunk et al. 2010). In this study the engineers' "goal" was to get the "correct answer". One engineer "persisted" until he "worked out the answer", another engineer says "I kept at it until I got the right answer" and a further engineer says she was "diligent", "methodical" and she would also "go back" over her work and she "filled in units" to verify that equations were "correct".

As discussed in section 3.3.1, students' self-perception of ability and expectancies for success are the strongest predictors of subsequent grades in mathematics (Schunk et al. 2010). Engineers say that confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. From the "satisfaction" of getting the "right answer" one engineer says "I got confidence in the fact that I was getting good results in mathematics and then I realised this is something that I could be good at". Another engineer asserts that the key to mathematics learning is "finding that you are able to do it". The sense of achievement one engineer experienced when he solved a difficult problem spurred him "to do more". This is consistent with Ernest's view in section 3.3.2 who reports that success at mathematical tasks leads to pleasure and confidence and a sense of self-efficacy, the resultant improved motivation leads to more effort and persistence (Ernest 2011).

In work engineers' enjoyment of mathematics also comes from their success when using mathematics and their confidence in mathematics and in mathematical solutions. When one engineer came up with "a minimal tweak" that made one of Sony's high volume products "comply with an international standard" it put him "on cloud nine for weeks". Another engineer enjoys using mathematics in work because it is "clean … it is completely logical … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it". It is the engineers who do not enjoy mathematics in work who are of the view that mathematics is "risky".

7.2.2.7-1-3 Sociocultural influences

Sociocultural influences are a big influence on engineers' mathematics learning and subsequent motivation to use mathematics. In section 3.3.2 Zeldin and Pajares (2000) say that students who are exposed early to mathematics-related content by relatives who work in mathematics based fields often find this domain comfortable and familiar. Their vicarious experiences with family members create a positive self-efficacy perception in the areas of mathematics and science (Zeldin and Pajares 2000). From the interview data it is apparent that some engineers' families provided support and scaffolding for their mathematics learning whereby engineers "regularly discussed maths problems" and other topics such as "methodology", "the right answer" and "negative views" about mathematics with their families.

In section 3.3.1 it is maintained that peer networks can heavily influence individuals' academic motivation. Peer networks are large groups of peers with whom students associate. Within the groups values are reinforced and individuals' academic motivation and students in networks tend to become similar which can lead to more or less engagement in school activities (Schunk et al. 2010). Collaborative learning, where a group of students work together dealing with different perspectives and a common goal, encourages interaction between students. The peer tutoring element of collaborative learning benefits both students who are tutoring as they are encouraged to clarify their own thinking and those who are being tutored as they can address their areas of misunderstandings. Collaborative learning opportunities encourage students to verbalise their ideas and challenge other students (Pietsch 2009). Engineers say that engaging in social or group learning of mathematics with peers or role models has many advantages for students preparing for the Leaving Certificate mathematics exam. They say that the "comfort and positivity" of peers towards numerate subjects; compensation for poor teaching, playing "football together because nobody else would play football" with "geeks"; turning Leaving Certificate mathematics into this "fun thing" and motivating students to "get an A in Leaving Certificate mathematics" are advantages of having friends who are positively disposed to mathematics learning. Engineers struggled with the transition from school mathematics to engineering mathematics where “lecturers don’t teach, they lecture”, “they tell you where the information is” and you “are very much left working it out for yourself”. One engineer says that in college “the social element of the maths was gone”. Another engineer engaged in peer learning where he says there were “a lot of us putting our heads together trying to get solutions”. A further engineer who “would not dare ask a question out loud in a lecture” attended tutorials where “the post-grads would come and just talk to you”. These views are supported in section 2.3.2 where the NCTM assert that communication is an essential part of mathematics education as it is a way of sharing ideas and clarifying understanding (National Council of Teachers of Mathematics 2000).

It is maintained in section 3.3.2 that sociocultural Influences strongly impact mathematics learning (National Research Council 1989; Schoenfeld 1992; Zeldin and Pajares 2000). As children become “socialised by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. Later, as parents, they pass this conviction on to their children (National Research Council 1989). Some engineers in this study are of the view that there is a general belief in society that mathematics is difficult and there is a stigma associated with being good at mathematics. One engineer is of the view that a “them and us culture” happens at quite an early age when “people decide that they can’t do it [mathematics]” and “that the people who do it are somehow different from them”. Being good at mathematics causes social problems for students, they feel “isolated”, they hide “the guilty pleasure of enjoying maths” and they try to change their personality or appearance so as “not to be branded a geek”.

7.2.2.7-1-4 Teachers

In section 3.31 Bandura (1986) presents that behaviour represents an interaction of an individual with the environment and that learning is greatly expanded by the capacity to learn vicariously. As such teachers are role models and their attitudes, emotions, beliefs and values about mathematics impact their students' learning (Bandura 1986). All twenty engineers are unanimous in their view that "teacher is biggest influence" on students' relationships with mathematics. The four engineers who don't express any enjoyment of their school mathematics and who also had low confidence in their mathematics ability all had poor mathematics teachers. One engineer stands out in terms of his low confidence in his school mathematical ability. He says that due to "bad" teaching, he developed an "inferiority complex about maths" and a "blockage" to learning mathematics in secondary school that "caught" him all the way through college and work. In her Leaving Certificate year, when another engineer moved away from her "manic depressive" teacher to a grind school, her new mathematics teacher "totally revitalised her feelings of what maths was about".

Engineers identify three teaching variables: teaching the value of mathematics; teachers' attitudes; and societal influences on teaching quality. There is a view that primary school is "the foundation for everything else", it is where the "damage or good stuff" is done and that many primary school teachers "have no concept how any subject relates to anything" in the real world. Engineers say that teachers fail to communicate the value of mathematics and they also fail to demonstrate real world applications to students. Instead teachers should "emphasis more the applications of maths … say that this is why we are doing it, the place of maths in the world and make that part of the taught and examined subject". Similarly in section 3.3.2 Ernest recommends that mathematics teachers should ask themselves, "what is mathematics" (Ernest, 2011) and Schoenfeld recommends that mathematics instruction should provide students with a sense of "what mathematics is and how it is done" and that as a result of their instructional experiences, students should learn to "value mathematics and feel confident in their ability to do mathematics" (Schoenfeld 1992). In section 2.3.2 the NCTM highlights the need to focus on "important mathematics" that will prepare students for continued study and for solving problems in a variety of school, home and work settings (National Council of Teachers of Mathematics 2000).

According to the engineers in this study, teachers are a huge influence on students' motivation. In section 3.3.2 a report by National Research Council (1989) in the U.S. maintains that "self-confidence built on success is the most important objective of the mathematics curriculum" and that the ability of individuals to cope with mathematics, wherever it arises in their later lives, depends on the attitudes toward mathematics conveyed in school and college classes. The report states that mathematics curricula must avoid leaving a "legacy of misunderstanding, apprehension, and fear" (National Research Council 1989). In section 3.3.1 it is maintained that constructivist teaching (theory contending that individuals construct much of what they learn and understand through individual and social activity) changes the focus from controlling and managing student learning to encouraging student learning and development (Schunk et al. 2010). Many engineers are of the view that teachers' attitudes to mathematics contribute to students' affective engagement with the subject and that the many "unqualified" mathematics teachers are neither confident nor positive in their teaching of mathematics. The engineers say that many teachers present mathematics as a "hard" subject in class and they opt for rote learning rather than understanding. Engineers believe that if students "feel they can't do maths they are just not going to do maths" and many students "going into secondary school have already decided to do ordinary level mathematics for their Junior Certificate exam". This view is reinforced in section 3.3.2 where a study found that "students' perceptions of their teachers' perceptions of their ability to do mathematics decreases as the students progress from elementary to high school" (Smith et al., 2009). Engineers are of the view that teachers need to be more positive about mathematics and "the idea that maths is actually something that a lot of people will enjoy" might get children started with mathematics and if they discover that they are "good at it" they might enjoy it more and "stick with it". Good teachers "should encourage students to stay with it [mathematics]" and with good teaching students would "grasp the maths, understand it and feel good about it rather than just learn it off by heart". The engineers' views are also consistent with Lampert (1990) in section 3.3.2 who says that students acquire beliefs about mathematics through years of watching, listening and practising mathematics in the classroom (Lampert, 1990). Furthermore, Pape, Bell and Yetkin (2003), in section 3.3.2, say that teachers' role is to "establish the context for mathematical development" and to scaffold students' developing skills by presenting tasks that encourage students to value and enjoy mathematics and to articulate their thinking (Pape et al. 2003). This view is also supported by Yara (2009) in section 3.3.2 who found that students' positive attitude could be enhanced by teachers' enthusiasms, resourcefulness and behaviour, thorough knowledge of subject matter and by making the subject interesting. The attitude of the teacher and the teacher's disposition to mathematics "could make or unmake" students' attitudes towards learning mathematics (Yara, 2009).

There is a strong view amongst the engineers in this study that society is tolerant of "bad" mathematics teachers in Ireland in both primary and secondary schools. One engineer presents that "society needs to set certain expectations for kids coming out of school" and mathematics teachers need to be accountable for achieving those expectations. This is a case of Schoenfeld's "vicious pedagogical/epistemological circle", discussed in section 3.3.2, (Schoenfeld 1992).

7.2.2.7-2 F2.2: Mathematics education contributes positively to engineer's work and confidence in mathematical ability and in mathematical solutions are the main motivators for engineers to use mathematics in their work

The majority of engineers note the positive contribution of mathematics education in their work. For some engineers mathematics is "essential" and for others it is a "useful tool". Mathematics is "valuable" in ten per cent of one engineer's work and "ten per cent of the engineers" in another engineer's company "need some of the learning from higher level maths" The range of values of mathematics education in engineers' work includes: "differential equations"; "kind of maths and figures, particularly statistics required in my industry"; "maths in a great depth"; "problem-solving techniques"; "appreciation of mathematics"; "mathematical logic"; "discipline of maths"; "estimation of engineering solutions"; "having a feel for an answer" and "some checking by maths". The five lowest *curriculum mathematics* users and one other engineer do not value their mathematics education in their work. However it is also noted that three of the lowest *curriculum mathematics* users do value estimation of engineering solutions in their work and the other engineer who doesn't see the value of his mathematics education in work says that engineers "have to be good at problem solving".

The majority of engineers enjoy using mathematics in their work. Much of the engineers' enjoyment of mathematics in work comes from their success when using mathematics and their confidence in mathematics and in mathematical solutions. Engineers like getting an "exact solution" and for one engineer mathematics is "a safety valve" in his work. Engineers tend to "double check" the mathematics before presenting a solution to co-workers. In engineering practice "maths is usually more of an individual activity than a team effort" and one challenge for engineers is their colleagues' attitude towards mathematics. One engineer says that mathematics in work is "clean … it is completely logical … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it". However some engineers say that mathematics is "risky" because "you have to stand on your own two feet" and "if you get it wrong it can look very bad". One engineer says he has "a nagging fear that" his mathematics is "wrong". Another engineer is of the view that "there is a certain respect for mathematics" in his company but that his "current CEO currently is very much a marketing man and so definitely the emphasis is on sales and marketing and away from the maths right now".

As discussed in section 2.7, there is a "belief among some practising engineers that the mathematics they learned in college is not applicable to their daily work", however there is limited published research on practising engineers' mathematics usage (Cardella 2007). In section 2.7.1, an investigation of engineering students' use of mathematics found that "recognising the value of mathematics as a tool likely prepares students to use mathematics in appropriate contexts" (Cardella 2007). In section 2.7.1 a study of civil and structural engineers found that undergraduate engineering students continue to need to know and learn mathematics (Kent and Noss 2002). Another study found that engineers require "at least a conceptual understanding of the majority of the math topics" (Ellis et al. 2004). Research literature in section 2.7 suggests that there is a greater need for *curriculum* *mathematics* in “breakdown situations”, where tools produce unexpected results (Alpers 2010a; Alpers 2010b; Alpers 2010c; Gainsburg 2006; Triantafillou and Potari 2006). There is very little mention in the available research literature about engineers’ motivation to use mathematics or their emotional feelings when using mathematics in work, one exception is Monica Cardella in section 2.7.1 who observed that “some undergraduate engineering students can become frustrated by the ambiguity and uncertainty that are normal for authentic engineering tasks (Cardella 2010).

7.2.3 Theme 3: Factors Influencing Engineering Career Choice

The findings outlining the factors influencing engineers' career choice are presented in this section. Theme 3 is presented as follows:

Page number

7.2.3.1 Engineering career choice influences

7.2.3.1-1 Feelings about mathematics is main influence on engineering career choice

The engineers' path to engineering education is included in Table A9-8, Appendix 9, Volume 2. The majority of engineers say that their feelings about mathematics were the main factor in their choice of engineering as a career (A, B, E, G, H, J, K, L, M, O, Q, S, T, and U). Examples of this include: choice of engineering was "very strongly influenced by my feelings about mathematics" and "if I hadn't been happy or comfortable with maths I probably wouldn't have picked engineering" (A); feelings about mathematics influenced choice of engineering "one hundred per cent" (B); engineering "wouldn't be much of a struggle" because "I was confident with maths" (H); with "ability and enjoyment of mathematics" engineering "just made sense"(L); "I looked at my CAO application and said I would like to do more maths, so I just ticked all these boxes for engineering , civil engineering, manufacturing engineering, chemical engineering, that's why, it was maths" (M); O wanted a career that "was maths related" because he "loved maths"; Q looked at careers associated with mathematics because she enjoyed mathematics and school mathematics was her "strength"; "to me maths was everything, maths was where I wanted to be and to me it was the key to the career that I wanted, I wanted to be an engineer … I didn't want to do anything else" (S); interest in engineering came from "confidence from having done higher level maths" (T); career choice was influenced "a very great deal" by "love" of mathematics, engineering and mathematics "were hand in hand, I had very much an aptitude for mathematics in school, that's the subject that I found easier, that's the subject that I didn't have to study and to me the engineering followed on from that"(U).

Some engineers whose feelings about mathematics impacted their choice of engineering viewed engineering as a continuation of their mathematics education. Engineering was "a logical progression" from A's school subjects. For B engineering was "a very natural progression from one education phase into the next education phase". G is of the view that "maths is one of those developmental things" in that the more mathematics one does the better they get at it and for G engineering was the next stage in his mathematics development. An attraction to engineering for H was that she "wanted to keep the maths skills up". She says she "liked the look" of the engineering curriculum and the first two years of her engineering course were "so maths orientated". J did not directly choose engineering, instead he chose to develop his mathematics skills and with these skills he "fell into" engineering. M says he choose engineering education because he wanted "to do more maths".

While some engineers were primarily attracted to the practical side of engineering, they were aware of the association between mathematics and engineering. F chose engineering because he "was interested in engineering things and mechanical things" and he "could see maths being used" in engineering which led him "to be interested in maths". N chose engineering because he "just loved building things", he was good at "the technical and practical side of things" and he says that while his choice "wasn't necessarily a love of maths", mathematics "was a requirement that I got to like afterwards". P's choice of engineering was also "very definitely" influenced by mathematics but he says "it was the more practical nature of engineering that appealed to me". P does note that "if you haven't enjoyed school maths, the probability that you will undertake a career in engineering has to be quite low". R chose engineering because she "always wanted to build bridges". She says her "emotions towards maths was only for the purpose of getting into UCD to do engineering". While R is now of the view that "you would have to like maths to want to do engineering", she says that when she was in secondary school she did not know how much mathematics there would be in engineering education.

D says his reasons to become an engineer had nothing "to do with love of maths". His career decision was based on his interest "in things, how things worked, building things and making things"; he says he was "just fascinated by how things work". D had "heard people say" that engineers needed to be good at mathematics and for him mathematics was "probably the biggest blockage" when choosing engineering given that he was not "great at maths" in school. He says he adopted a view that mathematics "is just one subject" and that one needs "other attributes to be a good engineer".

Seven of the twenty engineers came from families who had engineers. The influences of engineering family members on the participants' career decisions were more in the context of providing support for mathematics learning rather than promoting engineering as a career. D is the only engineer with a family member in engineering who didn't receive any home support with his mathematics education.

Of the engineers who say that their feelings about mathematics was the main factor in their choice of engineering E is the only engineer who did not receive support with her mathematics learning at home. However E presents herself as having very positive feelings for mathematics in both primary and secondary school. She says the "praise" she got from her mathematics teacher in primary school "egged her on". The engineers whose main reason for choosing engineering was not their feelings about mathematics (C, D, F, N, P, and R) did not get any particular family encouragement or home support with mathematics.

Farming backgrounds also contribute to engineers' interest in engineering careers. B's "interest in understanding things, taking things apart and trying to build new things" was born out of his uncle's workshop and farm. C developed "an interest in taking things apart" from his farming background and he says that engineering is perceived as "an acceptable profession for a farmer's son who is not going into farming". While K's teacher encouraged him to do "pure maths in Trinity", he felt that this "wasn't hands on enough". Having grown "up on a farm", K "was used to touching stuff" and he couldn't see himself "being purely abstract". R, who is also from a farming background, is of the view that "engineers from rural areas have more of an affinity or more of a feel for engineering". She says that "farms shape engineers" and that farmers "are probably the most non-sexist guys who ever lived because they don't care who does the work, who milks the cows, it can be male or female, they are all allowed to do equal work". T, whose father is both an engineer and a farmer, is of the view that farming and engineering are similar in that they each involve "hard work", "being a bit practical", "technology" and "how to do things more efficiently".

At the time of choosing their careers, engineering was viewed as a prestigious career (B, C, G, L, N, R, and S). B's "entry into the engineering profession" was "a due reward" for "excelling in maths". At the time of G's entry into engineering education, the points were high and he says the profession "had a lot of credibility". When L "did the leaving certificate in 1997 … points for engineering would have been high and there would have been that perception that it was a good career". R says that she "naturally went after the highest points course because it gave you a standard". She says she was "lucky" because the year she did the Leaving Certificate exam, "the points for engineering were the same as medicine". She says she was "up there at the top, it was ego as well". While S is of the view that "the people who enjoy maths are more likely to become engineers", he says that when he was in school "the public perception of engineering was much higher" and for him engineering "was a good career choice".

7.2.3.2 The engineering profession

In this section the engineers' views about the engineering profession are presented.

7.2.3.2-1 Engineering has poor image

While some engineers say that engineering was seen as a good career choice at the time of their entry into engineering education (B, C, G, L, N, R, and S), the majority of engineers say that the engineering profession currently has a poor image (A, B, C, E, F, G, L, N, Q, R, S, T, and U). Engineers' views include: engineering doesn't "have a fantastic image", it isn't "a sexy profession" and little is known about "the great industrial engineers of the past" (C); engineering is "not seen you know as a very glamorous career" and the "profession's brand" has been "watered down over time" (F); engineering is the most "underrated profession" and anybody can call themselves an engineer without having the necessary qualifications (G); when "an engineer does a good job" nobody notices, "they only notice when you do a bad job"(H); "the status associated with being an engineer has dropped in the last couple of years" (L); engineering has "an image problem" (N); "anyone carrying a spanner calls themselves an engineer" (S); "it is perceived that engineering isn't a job for a girl" or it is only for "dowdy girls" (T); and there is a relatively recent perception in U's company that engineers can only get so high in his company, whereas other disciplines can go higher.

There is a strong feeling amongst the engineers that engineering is undervalued and badly paid: "engineering isn't particularly highly valued" and "a poor solicitor" earns more "than a good engineer" (A); engineers actively seek to minimise the cost of engineering expertise when costing projects and the same principles are not applied to accountancy or legal costs associated with the same projects (D); people who "are in charge of the money", while "dependent on other people's skill sets", are better paid compared to engineers (H); in the consumer electronics business; there is "a very strict cost model" whereby companies are continuously "looking at cheaper ways of doing engineering" (M); "the anorak brigade of engineers" are "very happy" solving problems while the commerce people sit "in the bar" discussing how "to make money" (R); and "a chief executive of a company who has decided that his direct reports and his very senior management are all going to be from marketing and accounting disciplines … has decided engineers are not good at that thing [senior positions]" (U).

Views that the "engineering profession is badly understood" include: many young people have a "blurred picture" of engineering, they see an engineer as someone who is up to his or her "neck in equations for forty years" and not the "happy, successful engineer contributing to society" (G); "the term engineering to a secondary school student is associated with the construction industry rather than a lot of the other areas of engineering" (L); students "have no idea about the different types of engineers that exist" (T); and it would be "useful" to show school students "what an engineer does … without going too much into the mathematics of it" (U).

While there are concerns about the engineering profession in that it is undervalued and has a poor image, the engineers also express their views that the profession is badly represented and that engineers themselves are not particularly interested in promoting the profession. For example, "when there is a big success story or an achievement it is a scientific breakthrough, whereas when it fails it is an engineering disaster" (Q); when "an engineer does a good job" nobody notices, "they only notice when you do a bad job", thus as an "engineer your job is to stay below the radar" (H); "engineers "don't promote themselves enough … nor do they see the value in promoting themselves or engineering" (N); "engineers don't fight for their territory", they often appear invisible in major engineering projects while doctors on the other hand will regularly appear on television wearing a stethoscope around their necks (G); there is a need to get "engineers into positions of power and influence so that they become more significant role models in society" (B); and there are times when from a career point of view it suits an engineer "not to be painted as an engineer" because a chief executive of a company "has decided that his direct reports and his very senior management are all going to be from marketing and accounting disciplines … he has decided engineers are not good at that thing" (U).

There is a strong view that engineers' and technicians' roles are mixed up in engineering practice. Engineers are generally defined as those with a minimum level 8 engineering qualification while technicians have a level 6 engineering qualification.

7.2.3.3 Modern young people's career choices

In this section the engineers' views about modern young people's career choices are presented.

7.2.3.3-1 Current students maximise points usage

There is a strong view among the engineers in this study that current Leaving Certificate students opt for high points courses. When choosing careers, students maximise their points' usage and those who score high points in their Leaving Certificate exam are not inclined to choose low points courses (E, F, G, J, L, Q, R, S, and T). For example: "the really top guys all want to do medicine" just because they have the points to do so (F); Students choose subjects based on "the best set of points" they can get from them and very often they "drop" higher level mathematics in favour of home economics even though they mightn't like it" (J); students who currently get an "A1[27] in maths" are likely to score high points overall and they are unlikely to opt for an engineering course "that is only 350[28] points" (L); current Leaving Certificate students "don't choose careers, instead they choose college courses" (Q); "there is a lot of evidence of people picking a course consistent with the number of points that they feel they are going to get rather than what they are interested in" (S); and "points are the motivation" for many young people when choosing a career and "the course with the highest points is the one you want" (T).

Many engineers are of the view that engineering does not meet young people's career expectations for example: "engineering seems boring" and those who want a career in mathematics can opt for actuarial studies and get "big jobs" in "London, New York and become very successful" (E); and engineering education "has been dumbed down seriously through the intervention of the institute of technology route"[29]. Why would students, who have higher level Leaving Certificate mathematics do engineering when they could get "the same level 8 degree without higher level maths"? (R);

[27] A1 Grade: ≥ 90%

[28] Points score: Maximum number of points is 600 (up to 2011)

[29] Institute of technology route: In Ireland students who achieve high grades in technician courses (level 6) in institutes of technology can subsequently transfer to level 8 courses and receive exemptions from first two years of level 8 courses.

7.2.3.4 Discussion of theme 3

There are three main findings associated with theme 3, these are:

F3.1 Feelings about mathematics is the main influence on engineering career choice.

F3.2 Engineers say that the engineering profession currently has a poor image.

F3.3 Higher level Leaving Certificate mathematics is currently valued as a points earner and not as a stepping stone to engineering careers.

7.2.3.4-1 F3.1: Feelings about mathematics is the main influence on engineering career choice

Fourteen of the twenty engineers interviewed say that that their feelings about mathematics were the main influence in their decision to choose engineering careers. One engineer says he considered engineering as “kind of maths”. Another engineer says “to me maths was everything, maths was where I wanted to be and to me it was the key to the career that I wanted, I wanted to be an engineer”. For some engineers, engineering was “a very natural progression from one education phase into the next education phase”. For just two engineers, mathematics was an obstacle for entry into engineering education. One of these engineers had a secondary school mathematics teacher who was “plain ordinary bad” and because the other engineer, whose teacher was a “manic depressive”, knew she needed higher level mathematics for engineering and she had to learn mathematics “by hook or by crook” and also by “slight rote” for her Leaving Certificate exam.

With the exception of one engineer who had a very positive primary school mathematics experience, the engineers whose feelings about mathematics was the main reasons for choosing engineering careers all received support with their mathematics learning from their family from a young age. Engineers, whose main reason for choosing engineering was for reasons other than their feelings about mathematics, didn’t get any family encouragement or home support with mathematics. For one engineer, it was his “apprenticeship that opened up” engineering. Another engineer was fascinated by how things work. Another engineer was interested in “electrical things and mechanical things”. Another engineer “loved building things” and “maths was just part of it”. While another engineer “understood that maths was a very important element in the engineering curriculum”, he states that “it was the more practical nature of engineering that appealed” to him. A further engineer “always wanted to build bridges” and her “emotions towards maths was only for the purpose of getting into UCD [university] to do engineering”.

It is also noted that at the time of choosing their careers, seven engineers say that engineering was a prestigious career. One engineer’s “entry into the engineering profession” was “a due reward” for “excelling in maths”. “High points” and career “credibility” also influenced another engineer’s decision. When a further engineer commenced engineering studies, the entry points for engineering were on par with medicine and there was an “ego” associated with engineering at the time and she felt she was “up there at the top”.

In section 2.4 James and High (2008), following a review of literature about mathematics education, were unable to answer the following question: “is there a correlation between people choosing engineering as their field of study and those who enjoy applications of mathematics” (James and High 2008)? This study presents evidence that the answer to this question is yes there is a correlation. The majority of the engineers in this study say that their feelings about mathematics were the main influence in their decision to choose engineering. Engineers’ strong feelings about mathematics in the context of engineering career choice include: with “ability and enjoyment of mathematics” engineering “just made sense”; “I looked at my CAO application and said I would like to do more maths, so I just ticked all these boxes for engineering; “to me maths was everything, maths was where I wanted to be and to me it was the key to the career that I wanted, I wanted to be an engineer … I didn’t want to do anything else”; interest in engineering came from “confidence from having done higher level maths”; and engineering career choice was influenced “a very great deal” by “love” of mathematics.

There is a view in research literature that mathematics serves as a "gatekeeper" to engineering education (Winkelman 2009) in section 2.4 and Ifiok Otung questions the "wisdom of scaring away potentially successful engineers with a mathematical content that is rarely used during the career of 98% of practitioners" (Otung 2002). The answer to this question is outside the scope of this study. However two engineers in this study had bad school mathematics experiences and they were not scared away from engineering careers. For one engineer whose Leaving Certificate mathematics teacher was "plain ordinary bad", higher level mathematics was a "career requirement" and his interest in engineering as a career motivated him to continue with higher level mathematics in school. While he says that mathematics was "probably the biggest blockage" when choosing engineering and that mathematics "is just one subject" and that one needs "other attributes to be a good engineer", he also says he "was afraid "of some of the mathematics he encountered in engineering practice and when using mathematics he has "a nagging fear that" he has "got something wrong". When he encounters a mathematics problem, he "refers" to his colleagues. Another engineer also knew that she needed higher level mathematics for admission to engineering education and she says "I had to do it by hook or by crook in whatever way I could remember it to get a C in the honours exam". She is currently an engineering manager and she uses a high level of both *curriculum mathematics* and *thinking* in her work.

As noted in section 2.4 the main research finding in literature concerning mathematics in the context of career choice is that women's mathematical self-efficacy is significantly lower than men's perceptions of their capability to succeed in mathematics and this is a major influence on career choice (Correll 2001; Løken et al. 2010; Zeldin and Pajares 2000). Betz and Hackett (1981) suggest that women's lower self-efficacy expectations with regard to occupations requiring competence in mathematics may be due to "a lack of experiences of success and accomplishments, a lack of opportunities to observe women competent in math, and/ or a lack of encouragement from teachers or parents" (Betz and Hackett 1981). This is supported in this study as engineers say that the feeling of success is the main contributor to enjoyment of school mathematics and that confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. One engineer who got confidence from “good results” in mathematics “realised this is something” she could be good at”. Another engineer says the key to mathematics learning is “finding that you are able to do it”. The sense of achievement one engineer experienced when he solved a difficult problem spurred him “to do more” mathematics. Engineers whose feelings about mathematics impacted their choice of engineering were motivated to engage in more mathematics learning and they say that engineering education was “a logical progression” and “a very natural progression from one education phase into the next education phase”.

There are three interesting observations in this study: (i) the correspondence between engineers whose family supported their mathematics learning from a young age and the engineers whose main reason for choosing engineering was their feelings about mathematics; (ii) engineers, whose main reason for choosing engineering was not their feelings about mathematics, did not receive any family encouragement or home support with mathematics; and (iii) engineers who are especially critical of their mathematics teachers say that their feelings about mathematics did not influence their career choice. These observations further reinforce the relationship between students’ school mathematics learning experiences and engineering career choice. In section 2.4, Prieto, Holbrook, Bourke, O'Connor, Page and Husher (2009) maintain that students’ image of the engineering profession comes from their parents, family relations and school career advisor. They also maintain that students’ mathematics and science learning is compromised because “college graduates who become teachers have somewhat lower academic skills on average than those who do not go into teaching” and that significant percentages of middle school mathematics and science teachers do not have a major or minor in those subjects. They believe that enriching the mathematics and enabling sciences experience for students holds the key to increasing enrolments in engineering education (Prieto et al. 2009). Similarly Heywood (2005), in section 2.4, believes that interventions in schools can help teachers acquire knowledge that will better prepare and excite students about engineering careers. Heywood asserts that even though we live in a technological society, that “engineering departments possess a vast knowledge that is not readily available to school teachers". He suggests new types of degrees in which students undertaking an engineering program can also obtain teacher certification (Heywood, 2005).

It is observed, in section 7.2.2, that all twenty engineers in this study are unanimous in the view that "teacher is biggest influence" on students' relationships with mathematics. However the engineers also have a view that teachers fail to communicate the value of mathematics and they also fail to demonstrate real world applications to students. They say that that many primary school teachers "have no concept how any subject relates to anything" in the real world and that many "unqualified" mathematics teachers in the early years of secondary school are neither confident nor positive in their teaching of mathematics. Many teachers present mathematics as a "hard" subject in class and they opt for rote learning rather than understanding. Engineers believe that if students "feel they can't do maths they are just not going to do maths" and many students "going into secondary school have already decided to do ordinary level mathematics for their Junior Certificate exam" and are thus excluded from direct entry to level 8 accredited engineering courses. Engineers say that teachers should "emphasis more the applications of maths … say that this is why we are doing it, the place of maths in the world and make that part of the taught and examined subject … the idea that maths is actually something that a lot of people will enjoy" might get children started with mathematics and if they discover that they are "good at it" they might enjoy it more and "stick with it". Similarly a study, in section 2.4, found that "instrumentality", which is a "learner's tendency to ascribe worth and benefit to knowledge and skills in the domain, which in turn influences attention, engagement and investment", demonstrates strong influence on interest and the likelihood of pursuing postsecondary education (Hardré et al., 2009).

Engineers' view that teachers, task value (why should I do mathematics?), feelings of success and peer and societal influences are key motivators to engage in mathematics learning (F2.1) is similar to career choice theory where interests; abilities; and, values are key career choice factors. In section 2.4, it is suggested that career development is an evolutionary process comprising three periods: fantasy; tentative and realistic.

In the fantasy period, families respond with attitudes toward both the behaviours and the occupations role-played by young children. In the tentative period the career choices of eleven to seventeen year olds are based on personal criteria: interests; abilities; and values and also the attitudes of others towards those people and occupations. In the early years of adulthood, individuals in the realistic phase begin to balance the personal criteria with the opportunities, requirements, and limitations of the occupations presented in society. An individual's career choice is a compromise of interests and abilities, as well as satisfying values and goals as much as possible (Ginzberg et al. 1951).

7.2.3.4-2 F3.2: Engineers say that the engineering profession currently has a poor image

Engineers present strong views that engineering has a poor image; the engineering profession is undervalued and badly paid; there is little knowledge about what engineering is; engineering is badly represented and promoted; and engineers' and technicians' roles are mixed up. Engineers maintain that many students, while aware of the association between engineering and mathematics, do not know what an engineer does. One engineer says that many young people have a "blurred picture" of engineering where they see an engineer as someone who is up to his or her "neck in equations for forty years" and not the "happy, successful engineer contributing to society".

While at the time of choosing their careers, engineers say that engineering was a prestigious career and some engineers also say that the engineering profession is currently undervalued and badly paid, there is little knowledge about what engineering is and engineers' and technicians' roles are mixed up. This view is supported in section 2.6 where it is maintained that due to the inadequate body of work on engineering practice there are misconceptions as to what engineers actually do (Anderson et al. 2010; Cunningham et al. 2005; Tilli and Trevelyan 2008). In section 2.6 low enrolments in engineering education have been attributed to "a negative image of and inadequate information about, careers arising from the study of science and engineering" (Roberts 2002) and also to misconceptions, mystification and misunderstandings about what engineers do (Capobianco et al. 2011; Knight and Cunningham 2004; Oware et al. 2007a; Oware et al. 2007b; Prieto et al. 2009).

The view of one engineer, that sometimes it is better not to present oneself as an engineer as he himself works for "a chief executive of a company who has decided that his direct reports and his very senior management are all going to be from marketing and accounting disciplines" and who has also decided that "engineers are not good at that thing," is reinforced in the research literature. For example, in section 2.6, it is noted that the percentage of Siemens' managing board members who are engineers and scientists reduced from 64% to 25% in the period 2001 to 2010 (Becker 2010). "Today's engineers no longer hold the leadership positions in business and government that were once claimed by their predecessors in the 19th and 20th century and students "sense the eroding status and security of engineering careers and increasingly opt for other more lucrative and secure professions such as business, law and medicine" (Duderstadt 2008).

As noted in section 2.4, social cognitive theory assigns greater confidence in career choice with greater knowledge of occupation specialities and with a greater match between one's image of a career and one's self-identity (Lent et al. 2002). However studies show that young people's perceptions of engineers' work is that of fixing, building and that engineers are generally male (Capobianco et al. 2011; Oware et al. 2007b). Heywood is of the view that raising the status of design and technology in schools is difficult when students perceive engineering jobs as "unglamorous" (Heywood, 2005). Similarly, in section 2.6.2, Duderstadt attributes the poor image of engineering to the evolution of the profession from a trade and the way that "industry all too frequently tends to view engineers as consumable commodities, discarding them when their skills become obsolete or replaceable by cheaper engineering services from abroad (Duderstadt, 2008). There is also a view in literature that students' image of the engineering profession comes from their parents, family relations and school career advisor (Prieto et al. 2009). The need to impart greater knowledge about the engineering profession to students is further highlighted in section 2.4 where it is asserted that huge changes have occurred within engineering fields in the past thirty years (Heywood 2005) and in section 2.6 with the view that the role of the engineer has become quite broad (Williams, 2003, Lohmann et al., 2006, Chatterjee, 2005). For example, in section 2.6.1, it is noted that practice of an engineer who is a "disengaged problem solver" is outmoded (Sheppard et al. 2009) and that modern engineering practice is based on "distributed expertise" where engineering is a combined performance involving a range of people such as clients, suppliers, manufacturers, financiers and operators and as such a large proportion of engineers' time is spent on social interactions (Trevelyan 2010a). Heywood (2005) suggests that teachers need to acquire the knowledge that will better prepare and excite students about engineering careers (Heywood, 2005).

7.2.3.4-3 F3.3: Higher level Leaving Certificate mathematics is currently valued as a points earner and not as a stepping stone to engineering careers

Engineers say that while mathematics was once a stepping stone to engineering, mathematics now has a greater value to Leaving Certificate students as a points earner. Current students maximise their CAO points usage by opting for higher points courses rather than considering other career choice factors. Students who get "A1 in maths" are likely to score high points overall and they are unlikely to opt for an engineering course "that is only 350 points". This is supported by an analysis in section 2.4 illustrating that of the 8,420 students, who achieved the mathematics standard required for entry into level 8 engineering courses in Ireland in 2009, only 1,200 of these students chose places in such engineering and technology courses despite a strong demand by employers for engineers at the time (Devitt and Goold, 2010).

One engineer suggests that by including so much mathematics in the engineering subjects, universities are making engineering "elitist". Another engineer is of the view that engineering education "has been dumbed down seriously through the intervention of the institute of technology route" and she asks why would students, who have higher level Leaving Certificate mathematics do engineering when they could get "the same level 8 degree without higher level maths"? This engineer's own reason for choosing engineering was not related to her feelings about mathematics; instead she wanted to build bridges. While she expresses a valid opinion, her view also reinforces the perception of "elitism" in mathematics education, discussed in section 2.2.3, suggesting that only a "clever core" of students are capable of learning advanced mathematics (Brown et al. 2008; Ernest 2009; Hodgen et al. 2010; Matthews and Pepper 2007; Nardi and Steward 2003). In section 2.2.3 there is a discussion about the narrowness by which mathematics success is judged and the visibility of the "hierarchy" of mathematics grades ranging from students at the top of the class to the others who "sink to the bottom" (Boaler 2006). It is interpreted that the declining interest in engineering careers is compounded by "elitism" at the top of the mathematics hierarchy and also by a perceived inability to do mathematics at the bottom of the hierarchy. This reinforces the importance of task value of mathematics (why should I do mathematics?) which is queried in section 2.2.3 by Skemp: "why should anyone want to learn mathematics?" (Skemp 1987) and by Ernest: "what is the purpose of teaching and learning mathematics"(Ernest 2004b). However at the same time one of the biggest challenges facing engineering educators is students' lack of mathematics proficiency where drop-in mathematics clinics are now standard in many universities (Buechler 2004; Croft and Grove 2006; Fuller 2002; Gleason et al. 2010; Henderson and Broadbridge 2007; Henderson and Broadbridge 2008; Irish Academy of Engineering 2004; King 2008; Masouros and Alpay 2010; Reed 2003). It is noted, in section 2.4, that the mathematical ability of students entering engineering is a concern for both direct entry to engineering degree programs (level 8) and for students progressing to engineering via level 6 technician courses (Heywood 2005). These concerns reinforce the significance of investigating if there is a relationship between students' experiences with school mathematics and their choice of engineering as a career and also the significance of investigating the role of mathematics in engineering practice, both of which are the main aims in this study.

7.2.4 Theme 4: Engineering Practice, Roles and Activities

The findings outlining the engineers' views about their work and about engineering practice generally are presented in this section. Theme 4 is presented as follows:

Page number

7.2.4.1 Engineers' work

7.2.4.1-1 Engineers' work is diverse

A profile of engineers' job descriptions is presented in Table 7-2, Appendix 9, Volume 2. The twenty engineers interviewed in this study comprise a variety of engineering roles, disciplines and work. The interview participants work in a broad range of organisations that produce a variety of engineering products and services. The products produced by these organisations include: major engineering projects such as construction of pharmaceutical plants; electricity generation and distribution; gas distribution; telecommunications; pharmaceutical drug substances; hip and knee replacements; consumer electronics; light rail transport system; local authority services (e.g. water, sewerage, street lighting); information technology; software; and education and research. The engineers' work is also diverse in that it includes; process engineering; sales; engineering management; project management; people management; design; risk analysis; pricing; lecturing; research; consultancy and quality engineering. While ten of the engineers describe themselves as managers or project managers, many of these engineers also have technical roles. With the exception of B, F and G whose roles are mostly commercial in an engineering environment and P who is retired, all other engineers' work has a significant technical engineering component. Of the twenty engineers interviewed, A is the only contract engineer. He specialises in pharmaceutical process engineering. Two engineers J and S are involved in education and research. The work of the six youngest engineers E, H, K, L, Q, and T has a significant technical component.

Name	Gender	Mathematics Usage	LC Mathematics	LC Year	Company Sector	Product	Engineering Discipline	Engineering Role	Job Description
A	M	1.28	H	1990	Pharmace-utical	Pharmace-utical drug substances	Chemical	Design / Development	Process Engineer - making the process equipment do what it is supposed to do
B	M	1.52	H	1984	Telecomm-unications	Telecomm-unications	Electronic / Electrical	Technology Service Sales Manager	Sales manager - management of the commercial side of the public sector telecommunications contract
C	M	1.76	O	1985	Project Engineering	Engineering design projects	Mechanical	Design / Development	Department manager - management of team of mechanical engineers who develop capital projects for clients, also lead engineer on many projects
D	M	1.88	H	1966	Project Engineering	Engineering design projects	Mechanical	Project Management	Project manager - management of mechanical engineering side of pharmaceutical design projects
E	F	2.04	H	1997	Project Engineering	Engineering consultancy	Civil	Design / Development	Senior design engineer - analysis of water collection and distribution systems. Writing flood study reports and designing flood study measures
F	M	2.08	H	1985	Energy distribution	Gas supply	Mechanical	Project Management	Project manager - managing cost benefit analysis and risk analysis in the commercial department
G	M	2.09	H	1994	Electricity distribution	Electricity transmiss-ion	Electronic / Electrical	Commercial	Commercial manager - management of pricing for the wholesale electricity market in Ireland
H	F	2.33	H	1997	Project Engineering	Rail transport system	Civil, Rail, Water	Design / Development	Projects manager - design, tender, implementation and construction of projects on the rail line
J	M	2.67	A-L	1971	University	Education and bio-medical materials	Biomedical	Education, Research	Lecturer and researcher - lecturing “bio mechanics” to engineering students and research into bio-

									medical materials
K	M	2.68	H	1995	IT consultancy	Informat-ion technology	Electronic / Electrical	Information Technology Consultancy	Information technology consultant - determining the most economically advantageous tender for public sector contracts
L	M	2.90	H	1997	Project Engineering	Engineering consultancy	Electronic / Electrical	Design / Development	Project manager and electrical designer - managing and electrical design of major engineering projects e.g. Terminal 2 Dublin Airport
M	M	2.91	H	1991	Consumer electronics	Consumer electronics	Manufact-uring / Production	Design / Development	Programme manager - development and acquisitions of tooling and equipment for high volume manufacturing
N	M	3.34	O	1981	Local authority	Maintena-nce of city drainage network	Civil	Maintenance	Executive engineer - management of team who maintain the city drainage network and deal with any problems that occur
O	M	3.51	H	1979	Software	Internatio-nal version of Office software for iPhone and iPad	Software	Design / Development	Software senior test lead - management of people and projects with responsibility for software localisation
P	M	3.53	H	1963	Retired	Electrical/ electronics	Electronic / Electrical	General Management	Retired - career included engineering, marketing and general management with a variety of mainly US companies
Q	F	3.54	H	2003	Medical Devices	Hip and knee replacem-ents	Medical Devices	Design / Development	Quality engineer - process development and design and quality of products
R	F	3.60	H	1980	Local authority	Local authority services	Civil	Design / Development	Senior area manager - responsibility for unfinished housing estates
S	M	3.84	H	1980	University	Education	Electronic / Electrical	Education	Educator, university lecturer and researcher
T	F	4.17	H	2002	Electricity	Electricity transmiss-ion and distribution	Electronic / Electrical	Design / Development	Sub-station designer - design of power transmission and distribution stations around the country and also abroad
U	M	4.23	H	1984	Telecommu nications	Telecommu nications transmiss-ion network	Electronic / Electrical	Design / Development	Head of synchronise digital hierarchy (SHD) design - management of team of engineers who design the telecommunications transmission network in Ireland and who also manage the capacity in the network

Table 7-2: Profile of engineers' work.

7.2.4.2 Engineers' views about engineering practice

In this section the engineers' views about engineering practice are presented. A summary of engineers' views about engineering practice is included in Table A9, Appendix 9, Volume 2.

7.2.4.2-1 Engineering is much more than mathematics

The majority of engineers interviewed state that despite the perception that engineering is about mathematics, engineering involves much more than mathematics (A, D, E, F, H, J, L, N, O, P, Q, S, T, and U). This is supported by the following: "engineering is so much more than maths", "there is not a huge amount of maths involved a lot of the time" and the mathematics used "varies from job to job" (A); while "there was a lot of maths required to be an engineer" the "whole thrust is to reduce the figuring out to be done mathematically down to the minimum" in engineering practice (D); engineering practice is more about "the practicalities of engineering" than about higher level Leaving Certificate mathematics (E); "engineering is not pure science and pure maths" (F); engineers "don't sit in front of the computer and do maths all day" (H); in a "typical engineering company" only "a few people" do "maths at quite a high level", there are "people below them who need to understand and interpret what they are doing and then others who just need to know the big picture" (J); engineering is not about "writing down equations and working things out" (L); only a "minority of engineers require a very high standard of maths" (N); "in engineering maths is just a tool" (O); while "engineering is primarily applied mathematics in one shape or another", the "importance of one's *curriculum maths* will reduce and the importance of mathematical *thinking* will increase" over the lifetime of an engineer's career (P); there are two types of engineers, mathematical engineers who "understand processes" and "tool box" engineers who understand machines (Q); engineers tend to end up working in non-traditional engineering roles where they are not using maths on a regular basis" (S); engineering is "more about getting the basics right and building from there than an extremely high level of maths" (T); and "there are engineers in so many different functions across this company and I know well that their mathematics use varies very widely" (U).

7.2.4.2-2 Engineering is very broad

The majority of engineers interviewed are of the view that engineering roles are so broad that engineers can easily transfer from one role to another within an organisation (A, B, C, D, H, K, L, M, O, R, S, T, and U). This is supported by: "engineering is very broad" and "because the majority of engineers don't work in particularly specialised or specifically technical roles" many engineers "could fill quite a number of roles within organisations" (A); B's engineering career followed a path that has "variety, variety of environment, variety of context and variety of people"; "engineering is so broad" that for example a "mechanical engineer could safely migrate into a number of different discipline engineering functions" (C); there are "so many disciplines and aspects" to engineering (H); there are many branches of engineering and each branch is different and uses mathematics differently (L); some engineers "use mathematics to analyse data", some engineers "might use anecdotal evidence" and "in management teams some engineers are more logical and they are more likely to use some maths in some of the decision making" (M); in the context of mathematics "engineering disciplines aren't that specific" (O); "there is tremendous diversity in what engineers wind up doing" and "engineers, in many cases, despite their particular qualification their responsibilities tend to be a lot broader than what one might expect" (P); when R was a resident engineer working on a pumping station for water and sewerage treatment plants, there were civil, geotechnical, mechanical, electrical and structural engineering aspects to the project; "engineering is a very broad discipline and engineers even tend to end up working in non-traditional engineering roles where they are not using maths on a regular basis" (S); "engineering is such a varied profession" in that it ranges from research and development to project management and many of engineers in Ireland work in the "social side" of engineering doing "project management and problem solving, which are not directly related to maths" (T); and "there are engineers in so many different functions across this company and I know well that their mathematics use varies very widely … the knowledge that they have gained in one area, is nearly always useful in another area … of the ten engineers working for me, one has a B.Sc., the other nine have engineering degrees and of those I have three mechanical engineers, one civil and the rest are either, electrical or electronic and if you went out there, there is no way you could tell me which is which" (U).

The engineers are clearly of the view that engineering is very broad. They state that engineering roles are diverse and range from highly technical roles to the more "social side" of engineering such as project management roles. There is also the view that engineers' mathematics usage varies widely in engineering practice. Engineering practice has huge variety of work and there is a view that much of engineering work is multidisciplinary. The engineers' views about the transferability of engineers from one area to another within engineering practice confirms that engineering roles are not "particularly specialised or specifically technical".

7.2.4.2-3 Engineering is problem solving

Many engineers are of the view that engineering is problem solving (A, C, G, O, P, R, and T). This is supported by: engineering is "pretty much problem solving" (A); engineering is "taking a solution and refining it" (C); in engineering "maths is just a tool" and "a general problem solving methodology" (O); due to the project nature of engineering "to a large extent engineers are managers" where "problem solving and logical thinking are essential" (P); engineering is like "Lego" in that "you are just using everything you have and sticking them together to solve problems" and when "managing an area" for the local authorities R felt like "a social worker" because she was "sorting out everybody's problems" (R); the majority of engineers in Ireland spend ninety per cent of their working day doing "project management and problem solving, which are not directly related to maths"(T).

7.2.4.2-4 Engineering is a mindset/ bigger picture thinking/ decision making

There is a view that engineering is “bigger picture” thinking (C, D, H, J, M, N, and P). This includes: “a mind-set of how you go about things” (C); engineering is “the bigger picture” (D); engineering “is much more bigger picture thinking” and “engineers are expected to be rational and logical and to come up with the correct solution” (H); in a “typical engineering company” only “a few people” do “maths at quite a high level”, there are “people below them who need to understand and interpret what they are doing and then others who just need to know the big picture” (J); “in management teams some engineers are more logical and they are more likely to use some maths in some of the decision making” (M); engineers “see the overall picture; we are not just looking at one small aspect” and engineering “is a way of thinking”(N); it is important for engineers “to be able to analyse the available information and to form a view on how complete or incomplete that information is” and having “a feel for where the risks lie and can inform your approach to decision making” (P).

7.2.4.2-5 Engineering is using computation tools, it is reusing solutions and it is analysing data

Some engineers say that in engineering practice there is a tendency to reuse existing solutions and information rather than develop new solutions from first principles (F, G, K, M, O, and P). Examples of this include: engineers “wouldn’t expend resources developing solutions from first principles unless the solutions we have today aren’t working” (F); “the guys developing the algorithms from first principles are rare” (G); in modern engineering companies where “complexity equals time equals money” there is a “strong focus and a modern focus” on “reuse” and “design once, use many rather than design many use many” (K); engineers “use mathematics to analyse data” (M); and with “so much data everywhere” a knowledge of statistics is always useful in engineering practice (P).

7.2.4.2-6 Engineering is about practicality/ real world applications

There is a view that engineering is about real world applications and in many cases there is a greater need for "practicality" than for higher level mathematics in engineering practice (C, E, F, N, Q, and S). This is supported by: engineering practice is more about "the practicalities of engineering" than about higher level Leaving Certificate mathematics (E); "engineering is not pure science and pure maths, you really are using maths and science and applying them to the real world" (F); there are "two different types of engineering: there's the high maths person and there's the practical engineer and the majority of engineers do "the practical application of day to day stuff" (N); engineering takes place in the "the real world" and that "when you put a prototype into production and see how it is actually used in the real world, that's where the real engineering starts" (S).

7.2.4.2-7 Engineering is connection and integration of components

There is a view that engineering is more about interconnecting existing technology than developing new technology (G, H, K, O, and T). For example, this includes: engineers "integrate others' work" rather than develop a unit of technology from first principles" and while the integration of blocks of "suppliers work" adds significant value to the individual pieces of technology, "engineers don't really know what is under the bonnet" of an individual piece (G); engineers require an "understanding of the effect of one piece of work on another part of the system"(H); the focus on "connection and integration of components" lacks "understanding of how they [individual components] work"; much of engineering is about "making bigger blocks" and "marrying things together" and "the meeting point is often most likely where things go wrong" and where engineers are required (O); and T's industry doesn't "actually design the individual items of equipment it is more like tying them together".

7.2.4.2-8 Engineering is project management more than mathematics

There is some view that much of engineering practice is project management (L, T, O, and P). This includes: engineers are “more on the project management side of things” (L); majority of engineers in Ireland work spend ninety per cent of their working day doing “project management and problem solving, which are not directly related to maths” (L).

7.2.4.2-9 Engineering is communicating the solution

Communications is an important activity in engineering practice. For example, S asserts that engineers’ role is “to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate it to the decision maker”. U notes that “engineers who come into us from outside companies as salesmen, their job is to stand up in front of the likes of myself and tell me their story and why their equipment is so good and they often need to understand maths to do that”.

7.2.4.3 Use of resources in engineering practice

7.2.4.3-1 Computer solutions are part of engineering practice

In this section the engineers’ views about the use of resources in engineering practice are presented. The interview data clearly shows that computer solutions are widely used in modern engineering practice (A, B, C, D, E, F, G, H, J, K, L, M, N, O, Q, R, and U). Engineers maintain that computational tools have many advantages in engineering practice in that the tools bypass the need to write down the fundamental engineering equations and solve them and they offer a standard methodology for developing solutions within organisations. Most engineers say they use Excel. J has a view that using computational tools is “a different type of mathematics”. He says “the engineer should understand how the program is solving the equations and what it is doing, because it is always dangerous not to". Similarly H, R and U note that results produced by computational tools can easily be misinterpreted.

7.2.4.4 Discussion of theme 4

There are two findings associated with theme 4, these are:

F4.1 Engineers' work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions.

F4.2 Computer solutions are part of engineering practice.

7.2.4.4-1 F4.1: Engineers' work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions

The engineers interviewed in this study work in a variety of roles and disciplines and their organisations produce a variety of products. Many of the engineers' roles are a mix of technical and management. A majority of engineers view their work as "so much more than maths" and they say that there are tiers of mathematics requirements in engineering practice. While only a "minority of engineers require a very high standard of maths" other people "need to understand and interpret what they are doing and then others who just need to know the big picture".

There is a view that engineers don't work in particularly specialised or specifically technical roles and that engineers are easily transferrable from one role to another within an organisation. There is also a view that because engineering work is multidisciplinary that engineering graduates lose their discipline identity. One engineer says "there is tremendous diversity in what engineers wind up doing" and that "engineers, in many cases, despite their particular qualification, their responsibilities tend to be a lot broader than what one might expect". Another engineer maintains that "engineering is so broad" that a "mechanical engineer could safely migrate into a number of different discipline engineering functions". The breadth of engineering is also highlighted in section 2.6.1 where engineering is seen to encompass "physics, chemistry, biology, mathematics, psychology and more" (Chatterjee 2005) and include "engineering managers, entrepreneurs, financial analysts, salespeople, educators and a variety of other positions (Panitz 1998).

The engineers' view that engineering is about problem solving is consistent with research literature in section 2.6.1 and with the view that engineering is "at its core, problem solving" (Sheppard et al. 2006) and engineering is "the application of the theory and principles of science and mathematics to research and develop economical solutions to technical problems … the link between perceived social needs and commercial applications" (U.S. Department of Labor, 2007). According to the engineers a significant part of engineering is reusing existing solutions and interconnecting existing technology which is similar to a definition of engineering as "the process of integrating knowledge … connecting pieces of knowledge and technology to synthesize new products" in section 2.6.1 (Bordogna, 1992). There is a similar view that "modern engineers design products, processes and systems" that are sometimes state-of-the-art technology but engineering is mostly "applying and adapting existing technology to meet society's changing needs" (Crawley et al., 2007). Engineers' view that cost is a major factor in engineering solutions is supported by the view in section 2.6.1 that engineering is creativity constrained by cost, safety and other factors (Wulf and Fisher 2002).

Engineers say their work is "bigger picture thinking". Bigger picture thinking is logical thinking about the complete project. A study in section 2.6.1 found that a lack of understanding of the "big picture" in which a problem was grounded contributed to new engineers' uncertainty in their understanding of their work and to the value of their work in the organisation. One problem was multiple and conflicting goals and multiple solutions (Korte et al. 2008). Winkelman (2009), in section 2.5, contrasts the "open-endness" of design processes, where there are a multiplicity of possible solutions for a given problem, with undergraduate engineering mathematics where "a single correct answer is generally assumed" (Winkelman, 2009). One engineer in this study says that engineering takes place in the "the real world" and "you never know what the user is going to come back with … it could be something really simple that requires absolutely no maths". In section 2.5 it reported that engineering students "struggle between mathematical representation and the real-world manifestation of the concept" (Sheppard et al. 2009).

According to the engineers, data analysis is important in engineering. This view is supported by King (2008) in section 2.5 who reports that that modelling, data analysis, statistics and risk assessment are necessary for engineering practice in Australia (King 2008). It is noted in section 2.6.1 that workplace problems often lack data and are more complex and ambiguous with far more variables compared to school problems (Korte et al., 2008). One engineer believes that it is important for engineers "to be able to analyse the available information and to form a view on how complete or incomplete that information is" and that having "a feel for where the risks lie and can inform your approach to decision making". This is similar to the view expressed in in section 2.5 where engineers are required to be increasingly critical in "discerning information and making decisive judgments when confronting unexpected situations and novel problems (Radzi et al., 2009).

One engineer presents that an engineer's role is "to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate it to the decision maker". It is similarly maintained in section 2.6.1 that modern engineers work in teams and that engineers exchange "thoughts, ideas, data and drawings, elements and devices" with other engineers around the world (Crawley et al., 2007). However a study, in section 2.6.1, found that many graduates are unable to release the strength of their mathematics because they do not know how to communicate mathematics in the workplace. Furthermore no graduate believed they had studied mathematics communication at university (Wood 2010).

The engineers' views about engineering are quite similar to that of Dym, Agogino, Eris, Frey and Leifer (2005) in section 2.5 who say that system design and systems thinking skills include: thinking about system dynamics (anticipation of "unintended consequences emerging from interactions among multiple parts of a system"); reasoning about uncertainty (dealing with "incomplete information" and "ambiguous objectives" and application of probability and statistics); making estimates (one challenge of design is that as the number of variables and interactions grows, the system stretches beyond the designers' capability to grasp all of the details simultaneously and good system designers are usually good at estimation); and conducting experiments (design requires use of empirical data and experimentation) (Dym et al. 2005).

Many engineers engage in the "social side" of engineering where they spend ninety per cent of their working day doing "project management and problem solving" tasks. According to the research literature in section 2.6.1 modern engineering practice is based on "distributed expertise" involving clients, suppliers, manufacturers, financiers and operators and social interactions are at the core of engineering with a reliance on "harnessing the knowledge, expertise and skills carried by many people, much of it implicit and unwritten knowledge". Engineering practice relies on applied engineering science, tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience) and an ability to achieve practical results through other people (Trevelyan 2010a; Trevelyan 2010b). A study of new engineers in section 2.6.1 found that "learning from co-workers was the primary method of learning on the job" (Korte et al. 2008).

This insight into engineering work is important given that there is an inadequate body of work on engineering practice and there are misconceptions as to what engineers actually do (Anderson et al. 2010; Cunningham et al. 2005; Tilli and Trevelyan 2008). There is a view, in section 2.6, that engineers have done a poor job defining who they are and that engineers who design are called scientists, engineers who develop new products are called entrepreneurs, engineers who program computers are called IT professionals and engineers who work in industry are called managers (Chatterjee 2005). Also, in section 2.4, studies of young people's perceptions of engineers generally show that engineers' work is viewed as fixing, building, making or working with vehicles, engines, buildings and tools and engineers are generally male. Such misconceptions and stereotypes about engineering make it more difficult to attract students to engineering (Capobianco et al. 2011; Knight and Cunningham 2004; Oware et al. 2007a; Oware et al. 2007b; Prieto et al. 2009). Building a deep understanding of engineering practice into the curriculum has the potential to greatly strengthen engineering education (Trevelyan 2010a).

Engineers say that there are tiers of mathematics requirements in engineering practice which range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths". Data analysis is required to inform many engineering decisions. Also "bigger picture thinking" skills are a requirement in engineering practice. The is an important message from practising engineers as they present their views to questions such as the one posed by Ifiok Otung in section 2.4 who questions the "wisdom of scaring away potentially successful engineers with a mathematical content that is rarely used during the career of 98% of practitioners" (Otung 2002). There is also a message for teachers who engineers believe fail to communicate the value of mathematics and who also fail to demonstrate real world applications to students. "Real world" practicality is required in engineering practice and engineers are of the view that teachers should "emphasis more the applications of maths … say that this is why we are doing it, the place of maths in the world and make that part of the taught and examined subject".

7.2.4.4-2 F4.2: Computer solutions are part of engineering practice

Engineers say that computational tools are "a different type of mathematics" usage that offers speedy and standard solutions when interpreted correctly and they are widely used in engineering practice. The increasing availability of computerised tools and resources, as discussed in section 2.6.1, is contributing to the changing nature of engineering where IT tools are dominating modern engineering practice (Anderson et al. 2010). In section 2.6.1 it is observed that "the engineer today has at his or her disposal a vast array of modern problem-solving tools and methodologies, which can be applied without detailed knowledge of the underlying techniques" (Grimson, 2002). The view that much of the mathematics required in engineering practice is done by software and the challenge for engineers is to correctly interpret computer solutions is reinforced in the literature review. For example, in section 2.5 there is importance given to understanding the mathematics and scientific fundamentals behind the software tools and techniques engineers use and the "ability to validate quantitative outcomes of simulations" (King, 2008). However a study investigating mathematics graduates transition to the workforce noted that their undergraduate education did not teach them how to use standard computer products such as Excel, Visual Basic or SAS. The graduates found that they had to change their ideas of how mathematics is used in the real world particularly where assumptions are relaxed (Wood 2010).

7.2.5 Theme 5: Career Development Paths in Engineering Practice

In this section the engineers' views about career development paths in engineering practice are presented. Theme 5 is organised as follows:

Page number

7.2.5.1 Graduate engineers are not ready to engineer

There is a view that graduate engineers are not ready to engineer and that they tend to look for mathematical solutions rather than engineering solutions (C, E, F, G, H, L, M, N, P, R, S, T, and U) Examples include: as a a graduate engineer, the focus was on "the product" rather than on the "total solution" (G); "as a graduate you are just trying to get your head around what it is that's going on, not to mention make a decision on it" (H); "at the early stages of one's career one to a very significant extent is regurgitating what one learned in college" (P); "early stage engineers try to formulate every problem mathematically" and they tend "to shy away from problems they can't formulate mathematically" (S); graduate engineers "for their first two or three years are not really going to be given any major problems to solve" and after that "initialisation" period engineers become more "frontline" and are required "to make very important decisions" (T); and in U's company "the younger engineers are brought in, they are shown a particular area or a type of technology and they are given in effect a problem to solve," then that process of "solving individual problems for individual sites is repeated" and as the engineers "gain experience they need to start to look at the bigger picture" (U).

7.2.5.2 Majority of engineers become managers

The majority of engineers move into management roles (A, B, C, D, F, G, J, K, M, N, O, P, R, S, and U). The data supporting this view includes: there are few positions in Ireland that require a senior technical person and engineers who are not "specialised" move into management roles (A); engineers get "side-tracked away from the design authority type job and they get persuaded for various reasons, not least for financial reward and compensation into project managers, programme managers, commercial managers and contract managers". Engineers succeed in these jobs because "of the discipline and the rigour and structure that they gained in their education path" (B); C has "gone further away from the discipline of engineering and into a more managerial position"; as engineers' careers develop, they move away "from actual engineering" into "supervision and management" roles where they focus on issues such as money, time and client relationships" (D); "the natural career progression" for F is a "move into lower middle management and into commercial roles" where engineers "get away from the pure engineering design, number crunching part of engineering to a management role in an engineering company". "Career progression is monetary" (F); when G was a graduate engineer his focus was on "the product" but since he has progressed onto a management role the focus is now on "the human side of the problems" and on "who" will solve a particular problem; the business side of M's organisation prefers engineers "to become programme managers" rather than do "the design tasks" and that consequently many of their graduate engineers are "going into the data driven type roles" (M); N describes his role as progressing towards management and taking charge of people; O "became a manager for the wrong reasons" because he "thought money was everything"; "engineers to a very large extent are influenced to move into management by the necessity to obtain financial reward" (P); "as you go along in your career" it is "more managing people and getting other people to think and getting other people to develop the solutions" (R); and some engineers in U's company "want to become technical experts in their area and there are others that want to go the management route and become more senior managers".

It is the fifteen oldest engineers who have strong views on engineers' career paths leading to management careers. The youngest of this group of fifteen engineers is K who is currently making the transition from engineering to project management. The five youngest engineers do not have any views on their careers moving towards management.

7.2.5.3 *Curriculum* mathematics usage declines as engineers' careers progress

There are mixed views about engineers' *curriculum mathematics* usage over the course of engineering careers. One view is that *curriculum mathematics* usage declines as engineering careers progress (B, C, D, F, L, N, O, and P). B is of the view that when engineers move into management "their reliance on maths degrades very rapidly" (B). C says he gets "the graduates" to do any "maths" that needs doing in his work "because they are closer to college"; "the higher up" engineers move in their careers, "the less mathematics they need" (D); "crunching numbers would be seen as something you do the first couple of years you are out of college" and engineers who "graduate up through the management chain" don't use "maths on a daily basis" instead they manage people who use mathematics (F); when N came out of college, he did "a lot of high level maths" because he was "fresh maths wise" and as "you progress towards management and take charge of people, your level of maths decreases; "there are degrees of involvement with mathematics as you progress through the range of activities in which an engineer may be involved" over the lifetime of an engineering career and "the longer you're out of college, the less likely that you're going to be working directly with mathematics … at the early stages of one's career one to a very significant extent is regurgitating what one learned in college … the higher one rises in responsibility, the less hands on engineering one needs to do and the more general responsibility one has … the relevance and importance of mathematics in one's everyday activities declines" (P). P worked as an engineer for six years, he then "moved into marketing" and subsequently into "general management" of an engineering company. In general management, P says his usage of mathematics was "as an analytical tool to inform a decision making process". R is of the view that as you go along in your career, it's more managing people and getting other people to think and getting other people to develop the solutions". R believes that Chartered Engineers are mostly managers who understand mathematics and who "are actually using more numbers than younger engineers" as they are managing budgets. She says "if you got a younger engineer you probably would get the functions higher and the numbers might go down a bit". S maintains that "as engineers grow older and wiser they realise that the bigger and more important problems are more multidimensional than just the little mathematical dimension". He says that "many engineers end up in management where they wouldn't necessarily be using maths regularly but they might have to talk to people who are using maths".

7.2.5.4 Discussion of theme 5

There are two findings associated with theme 5, these are:

F5.1 Graduate engineers are not ready to engineer.

F5.2 Majority of engineers become managers.

7.2.5.4-1 F5.1: Graduate engineers are not ready to engineer

The majority of engineers maintain that graduate engineers are not ready to engineer. There is a view that "at the early stages of one's career one to a very significant extent is regurgitating what one learned in college" and "early stage engineers try to formulate every problem mathematically" and they tend "to shy away from problems they can't formulate mathematically". One young engineer says that "as a graduate you are just trying to get your head around what it is that's going on, not to mention make a decision on it". In one company "the younger engineers are brought in, they are shown a particular area or a type of technology and they are given in effect a problem to solve", that process of "solving individual problems for individual sites is repeated" and as the engineers "gain experience they need to start to look at the bigger picture".

Research literature supports the finding that graduate engineers are not ready to engineer. For example, in section 2.5, there is a view that “many of the engineering students who make it to graduation enter the workforce ill-equipped for the complex interactions, across many disciplines, of real-world engineered systems” (Wulf and Fisher, 2002). One engineer in this study maintains that graduate engineers require “two or three years” of an “initialisation” period after which they are required “to make very important decisions”. A similar view, reported in section 2.6.1, is that it takes up to three years for a novice engineer to become reasonably productive in a commercial context. It is maintained that the diversity of engineering career settings and the complexity of engineering environments make it difficult for engineering educators to prepare students for the workplace (Trevelyan, 2011). One engineer who has a senior role in a large telecommunications company says that, in his company, “the younger engineers are brought in, they are shown a particular area or a type of technology and they are being given in effect a problem to solve”. That process of “solving individual problems for individual sites is repeated” and as the engineers “gain experience they need to start to look at the bigger picture”. Adjusting to the workforce can be problematic for many engineering graduates as they discover what they learned at university needs to be contextualised for work (Wood, 2010). A study of new engineers, in section 2.6.1, found that “workplace problems often lacked data and were more complex and ambiguous with far more variables” compared to school problems. A challenge for many new engineers was the accuracy of their methods which often depended on other people’s judgement rather than as derived from data. The new engineers presented that their work involved “a large amount of social interaction and social influence”. They had to learn the constraints of the social system within their work groups and the new engineers “relied on their co-workers and managers to learn the subjective aspects of their work”. The engineers say that “learning from co-workers was the primary method of learning on the job” (Korte et al., 2008). It is asserted, in section 2.5, that there is no relation between early stages of curriculum and career and that engineering education lacks professional skills development (Duderstadt 2008). The requirement for additional emphasis on project activities, summer training and closer links between engineering industry and academic institutions is noted in section 2.5 (Baytiyeh and Naja 2010).

However according to Trevelyan (2011) in section 2.6.1, the "scarcity of systematic research on engineering practice" makes it difficult for educators who wish to design learning experiences to enable students to manage the transition into commercial engineering contexts more easily (Trevelyan, 2011).

7.2.5.4-2 F5.2: Majority of engineers become managers

A majority of engineers believe that engineers ultimately become managers. One engineer says "as you go along in your career" it is "more managing people and getting other people to think and getting other people to develop the solutions". There is a strong view that "engineers to a very large extent are influenced to move into management by the necessity to obtain financial reward" and "in most cases promotion tends to increase the level of administrative responsibility and decrease the level of technical responsibility". This view is supported by the team nature of engineering practice discussed in section 2.6 (Crawley et al., 2007) and with the view that engineering involves diverse and multidisciplinary teams and a combined performance involving a range of people such as clients, suppliers, manufacturers, financiers and operators and as such a large proportion of engineers' time is spent on social interactions (Trevelyan, 2010a). Research literature, in section 2.6.1, notes the importance of the coordinated efforts of a group of people in engineering practice where the most significant constraints on engineers' work are organisational business practices relating to time and budgets (Anderson et al., 2010). A study of engineers who had been practising for no more than ten years revealed the strong need for integrating "managerial, leadership, teamwork, creativity and innovation skills, as well as knowledge of business policies in classroom activities" (Baytiyeh and Naja 2010).

There are mixed views amongst the engineers whether *curriculum mathematics* usage changes as one's engineering career progresses. One engineer believes that "the higher one rises in responsibility, the less hands on engineering one needs to do and the more general responsibility one has … the relevance and importance of mathematics in one's everyday activities declines". Another engineer is of the view that "as you go along in your career it needs less mathematics, it's more managing people and getting other people to think and getting other people to develop the solutions”. She believes that Chartered Engineers are mostly managers who understand mathematics and who “are actually using more numbers than younger engineers” as they are managing budgets. Another engineer says that “as engineers grow older and wiser they realise that the bigger and more important problems are more multidimensional than just the little mathematical dimension” He says that “many engineers end up in management where they wouldn’t necessarily be using maths regularly but they might have to talk to people who are using maths”. While much of the research into engineers’ mathematics usage investigates engineering students rather than experienced engineers, one study of civil and structural engineers, in section 2.7.1, found that younger engineers do most of the analysis, especially computer-based analysis while older engineers do the broader design tasks. One engineer in that study was of the view that as engineers grow up, while they may no longer be using the mathematics they started out using they are still using the understanding that they derived earlier in their experience (Kent and Noss 2003).

7.2.6 Theme 6: Engineering Practice, *Curriculum Mathematics* Usage

The findings outlining the engineers' views on their *curriculum mathematics* usage in engineering practice are presented in this section. Theme 6 is presented as follows:

Page number

7.2.6.1 *Curriculum mathematics* has a diversity of uses in engineering practice

The engineers' *curriculum mathematics* usage, as measured in the statistical survey, increases from engineer A up to engineer U and is illustrated in Table 7-1. A summary of engineers' *curriculum mathematics* usage is presented in Table 7-3, Appendix 9, Volume 2.

Fourteen of the twenty engineers interviewed say they use some higher level Leaving Certificate mathematics (F, H, and P) or some engineering level mathematics (D, G, J, K, L, N, Q, R, S, T, and U) in their work. *Statistics and probability* is the most popular domain with sixteen of the twenty engineers using *statistics and probability* in their work (A, C, D, E, F, G, H, J, K, L, M, O, P, Q, R, and U). Ten engineers use *statistics and probability* at higher level Leaving Certificate or engineering levels. Thirteen engineers use *algebra* in their work (G, J, K, L, M, N, O, P, R, S, T, and U). Eleven engineers use *geometry and trigonometry* (C, E, H, J, L, N, Q, R, S, T, and U), ten engineers use *number* (A, B, L, M, O, P, Q, R, S, U) and eight engineers use *functions* (J, K, L, P, R, S, T, and U). Seventeen engineers rate their *curriculum mathematics* usage in their work as either type 2 (*connecting*) or type 3 (*mathematising*). Only three engineers (A, H, and L) of the twenty engineers rate their highest *curriculum mathematics* usage type as type 1 (*reproducing*). Of the three types of *curriculum mathematics* usage, type 3 (*mathematising*) is the highest usage type for nine of the twenty engineers (J, K, O, P, Q, R, S, T, and U).

The interview data presents a diversity of ways engineers use *curriculum mathematics* in their work. For A, “mathematics is not a major element” of his work. Any higher level mathematics in C’s company is “done by consultants”. D is of the view that “very few engineers work in areas where they are challenged mathematically”. E would “prefer to use maths more” in her work. F is of the view that practising engineers “rarely go back to the first principles” and *statistics and probability* is “all over” his work, for example the future capacity of a particular gas field is determined from an existing statistical “production profile” of the gas source. H does “very little actual maths calculations”; her job is “more about interpreting stuff” and being “able to understand data”. She describes a project concerning “noise monitoring” on one of the rail lines where the “consultants” produced “quite technical” reports and she relied on her knowledge of statistics to develop the “criteria for success or failure” in relation to rail noise levels. She also uses “basic *geometry and trigonometry* to work out site levels”. Any *algebra* she requires is done using Microsoft Excel and calculus “is a vague and distant memory”. While *curriculum mathematics* “is essential” to J’s biomedical research work he also admits that there are “great chunks of the subject” which he has “never needed to use”. He describes his usage of mathematics when teaching students about the properties of materials as “a few differential or integral equations now and again and a bit of *algebra*”. In his research work, J needs to express his ideas in “mathematical form in order to make predictions and to compare them with experimental data”. He says he uses *statistics and probability* “all of the time” because he is “dealing with experimental data and trying to understand it”. J describes his usage type as *mathematising* or as he puts it “formulating the problem, then solving it in some particular case and then relating that back to the real world comparing to maybe experimental data or things like observations”. J says that he goes “for the messy inexact solution most of the time” because a lot of his research work is “dealing with results and data which are very scattered” and any new theory about how the human body works is likely to be “a very approximate theory”. He says that “precision is very important in the sense that you need to know how precise your solution is, you need to know how accurate your data is and of course a lot of statistics analysis would tell you that”. L is of the view that he will never use the level of mathematics he took in college. As a programme manager, M has to "look at data, make decisions and give directions". There is "a lot of implicit *number* work … a very basic bit of *algebra* … some *statistics and probability*" in O's work. He describes his usage as "transferring things into a mathematical problem". O previously worked in "search engine development", where he says the work was "statistical in nature as the algorithms are designed to try and figure out ultimately what users mean when they type in something to search for". He says that a lot of these developers "would have a PhD in statistics".

The top six *curriculum mathematics* users note the importance of mathematics in their work. P is of the view that an appreciation of *statistics and probability* was necessary at all levels in manufacturing companies where quality control engineers used "a statistical approach to analyse the data" and managers needed to "understand the solutions other people were implementing". It is only at work that Q is seeing the application of the mathematics she studied in college. She says "*statistics and probability* and *number*, I do loads of that". R says "I am the only one in my fifty two people staff that can actually do something from first principles". She says that because she "did higher level Leaving Certificate mathematics", unlike her colleagues, she does not need a consultant to do her job. When R was an "area engineer" she says her "maths usage was between a little and a little bit above depending on the jobs". She says that there were eight to ten years of her engineering career where her mathematics usage was very little and because her "brain was so underutilised due to the repetitive nature of the surface stressing programme, the hedge cutting notices, the town councils, listening to them and the queries and the parish pump politics", she "needed something more". In R's current job she says her "maths usage is totally jumped up" … "*functions* would be used every single day … *geometry and trigonometry* in land valuing … *statistics and probability* in traffic management, traffic statistics, accident statistics and that kind of thing … *algebra* is needed quite a lot for design purposes, flows and streams, designing storm-water pipes that sort of thing … *numbers* are used in managing budgets". In his work S uses "*functions, algebra, numbers, geometry and trigonometry* but not so much *statistics and probability*". In U's job where "there are so many different layers in telecommunications networks, so many different paths … there are about 1,400 exchanges in the country and there is a connection from every one, to in effect every other one but not by direct line … there are about fifteen different telecommunications networks" in Ireland, he has to calculate "how big a pipe" is required for a particular telecommunications route where the units for "the different parts of the pipe are not always the same". He says "I have to convert between the different units, depending on which network I am doing … turning things like man hours into megabits per second". U says that when "calculating basic figures and basic numbers" in work, he has "to be able to do that and often do that at speed". U illustrates the importance of mathematics in telecommunications by explaining that calculating the number of bits in a byte and the number of bytes in a telecommunications pipe is got by "working out 2^n continuously". He says that "in large pipes, the number of bits per second that go down them become so ferociously large that megabytes, gigabytes and terabytes are not large enough and you get to numbers that you cannot print, and if you did nobody would understand". U says "*statistics and probability, geometry, trigonometry, number, algebra, functions* are all equally important" in his work. He adds "I can honestly say that in the last month I've used all of those in some way … for example, yesterday we had to find out why synchronisation wasn't working across the country and it turns out that the distance involved was longer that the recommended distance because the route being taken was longer than the perceived route".

Name	Mathematics Usage	Domain	Type	Level
A	1.28	"Numeracy", "Statistics"	Reproducing	"Leaving Cert ordinary level"
B	1.52	"Numbers"	Reproducing and Connecting	"Leaving Cert ordinary level"
C	1.76	"Statistical analysis" and "geometry"	Reproducing and Connecting	"Leaving Cert ordinary level"
D	1.88	"Statistics and probability"	"Never got beyond connecting"	Engineering level
E	2.04	"Geometry, trigonometry, statistics and probability"	Reproducing and Connecting	"No higher than Leaving Certificate ordinary level"
F	2.08	"Statistics and probability"	"somewhere between connecting and reproducing"	Leaving Certificate higher level
G	2.09	"Matrix algebra" and "statistics"	Reproducing and Connecting	Engineering level
H	2.33	"Statistics" and probability and "basic geometry and trigonometry"	Reproducing	Leaving Certificate higher level
J	2.67	"Statistics", "geometry, algebra and functions"	Mathematising	"Either A-level or something I learned during my degree"
K	2.68	"Statistics and probability and some algebra and functions"	Mathematising	"Between higher level Leaving Cert and engineering level"
L	2.90	"Statistics" "Algebra" "Functions"	Reproducing	"Statistics at either Leaving Cert or engineering level, algebra at leaving cert level and functions at leaving certificate level
M	2.91	"Numbers, statistics and probability and probably algebra"	Connecting	Leaving Certificate ordinary
N	3.34	"Geometry, trigonometry and algebra"	Connecting	"Some of this is at engineering level"
O	3.51	"a lot of implicit number work … a very basic bit of algebra … some statistics"	"Somewhere between type 2 connecting usage and type 3 mathematising usage"	Leaving Certificate ordinary
P	3.53	"Algebra, functions, numbers, statistics and probability"	"Early on in my career the usage would have been type 3, mathematising"	"Minimum level of higher level Leaving Certificate"
Q	3.54	"Statistics and probability and numbers". "Geometry and trigonometry"	"Statistics and probability and numbers, I do loads of that and that's up in mathematising". "Geometry and trigonometry … at least connecting"	Engineering level
R	3.60	"Functions would be used every single day … geometry and trigonometry … statistics and probability … algebra … numbers"	"Usage type varies from reproducing to connecting and to mathematising"	"Usage of statistics and probability and geometry and trigonometry is at engineering level, algebra and numbers is at higher level Leaving Cert and functions is at Junior Cert level.
S	3.84	"Functions, algebra, numbers, geometry and trigonometry but not so much statistics and probability"	"Getting the students to reproduce the mathematics and make connections"… In research work, the usage type would be connecting and "trying to express problems in maths or formulate then into maths with a view to solving them"	"Higher level Leaving Certificate and above"
T	4.17	"Algebra, geometry … a lot of calculus … and very little statistics"" in her work.	"Reproducing and connecting … some mathematising but not at a very high academic level"	Engineering level
U	4.23	"Statistics, geometry, trigonometry, numbers, algebra, functions are all equally important … I can honestly say that in the last month I've used all of those in some way"	"Reproducing is the major function … there is a good part of connecting … mathematising is a lot more rare and it's more about solving problems that our field crews cannot solve"	"Leaving Certificate and engineering level maths".

Table 7-3: Engineers' *curriculum mathematics* usage.

7.2.6.2 Discussion of theme 6

There are two findings associated with theme 6, these are:

F6.1 Engineers use a high level of *curriculum mathematics* in their work.

F6.2 *Statistics and probability* are important in engineering practice.

7.2.6.2-1 F6.1: Engineers use a high level of *curriculum mathematics* in their work

Fourteen of the twenty engineers interviewed use aspects of either higher level Leaving Certificate mathematics or engineering level mathematics in their work. The twenty engineers' *curriculum mathematics* usages, as determined in the survey analysis, range from a score of 1.28 (A) up to a score of 4.23 (U) based on a total score of 5 for usage of all five domains, all five academic levels and all three usage types. The maximum *curriculum mathematics* usage score of 5 represents a significantly large volume of mathematics and while many engineers use considerably less than the maximum in their work, their *curriculum mathematics* usage is significant. Engineers confirm that they use aspects of higher level Leaving Certificate and engineering level mathematics in their work and much of the usage is at the higher usage type (*mathematising*).

The majority of engineers' mathematics usage is at either *connecting* or *mathematising* and it is notable that only three of the twenty engineers rate their *curriculum mathematics* usage no higher than type 1 (*reproducing*), which is usage of mathematics through knowledge of facts and concepts. Type 2 (*connecting*) and type 3 (*mathematising*) mathematics usage is using mathematics at the level of problem solving, which is a significant part of engineering practice. The dominance of type 2 and type 3 *curriculum mathematics* usages in engineering practice is an important finding in the context of mathematics teaching whereby nineteen of the twenty engineers interviewed are of the view that mathematics learning requires understanding, not information retention (F1.1). Engineers say that unlike other school subjects where learning is about "information retention" and "regurgitation", mathematics learning is a "process" of problem solving and/ or application of mathematics and "understanding" is an essential part of learning. Engineers further emphasise the importance of understanding in mathematics learning in their views about their good mathematics teachers who "connected with people through maths", who "pitched maths at our level", and who "made sure that we understood something before moving on to the next topic" (F1.2)

Of the twenty engineers interviewed, A, who rates lowest of the twenty engineers in his use of *curriculum mathematics* in work, is the only engineer whose mathematics usage does not exceed both Leaving Certificate ordinary level and *reproducing* type. However A is of the view that engineers in general use just ten per cent of the mathematics learnt in university and the difficulty for engineering education is "figuring out which ten per cent for each individual". Similarly while another engineer says that *curriculum mathematics* "is essential" to his research work, he also admits that there are "great chunks of the subject" which he has "never needed to use". Similarly other engineers estimate that "ten per cent of the engineers on site here would need some of the learning from higher level maths" and mathematics is "valuable" in the ten per cent of their work. From the sample of engineers interviewed, it is not possible, with a sample size of twenty engineers, to determine if specific mathematics domains are used more by specific engineering disciplines and engineering roles, as shown in Table A9-12, Appendix 9, Volume2.

For the top six *curriculum mathematics* users, mathematics is essential in their work and it is used in a diversity of ways. For example, one engineer says "functions would be used every single day … geometry and trigonometry in land valuing … statistics and probability in traffic management, traffic statistics, accident statistics and that kind of thing … algebra is needed quite a lot for design purposes, flows and streams, designing storm-water pipes that sort of thing … numbers are used in managing budgets". In another engineer's job where "there are so many different layers in telecommunications networks, so many different paths … there are about 1,400 exchanges in the country and there is a connection from every one, to in effect every other one but not by direct line … there about fifteen different telecommunications networks" in Ireland, he has to calculate "how big a pipe" is required for a particular telecommunications route where the units for "the different parts of the pipe are not always the same". He says "I have to convert between the different units, depending on which network I am doing … turning things like man hours into megabits per second".

In this study engineers present the need for and ways they use *curriculum mathematics* in their work. This knowledge is important given that there is an inadequate body of work on engineering practice (Anderson et al., 2010, Tilli and Trevelyan, 2008, Cunningham et al., 2005) and students and teachers generally lack an understanding of what engineers do (Courter and Anderson, 2009, National Academy of Engineering, 2008). Despite a belief among some practising engineers that the mathematics they learned in college is not applicable to their daily work (Cardella, 2007), the interview data illustrates that both higher level Leaving Certificate mathematics and engineering level mathematics are required in many engineers' work and that much of engineers' mathematics usage is either *connecting* or *mathematising*. It is similarly observed in this study that while the majority of engineers are of the view that engineering practice is much more than mathematics they themselves are users of high level mathematics where their usage is connecting and mathematising. This is shown in Table A9-13, Appendix 9, Volume 2.

7.2.6.2-1 F6.2: *Statistics and probability* are important in engineering practice

From the interview data, there is evidence that *statistics and probability* are important in engineering practice. Sixteen of the twenty engineers use *statistics and probability* in their work and some use it at a very high level. For one engineer *statistics and probability* is "all over" his work. Another engineer says he has to "look at data, make decisions and give directions" to his team of engineers. Engineers say that an ability to understand data is required in engineering practice. For example, one engineer's mathematics usage is "more about interpreting stuff" and being "able to understand data" than doing "calculations". Another engineer says that he uses statistics "all of the time" because he is "dealing with experimental data and trying to understand it". The importance of *statistics and probability* in engineering is also noted in the research literature, in section 2.5, where it is stated that with the current advancement in knowledge and technology engineers are required to be increasingly critical in “discerning information and making decisive judgments when confronting unexpected situations and novel problems” (Radzi et al., 2009). In section 2.5 the Australian Learning and Teaching centre found that data analysis, statistics and risk assessment are deemed necessary for engineering practice (King 2008).

There is a view that an appreciation of *statistics and probability* is necessary at all levels in manufacturing companies where quality control engineers use “a statistical approach to analyse the data” and managers need to “understand the solutions other people were implementing”. One engineer, whose work involves the design and development of medical devices, says that “statistics in particular is very specific” to her industry. She says that “some of the statistical analysis” she uses in her work is “more heavy weighted in the higher end of engineering and in theoretical maths than a graduate coming out of college would grasp”. Another engineer, who works in a local authority (city council), uses *statistics and probability* in “traffic management, traffic statistics, accident statistics and that kind of thing”.

While it is noted that estimation is important in engineering practice, it is also observed that some engineers do not consider estimation of engineering solutions to be mathematics. For example, one engineer says his job does not require higher level mathematics however he also says that “having a feel for an answer or solution is more useful” than having an answer “correct to eight decimal places”.

Another engineer “had to go back and study statistics” because his mathematics teacher omitted the statistics option from his Leaving Certificate teaching and because he needed statistics for his job. It is also claimed, in section 2.5, that engineering graduates are not good at estimation and that engineering curricula underemphasise the application of probability and statistics, (Dym et al. 2005). In section 2.711 Cardella and Atman also found that engineering students struggled to deal with uncertainty (Cardella and Atman 2005). Furthermore in a study of the early work experiences of recent engineering graduates it was found that interpreting data was a new experience for many engineers (Korte et al. 2008).

7.2.7 Theme 7: Engineering Practice, Mathematics *Thinking* Usage

The findings outlining the engineers' views on mathematics *thinking* usage in engineering practice are presented in this section. Mathematics *thinking* usage is usage of mathematical modes of *thinking* learned and practised through mathematics, e.g. methods of analysis and reasoning, logical rigour, problem solving strategies (e.g. problem decomposition and solution re-integration), recognition of patterns, use of analogy, and a sense of what the solution to a problem might be. Theme 7 is presented as follows:

Page number

7.2.7.1 Elements of mathematics *thinking* usage required in engineering practice

In engineering practice mathematics *thinking* usage comprises of: problem solving; big picture thinking; decision making; logical thinking; estimation and confirmation of solution.

7.2.7.1-1 Problem solving

For a majority of engineers interviewed, problem solving is a major part of the engineers' mathematics *thinking* usage (A, B, D, E, F, G, H, J, K, L, M, N, O, Q, R, S, T, and U). This is supported by the following: *thinking* involves "solving a problem" and solving "complex problems faster" (A); *thinking* usage is leading a team to "a solution that will address customer requirements" (B); *thinking* "comes down to problem solving" (D); "*thinking* usage … mathematical ways of problem solving … typical engineering approach is to deconstruct problems into a series of small problems and to connect the "bite size" solutions together to form the overall solution" (F); "analysis phase of problem solving" is part of *thinking* usage (G); "abstracting a problem" (J); "looking at large complex problems … looking at how to decompose them, how to restructure them, how to make it simpler to attack those problems is my *thinking* usage" (K); "analysing problems and selecting the path forward" (M); "every problem would need to be solved logically"(N); "like a maths problem in your Leaving Cert" where "if you can't figure it out then you work around it and you get your brain going in different ways" (O); "breaking everything down and building it back up again just like maths" (R); "stepping back from a problem and discovering if there is another way of going at this problem" (S); "you might start with a number of possible solutions or a number of possible problems or a number of possible reasons for the problem and then you move from there to the likely solution based on your experience of different problems and on cause and effect" (T); "*thinking* and problem solving" and "how many problems can I solve with a particular budget" (U).

7.2.7.1-2 Big picture thinking

Many engineers define their mathematics *thinking* usage as "big picture" *thinking* (A, B, D, F, G, H, P, R, S, and U). Big picture *thinking* is the term the engineers use to describe the "overall concept of a situation". It is about defining a problem or identifying a question that meets the "objective" which is usually determined by "customer requirements". Big picture *thinking* is also about "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer. Big picture *thinking* is taking the "the real world" into consideration. Engineers describe this as follows: "figuring out what the questions should be and what the answer means" (A); "formulate an overall concept of a situation or of a problem" and "lead a team towards a solution that will address customer requirements" (B); "sight of the objective" whereby "if engineers' effort towards the objective increased by ten per cent, company profits would double" (D); "in the real world", there are "four or five different answers" and there is a need to think about "which is the best answer for all the participants … in the gas industry, you have to always err on the side of safety" (F); "pattern recognition … "problem definition … getting a feel for the solution … looking at the human side of problems" (G); "be very aware of the big picture … have a real tangible understanding of the effect of one piece of work on another part of the system", the rail network is an "integrated system" and one cannot look at one part "in isolation", instead the engineer has to look at the "knock on effect" of that part and "logic it out to see if there is a risk" to the entire system (H); "estimate the risks of not meeting a guaranteed performance level" and decide if "the additional cost of a piece of equipment is justified by the reduced performance risk" (P); "horse trading … bargaining … you have to give something, get something … logically figure out what is the optimum … without being too smart and losing the lot" (R); "an ability to think laterally … it's a bit like fresh eyes, or a fresh perspective … thinking outside the box … engineering should be about trying to identify the right question, because a lot of the times, people are obsessing over the wrong question" (S); and apply *thinking* "not just to engineering, but also to finance, to manpower and to people" (T).

7.2.7.1-3 Balance of judgement/ decision making/ structuring an argument

A majority of engineers say that "decision making" is part of their mathematics thinking usage (B, C, E, H, J, M, P, Q, R, T, and U). Decision making is about structuring an argument, balance of judgement and weighing up "the pros and cons". Examples of *thinking* usage include: "decision making … "being balanced" (B); weighing up "the pros and cons … decision making" (C); "logical thinking" helps E to "make a decision"; "decision making" (H) and (J); in M's work, where he might have to decide what vendor gets "a million dollar business" contract, in order "to make the best decision for the business" M would score the different factors of each vendor's quotation and develop "mathematical templates with weighted models" to come up with a "numerical reason" why he chose a particular vendor; "over time" P's "work was primarily about decision making" whereby he would "take whatever the available information was, try to represent it mathematically" and when "confronted with a selection of options" he would have to choose between them. "In the business of delivering a turnkey engineering solution for a fixed price", P might have to "estimate the risks of not meeting a guaranteed performance level” and decide if “the additional cost of a piece of equipment is justified by the reduced performance risk”; Q’s thinking usage involves “looking at something and gathering the information and … decision making … structuring an argument”; R’s *thinking* usage includes decision making whereby she has to “logically ... figure out what is the optimum” she can get from property developers “without being too smart and losing the lot”; T’s decision making resembles “direct maths” in that she always verifies her decisions in work and when looking at risk factors she sees the “risk as the massive divider under the line” and the benefit is the “numerator on top of the line”. She says that her *thinking* is based on “balancing things” just like “maths equations”; and U says that he tries to get all his team “involved in the decision making”.

7.2.7.1-4 Logical thinking/ critical analysis/ reasoning

Many engineers describe their mathematics *thinking* usage as “logical” *thinking*, “critical analysis” or “reasoning” (D, E, F, J, K, L, M, N, O, R, T, and U). D says that in the course of his career his “whole way of analysing things, reasoning and organising got better as time went on”. E describes her *thinking* usage as “a logical process” and she says that “logical *thinking*” helps her to “make a decision”. F says that, in his job, he is required to think in “a logical way”. “Logical *thinking*”, “critical analysis”, “reasoning” and “common sense” are all part of H’s *thinking* usage. J includes “logical *thinking*” and “critical analysis” as part of his *thinking* usage. K is of the view that engineers in general are “very logical” in their work. L describes his *thinking* as “being more logical about things and you apply that in your work”. M asserts the reason he is a programme manager is because he approaches problems “very logically”. N says that in his work “every problem would need to be solved logically” and “by solving maths problems … your brain gets triggered in these logical deductions”. O says that in his work he has to “figure out things in a logical way” and “reason out problems”. R has to “logically ... figure out what is the optimum” she can get from developers. She says “it is not straight black and white … it is a logical analysis”. T describes her *thinking* usage as a “reasoning” process and “a logical way of *thinking*”, she says “you might start with a number of possible solutions or a number of possible problems or a number of possible reasons for the problem and then you move from there to the likely solution based on your experience of different problems". U says one of his team of engineers does "critical analysis practically the whole time".

7.2.7.1-5 Estimation/ feelings for a solution/ coming up with a reasonably good answer quickly

For many engineers estimation is an important part of mathematics *thinking* usage and also an important part of their work (A, B, C, E, F, G, H, O, Q, R, and U). They say that estimation is having "a feel" for the solution and coming "up with a reasonably good answer quickly".

While A says his job does not require higher level mathematics, he is of the view that "having a feel for an answer or solution is more useful" than having an answer "correct to eight decimal places". B says that "the engineer through his education journey is able to bring that real world practical approximation process into play both for speed of response plus a commercial pragmatism". He is of the view that "so much of the value an engineer brings to his job and brings to society is to be able to do a reasonableness test to conceive a solution and within a good level of probability to be able to say yeah, that will meet the need, but then not being afraid to modify that and evolve that in subsequent observations or in practice". C suggests that *thinking* could be "refining your estimate". D is "much more confident" in his work about "having the principles right and conclusions right from a good understanding of the problem with some checking by maths rather than doing a big long calculation, coming up with the answer and saying bang, there's the answer". For much of E's work "an estimate is probably good enough". In F's work, it is not practical to get revenue projections "one hundred per cent right", these might be "anywhere between eighty and one hundred per cent" correct or with "a bit more analysis" they might be "between ninety five and one hundred per cent" correct. G says that getting a "feel" for the solution is part of his *thinking* usage and that as he gets further on in his career, "there is a lot more judgement" and he is "less inclined to do things from first principles". H asserts that in her work she can "look at the figures very quickly and make decisions”. O notes that in his work “estimating things is so powerful because you can come up with a reasonably good answer to something very quickly”. Q says her *thinking* usage involves “how do I get something done in the quickest way”. R says that when working with local authorities “you have a feel for what’s going on … if you asked me what would hold up this roof, I would probably give you the right size, the right specification … I will give you the answer now”. U says “there is a certain amount of estimation” in his work.

7.2.7.1-6 Confirmation of solution/ discipline/ rigour

A majority of engineers say that confirmation of solution, discipline and rigour are part of their mathematics *thinking* usage (A, B, E, F, J, K, L, M, N, Q, R, and U). If A was not “comfortable” with the answer when solving a problem, he would check it. B notes that “decision making” requires a discipline of “not being forced into an early conclusion”. He says he “has the confidence to actually check the answer to make sure that it is within tolerance”. E says she uses “maths just to check that some program” is working. F uses “rigour” when analysing risks. J “would always try to get the solution two different ways” this might be “just adding the columns of figures from top to bottom instead of bottom to top”. K would “double check on everything”. L maintains that “confirmation of solution” is important in his work. M’s work is “task oriented” and when “looking at schedules, analysing problems and selecting the path forward” he needs to be disciplined. In N’s work environment there is a “need to get things very right”. He is of the view that in both “engineering *thinking*” and mathematics “you have to be exact”. N says he has “to get it right at the end of the day” and that “almost there” or “down the right track” is not good enough. Q’s *thinking* usage includes “confirmation of solution”. R says that when working with local authorities “you have a feel for what’s going on … if you asked me what would hold up this roof, I would probably give you the right size, the right specification … I will give you the answer now and I will go back two days later and I will just put it on paper with proper calculations”. U says that “checking and double checking” is part of his *thinking* usage.

7.2.7.2 Mathematics education contributes to *thinking* skills

The majority of engineers say that their mathematics education contributed to the development of their *thinking* skills (A, B, E, F, H, J, K, L, M, N, O, P, R, S, and T). A is of the view that the association between engineering and mathematics is "indirect". He states that "it's not necessarily the formal learning of the maths, it's not the bits that you actually go and use, it's how you use them" that develops "mathematical *thinking*". B is of the view that "the engineer through his education journey is able to bring that real world practical approximation process into play both for speed of response plus a commercial pragmatism, knowing he has the confidence to actually check the answer to make sure that it is within tolerance but that he is able to get on and be operational and be responsive on a day to day basis". E says that, as a result of her mathematics training, she is "organised" when she takes a logical approach to "problem solving". F believes that one "never really moves away from" the "logical mind-set" developed "from having done science and maths subjects" in school. H says what "the grounding in maths helps you do, is to look at the figures very quickly and make decisions". Even if J has "forgotten ninety per cent" of what he was taught in school and college, his mathematics education has given him "an approach to a problem which is different from somebody who doesn't have maths training". K suggests that mathematics education, where "you had to do things in a particular order, in a particular sequence and you had to explain each step … teaches logical *thinking* and teaches that everything must follow a particular sequence". He says he approaches his work the same way he "took on a maths question" in school in that he had "to have everything right and accurate" and he would "double check on everything". L is of the view that a mathematics education gives students the ability "to think like an engineer". He says "the sort of person who does honours maths ends up *thinking* and acting in a certain way" and he or she develops the ability "to think like an engineer". M maintains that "the discipline" he got from higher level Leaving Certificate mathematics "is a big advantage to how I approach my work on a day to day basis". He is of the view that the discipline of "organising your study and the time it took to do your honours Leaving Certificate maths" is "something you bring through college and into to your working life". N is of the view that what he takes "from maths, it's not the actual maths that you do; it's the logical format that you go through". He says that mathematics problems gave him "the mentality to think". O says his *thinking* at work is like a "maths problem in your Leaving Cert" in that "if you can't figure it out then you work around it and you know you get your brain going in different ways". He says that the "practice of doing that … transfers into other things that you do". P presents that engineers throughout their careers have "varying degrees of involvement with mathematics". He says that in "the early stages of one's career, one to a very significant extent is regurgitating what one learned in college and that as one progresses through one's career one tends to use mathematics as an analytical tool to inform a decision making process". R says she likes "the idea of breaking everything down and building it back up again just like maths". S believes that "mathematical training is good for your brain and probably enables you to tackle new problems". He believes that while Chartered Engineers "mightn't be using maths every day … they are reaping the benefit of having had the training". T describes her *thinking* as "indirect maths", this she says is her "way of working" and it comes "from having done maths". She says "when you do maths, you develop a logical way of *thinking* and you approach every problem the way you would approach a maths problem". She says that her *thinking* is based on "balancing things" just like "maths equations".

7.2.7.3 Engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage

All engineers rate their *thinking* usage very highly and greater than their use of *curriculum mathematics*. A rates his "*thinking* usage" as "quite a lot" and he also rates his *thinking* usage higher than his *curriculum mathematics* usage (very little). He believes that his *thinking* usage is increasing as he is "still in a technical role". B rates his "*thinking* usage in his work" in the range "quite a lot" to "a very great deal" and considerably greater than his *curriculum mathematics* usage in his current work. He says his *thinking* usage is "still increasing" otherwise he would lose his "value". C maintains that "*thinking* is everything" in his work and he rates his *thinking* usage as "quite a lot" to "a very great deal". C's *thinking* usage is considerably greater than his *curriculum mathematics* usage. C says his "*thinking* usage" is "increasing all the time" and "the higher up you're going in an organisation, you're *thinking* of permutations all the time, what if, what if, for layout, for personnel, for ….". D rates his "*thinking* usage" in work as between "quite a lot" and "a very great deal" and "much higher" than his *curriculum mathematics* usage. He says it got "steadily better" in the course of his career and that his "whole way of analysing things, reasoning and organising got better as time went on. E rates her *thinking* usage as "considerably more" than her use of *curriculum mathematics*. F rates his *thinking* usage in the range as "quite a lot" and considerably greater than his use of *curriculum mathematics*. G rates his *thinking* usage as "between a little and quite a lot" and also greater than his *curriculum mathematics* usage. H rates her "*thinking*" usage in work as "a very great deal" compared to her *curriculum mathematics* which she rates as "a little". She says when she started working as an engineer she had "no common sense" but her *thinking* usage is increasing with engineering experience. J rates his *thinking* usage as "a very great deal" compared to his *curriculum mathematics* usage which he rates as "quite a lot". K rates his *thinking* usage as "a very great deal" and much greater than his *curriculum mathematics* usage. L rates his *thinking* usage as "quite a lot" and significantly greater than his use of *curriculum mathematics* in his work. M rates his *thinking* usage in work as "quite a lot" compared to his *curriculum mathematics* usage which he rates at just "a little". He says that earlier in his career his *curriculum mathematics* usage was higher and this *thinking* usage was lower. N rates his *thinking* usage in the range "quite a lot" to "a very great deal" and greater than his *curriculum mathematics* usage and increasing. O rates his *thinking* usage as "quite a lot" and "a lot greater" than his direct usage of *curriculum mathematics*. P rates his *thinking* usage in his work as "a very great deal". He presents that engineers throughout their careers have "varying degrees of involvement with mathematics". He says that in "the early stages of one's career, one to a very significant extent is regurgitating what one learned in college and that as one progresses through one's career one tends to use mathematics as an analytical tool to inform a decision making process". In his drawing, P presents his views how his *curriculum mathematics* and *thinking* usages varied with time spent in an engineering career, Figure 7-1. He says that "experience replaces mathematics as an important element in an engineer's capability". P believes that "there are a lot of people out there who have ten years' experience and there are a lot of other people out there who have one year's experience ten times over" and so "some people will arrive at the intersect point on the chart more quickly than others".

Figure 7-1: Representation of one engineer's *curriculum mathematics* and *thinking* usage.

Q rates her *thinking* usage as "a very great deal" and ahead of her *curriculum mathematics* usage which she rates as "quite a lot". Q says modes of *thinking* are in "everything" she does in her job. When she first joined the company she had to study advanced statistics and while her *curriculum mathematics* usage increased "the rate of increase of the modes of *thinking* is actually bigger" in her current job. Q is of the view that she has become "an independent thinker rather than checking what you are meant to do with other people". R rates her current *thinking* usage as "a very great deal" and greater than her *curriculum mathematics* usage. While S rates both his *curriculum mathematics* usage and his *thinking* usage as "a very great deal", he says that *thinking* usage is "where it's all at … to me this is absolutely critical". He says that while *curriculum mathematics* is "very useful for elements of problems particularly in engineering, it is not necessarily the full solution". He adds that *curriculum mathematics* "is probably useless in identifying what problem or what question you should be asking" and that "you are going to be confronted with problems which are just bigger and more abstract than you have the maths for". T rates her *thinking* usage as between "quite a lot" and "a very great deal". She says her *thinking* usage and her *curriculum mathematics* usage are both high and each has increased over her engineering career which is just five years. She is of the view that her *thinking* usage is currently slightly ahead of her *curriculum mathematics* usage because she was recently assigned "more responsibility" at work. U rates his *thinking* usage a "quite a lot" because he has "to apply the maths not just to engineering, but also to finance, to manpower and to people". In the case of budgets he says "budgets can be spread across so many different functions and areas that you have to get all of the figures and understand what they are and what they mean". For U whose *curriculum mathematics* is highest of all the engineers interviewed, he says his *thinking* usage is "probably higher than his *curriculum mathematics* usage because he is "doing a lot more management orientated as opposed to problem solving oriented or design orientated tasks …. dealing with a financial document as opposed as to trying to solve a problem".

7.2.7.4 Discussion of theme 7

There are three findings associated with theme 7, these are:

F7.1 Engineers' mathematics *thinking* usage is problem solving, big picture thinking, decision making, logical thinking, estimation and confirmation of solution.

F7.2 Mathematics education contributes to engineers' *thinking* skills development.

F7.3 Engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage.

7.2.7.4-1 F7.1: Engineers' mathematics *thinking* usage is problem solving, big picture thinking, decision making, logical thinking, estimation and confirmation of solution

Engineers' mathematics *thinking* usage comprises of: problem solving; big picture *thinking*; decision making; logical *thinking*; estimation and confirmation of solution. For a majority of engineers interviewed, problem solving is a major part of their mathematics *thinking* usage. Engineers say that engineering problems have many answers and that their job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer. The typical engineering approach to problem solving is to "deconstruct" problems into "a series of small problems" and to connect the "bite size" solutions together to form the overall solution. Engineers say that big picture thinking is taking the "the real world" into consideration where engineers need to "have a real tangible understanding of the effect of one piece of work on another part of the system" and "engineering should be about trying to identify the right question, because a lot of the times, people are obsessing over the wrong question".

Decision making and logical *thinking* are important in engineers' work. Engineers say that decision making is about structuring an argument, balance of judgement, weighing up "the pros and cons" and "being balanced". Logical *thinking* includes the "whole way of analysing things", "organising", "critical analysis", "reasoning" and "common sense". Engineers, in this study, present that that "speed of response" is important in engineering practice and that mathematics education contributes to an engineer's ability to think quickly. For example, one engineer says that what "the grounding in maths helps you do, is to look at the figures very quickly and make decisions". Trevelyan (2010), in section 2.6.1, also maintains that engineering performance is time, information and resource constrained. Seldom is there complete information available and the available information has some level of uncertainty (Trevelyan, 2010a). While there is a view amongst the engineers that they need to do their work "in the quickest way" and "an estimate is probably good enough", engineers also say that "confirmation of solution, discipline and rigour" are important in their work.

The engineers' views about their mathematics *thinking* usage are similar to the findings of a study of new engineers, in section 2.6.1, whose engineering work is described as a "problem-solving process or way of *thinking*", where they tried to "organise, define, and understand a problem; gather, analyse, and interpret data; document and present the results; and project-manage the overall problem-solving process." The new engineers presented that "workplace problems often lacked data and were more complex and ambiguous with far more variables" compared to school problems. One problem for the new engineers was that workplace problems often had multiple and conflicting goals and multiple solutions. Another problem for the engineers was their "not knowing the "big picture" in which a problem was grounded". The engineers found that their lack of understanding of the big picture contributed to the uncertainty and ambiguity in their understanding of their work and to the value of their work in the organisation (Korte et al., 2008).

In section 2.7.1.1 Cardella and Atman found that engineering students thought about mathematics in terms of core knowledge rather than as a *thinking* process and they were unable to apply many mathematical skills they had learned (Cardella and Atman 2005). There is also a view in section 2.5 that students "generally find it difficult to relate math to real objects around them or to engineering practice" and they "struggle to make the connection between mathematical representation and the real-world manifestation of the concept" (Sheppard et al. 2009). In section 2.5 there is a view that applying mathematics to solve complex engineering problems is an essential but often missing skill for young engineers. It is advocated that mathematics should be taught in the context of engineering with a focus on: "the development of thinking and understanding; the development of engineering and mathematical language; the development of the confidence required to tackle large engineering projects and persist in finding solutions" (Janowski et al., 2008). There is some evidence in the research literature in section 2.2.2 that problem solving strategies can be taught (Pólya, 1945, Schoenfeld, 1992). According to Ernest (2011), problem solving also has a metacognitive aspect. Metacognitive activities include "planning, controlling and monitoring progress, decision making, choosing strategies, checking answers and outcomes and so on" (Ernest, 2011).

Mathematical *thinking* is described, in the research literature, in section 2.2, as a form of mathematics considered necessary in many workplaces. Schoenfeld (1992) maintains that there is a considerable difference between school mathematics and the way experts engage in mathematics. His five aspects of mathematical thinking are: the knowledge base; problem solving strategies; effective use of resources; mathematical beliefs and affects and engagement in mathematical practices. (Schoenfeld 1992). Similarly Ernest (2011) is of the view that problem solving in mathematics is the process of doing mathematics and it differs from "textbook" problems which are a reinforcement of knowledge. He identifies two forms of mathematical knowledge: explicit and tacit (personal know how) (Ernest, 2011). Trevelyan, in section 2.6, also discusses tacit knowledge and he is of the view that engineering practice relies on applied engineering science, tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience) and an ability to achieve practical results through other people (Trevelyan 2010a). He says that engineering "relies on harnessing the knowledge, expertise and skills carried by many people, much of it implicit and unwritten knowledge" (Trevelyan 2010b). Trevelyan is also of the view that building a deep understanding of engineering practice into the curriculum has the potential to greatly strengthen engineering education (Trevelyan, 2010a).

7.2.7.4-2 F7.2: Mathematics education contributes to engineers' *thinking* skills development

There is a strong view that mathematics education contributed to the engineers' *thinking* skills development. The engineers view the association between mathematics and *thinking* as "indirect" in that it is how engineers use mathematics rather than the actual mathematics they use. They say that when learning mathematics that: doing "things in a particular order … teaches logical *thinking*"; the practice of working around a problem and getting "your brain going in different ways … transfers into other things that you do"; the emphasis on getting the right answer teaches one to "double check on everything" and the discipline of "organising your study and the time it took to do your honours Leaving Certificate maths" is "something you bring through college and into to your working life". There is also a view that mathematics education contributes to an engineer's ability to think quickly. For example, one engineer says that "the engineer through his education journey is able to bring that real world practical approximation process into play both for speed of response plus a commercial pragmatism, knowing he has the confidence to actually check the answer to make sure that it is within tolerance but that he is able to get on and be operational and be responsive on a day to day basis". Another engineer also looks at speed of decision making and she says that what "the grounding in maths helps you do, is to look at the figures very quickly and make decisions". A further engineer calls this speed of response "a feel for what's going on" and she says "I will give you the answer now and I will go back two days later and I will just put it on paper with proper calculations".

Cardella and Atman (2005), in section 2.7.1, are also of the view that mathematics courses benefit engineering students by the material and the *thinking* processes and strategies learned. They say that students might not be aware of their use of mathematics but that "if engineering students believed that mathematics was more about a way of *thinking* than about particular content knowledge, they might value mathematics more, be more motivated to learn mathematics and might be more predisposed to apply mathematical thinking" (Cardella and Atman 2005).

7.2.7.4-3 F7.3: Engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage

A significant finding in this study is that all engineers rate their mathematics *thinking* usage higher than their *curriculum mathematics* usage in their work. For one engineer *thinking* usage is the "value" he brings to his job and another engineer says that thinking usage is "where it's all at … to me this is absolutely critical" and that while *curriculum mathematics* is "very useful for elements of problems particularly in engineering, it is not necessarily the full solution". There is a view that early in the engineers' careers *curriculum mathematics* usage is higher and *thinking* usage is lower and that *thinking* usage increases for technical, commercial and management roles over the course of engineering careers. One engineer is of the view that the "higher up you're going in an organisation" the more "permutations" there are to consider. The engineer, whose *curriculum mathematics* is highest of all the engineers interviewed, has a similar view. He says that his *thinking* usage is "probably higher than his *curriculum mathematics* usage because his role is management orientated and he has "to apply the maths not just to engineering, but also to finance, to manpower and to people". The engineer whose *curriculum mathematics* usage is second highest in the group of engineers interviewed and who has not yet reached her thirtieth birthday, says that while her *thinking* usage and her *curriculum mathematics* usage are both high, her *thinking* usage is currently slightly ahead of her curriculum mathematics usage. Engineers present that their mathematics *thinking* usage comprises of: problem solving; big picture thinking; decision making; logical thinking; estimation and confirmation of solution.

The finding that engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage supports the view in section 2.2 that mathematical *thinking* is a form of mathematics considered necessary in many workplaces. It is related to tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience) and differs from school mathematics (Schoenfeld, 1992, Ernest, 2011, Trevelyan, 2010a, Trevelyan, 2010b). This is further supported by the finding that graduate engineers are not ready to engineer (F5.1). According to one engineer, when graduate engineers enter engineering practice are "given in effect a problem to solve", the process of "solving individual problems for individual sites is repeated", as the engineers "gain experience they need to start to look at the bigger picture". According to a study, reported in section 2.6.1, new engineers rely "on their co-workers and managers to learn the subjective aspects of their work" (Korte et al., 2008). Engineers are required to be increasingly critical in "discerning information and making decisive judgments when confronting unexpected situations and novel problems" (Radzi et al., 2009). There is also support in the research literature, in section 2.5, to better incorporate mathematics-oriented critical *thinking* skills including analytic skills, problem-solving skills and design skills into engineering curricula (National Academy of Engineering, 2005).

7.2.8 Theme 8: Engineering Practice, Communicating Mathematics

The findings outlining the engineers' views on communicating mathematics in engineering practice are presented in this section.

Page number

7.2.8.1 Communicating mathematics is part of engineers' work

For the majority of engineers communicating mathematics is part of their work (B, C, D, E, F, G, H, K, L, M, N, P, R, S, T, and U). Much of B's work is about interpreting "financial reports and statistics reports". D is of the view that engineers, in the course of their work, need to write down the mathematics, they "need to come to conclusions and express these conclusions" and they need to "write reports". Writing "flood study reports" is part of E's work. H says she has to take her "logical thinking" and "try to explain" to her colleagues "how you have come to your conclusion and try and get them to understand your logic". She adds that while mathematics is "the science to your argument" one needs "to know enough to know when someone is pulling your leg". K regularly uses mathematics when "making an argument" at work and when determining the "most economically advantageous tender", he has to use mathematics to explain to clients how companies might be "over compensating for lack of functionality by tweaking their prices". K presents his "rock solid argument" in a report which has an "executive level" containing the "executive summaries" and the detail is in the appendix. In his work, M says there is "a logical approach to making arguments" and that he needs to have "backup for his decisions because there could be legal implications". He uses "mathematical templates" to establish "a numerical reason" for recommending a particular vendor and he has "to tell the unsuccessful vendors and the management team how he made this decision". N recalls an incident whereby “a block fell down and killed somebody in Washington Street[30]”. He says that the engineer he “called in to do an analysis of the incident” had “only a basic level of maths” but he was “very good at report writing”. N adds that if this engineer “needed high level calculations he knew who to contact in UCC[31] to get help with the maths”. P notes in his most recent work as a general manager of an engineering company, that he needed “a certain minimum understanding of *statistics and probability*” … to understand the solutions other people were implementing … in order to be able to understand the reports that they prepare”. R asserts that “there is a skill in communicating maths” and that during the Celtic tiger economy[32] “a lot of consultants would have sent us in the same traffic management plan for different projects and you would have to be able to calculate out and just make sure that it was actually the project they were talking about”. S asserts that “real world” engineers have to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker”. In her work T says that when “dealing with engineers” she finds that using mathematics “is the best way to make an argument”. “Documents” are central to U’s work and he has to put the mathematics in his job “into a form that a non-engineer will understand”. U also notes that “engineers who come into us from outside companies as salesmen, their job is to stand up in front of the likes of myself and tell me their story and why their equipment is so good and they often need to understand maths to do that”.

Many engineers note the importance of communicating mathematics well (F, G, H, K, M, R, S, and U). F states that he sees “maths popping up in the engineering world all the time”. He believes that “someone could tell you an awful lot of balderdash if you weren’t aware of what the mathematics meant” and that with an understanding of mathematics “you are less likely to be hoodwinked”. G says that “engineers lack the emotional intensity that they need to communicate to get a point across to people or to realise the impact of what they do on people’s lives”. He says that “others seize that opportunity and that is why engineers are so often in the background”. H is of

[30] Washington Street: Major street in Cork city.
[31] UCC: University College Cork.
[32] Celtic tiger economy: Period of rapid economic growth in Ireland between 1995 and 2007.

the view that people who do well in engineering are those that are good at "communicating and making arguments". She says that while mathematics is "the science to your argument" one needs "to know enough to know when someone is pulling your leg". K says that by communicating mathematics effectively he can present a "rock solid argument" in a report. M notes that by rolling out IT solutions worldwide "one guy has to do the maths at the start" and "everybody else gets the benefits of the analysis". R notes the importance of the "skill in communicating maths". She says that if one doesn't "bring the problem and the solution to people in their language" mathematics becomes "elitist". S is of the view that "if engineers are to survive then they need to somehow harness communication skills". He believes that "real world" engineers have to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker". S adds that without this full solution "the decision makers" might ignore the engineer" and instead "use their own intuition". S is of the view that communication between the engineer and the manager is "very important" and that "while the engineer is more enthusiastic about the mathematical detail, the manager is probably more wiser to the ways of the world and if they talk in the right language to each other, they are more likely solve the bigger, broader problem and come up with a better solution to it" than if they were to tackle the problem separately. U suggests that communication "can be very biased" and when people do not go "into the detail behind the headline" the message can "be used and indeed abused". He admits to having on at least one occasion abused the message! He also says that when communicating mathematics he has "a certain amount of licence to get away with things" because he can include something his "audience will not understand" and are "afraid to ask" and he "will get away with it" if there aren't "bright sparks" in his audience.

7.2.8.2 Compared to other professions engineers are not good communicators

There is a view that engineers are poor communicators (A, C, G, M, N, S, and U). A notes that "the only maths" that appears in newspapers is either "statistics or data". Compared with the legal profession, C is of the view that engineers are not confident when communicating and compared to doctors, engineers are not as “arrogant”. G says that “engineers lack the emotional intensity they need to communicate to get a point across to people or to realise the impact of what they do on people’s lives”. M doesn’t see any link between “communicating and maths”. He views mathematics as rather isolating “sitting on my own at my desk, looking at my screen doing maths”. M says that in work “there isn’t time to get into maths, people have to trust you” … if you try to present the maths behind something, you would probably see people nodding off”. N suggests that engineers are not good communicators generally, for example, he says “the architect sells the beauty of the bridge more than the engineer sells the strength of the bridge”. S maintains that “engineers’ difficulty communicating mathematics happens after engineers spend a lot of years to get to the point where they can grapple with an abstract concept and then suddenly they have to try and communicate that concept to a decision-maker who wouldn’t be up to speed in the particular branch of science or maths”. S adds that “it is not reasonable to expect the accountant, the manager and whoever else is in the team to get up to the level of maths that the engineers are at, so the only way the communication is going to happen, is if the engineers develop their interdisciplinary communication skill”. U presents that the challenge of producing a document is taking his calculations and “turning them into ordinary English for finance speak or marketing speak or sales speak or whatever is necessary as it comes up”. He adds that while the document might appear as “clear as anything” to him that’s not what is seen “when other people go to read it”.

While some engineers are of the view that engineers are poor communicators, there is also a view that communicating mathematics is a precise skill. C states that very often engineers communicate very effectively with each other using “just drawings”. K presents his “rock solid argument” in a report which has an “executive level” containing the “executive summaries” and the detail is in the appendix. M asserts that in high volume manufacturing, it is important that “everybody is using the same system”. He says that “IT solutions are a big part” of multinational companies whereby “one guy has to do the maths at the start”, he might be “writing formulas, embedding them using macros and running algorithms in the background that I don’t need to see to be able to use the tool". M says that the tools are often developed in "Singapore" and subsequently the "IT solutions are rolled out worldwide … everybody else gets the benefits of the analysis". R believes that the "skill in communicating maths" is about bringing "the problem and the solution to people in their language". S says that engineers "need to distil out the key messages and translate them into a language which these other people can relate to and that's a big ask in itself". U is of the view that converting mathematics into ordinary English "nearly is the craft of journalism". He says that "documents" are central to his work in that he has to put the mathematics in his job "into a form that a non-engineer will understand". U explains that behind each of the graphs in his documents "are lists of embedded tables" that readers don't need to see and "they don't want to know that level of detail" but because "those numbers have to be populated from numerous different sources" U has to "work with those sources and calculate across different sources to get the figures to draw up the graphs".

H, M and U note that Excel is one useful way of communicating mathematics. H is of the view that "Excel is amazing" because it is "easy to understand" and all engineers use it. M uses Excel to send schedules to external contractors and he notes that Excel is very "user friendly". U says that Excel "is a very good format for putting documents in to give to other functions such as finance, HR and sales etc." U says that when producing documents he "can throw all the numbers into Excel and get Excel to work out the standard deviation", but if he had "to stand up in front of people and explain what is meant by the standard deviation" and if he couldn't, his calculations would be "meaningless".

While some companies use consultants to do mathematics, engineers say that this can lead to difficulties communicating mathematics. N is of the view that "administrators are taking over" engineering functions in his company even though "they don't know what the consultants are telling them". R says there was a misuse of consultants during the Celtic tiger economy and at that time consultants produced "the same bunch of figures" for different projects. She says "a lot of consultants would have sent us in the same traffic management plan for different projects and you would have to be able to calculate out and just make sure that it was actually the project they were talking about".

7.2.8.3 Discussion of theme 8

There are two findings associated with theme 8, these are:

F8.1 Communicating mathematics is an important part of engineers' work.

F8.2 Compared to other professions engineers are not good communicators.

7.2.8.2-1 F8.1: Communicating mathematics is an important part of engineers' work

A significant finding is that communicating mathematics is an important part of the majority of engineers' work**.** Engineers communicate mathematics when: expressing engineering concepts; expressing conclusions; writing reports; making arguments; explaining how "you have come to your conclusion"; justifying some decisions; rolling out IT solutions; reading reports; verifying consultants' work; communicating a concept to a decision-maker; asking the finance people to provide money and selling products. Engineers say they communicate mathematics to a range of people including: other engineers; a variety of technical people on project sites; colleagues in Ireland and Singapore; clients; managers; vendors; contractors; consultants; administrators; customers; decision makers; accountants; finance people and human resources people.

Engineers view effective mathematics communication as a means of enabling a number of people to get "the benefits of the analysis". Communicating mathematics effectively enables engineers to produce "rock solid" arguments and it is a means to prevent other people "pulling your leg". Engineers say there is "skill in communicating maths". It is the "craft" of putting the mathematics "into a form that a non-engineer will understand". While many engineers use Microsoft Excel to communicate with other engineers, engineers also need to be able "to stand up in front of people and explain what is meant by" the particular mathematics used. Consequences of poor mathematics communication skills are that calculations are "meaningless" and the message can be "biased" or "abused".

The importance of communicating mathematics is evident from the research literature. For example, it is reported, in section 2.2.1, that there are three components to doing mathematics, these are: processing, interpreting and communicating mathematical information in ways that are appropriate for a variety of contexts (Evans, 2000). Similarly in section 2.2.1: mathematics oriented *thinking* skills, which are so important in engineering practice, include: "the ability to interpret information presented in a mathematical manner and to use mathematics accurately to communicate information and solve problems" (Radzi et al., 2009); mathematical literacy reflects the skills needed in business and the communication of mathematically expressed decisions and judgements within businesses (Hoyles et al. 2002); individuals need to be able to understand and use mathematics as a language that will increasingly pervade the workplace (Hoyles et al. 2010); and one mathematics competency is "communicating in, with, and about mathematics" (Niss 2003).

In 2.6.1 it is reported that engineers' practice of modelling a problem in "objective, mathematical terms" is outmoded and that engineers are now "immersed in the environment and human relationships from which perception of a problem arises in the first place" (Sheppard et al., 2009); that modern engineers work in teams and they exchange "thought, ideas, data and drawings, elements and devices" with other engineers around the world (Crawley et al., 2007); that engineers spend 60% of their time explicitly interacting with other people (Tilli and Trevelyan, 2008); and that a major part of engineers' work is to explain, often at a distance and through intermediaries, how the products of their work need to be designed, built, used and maintained effectively (Trevelyan, 2010a).

Practising engineers' requirement to communicate mathematics is also apparent in the research literature. In section 2.7.1 a study of civil and structural engineers working in a large engineering design consultancy in London, observed that mathematics is used as a "communication tool" between the designer and the specialist whereby the "specialists" are able to: "synthesise complex problems down to something very small, which can be expressed mathematically … the specialist can give you a set of equations, which you can adjust, change the parameters. So the maths is used as a communication tool, he's digested a situation into a model which is accessible to the general engineer, with a general mathematical background" (Kent and Noss, 2002). There is a view in the research literature, in section 2.5, that communication and team work contribute significantly to the gap between engineering education and engineering practice (Tang and Trevelyan 2009). In section 2.7.1 it is further recommended that engineering students should learn how to communicate with "others who can provide mathematical expertise" (Cardella, 2008).

7.2.8.2-2 F8.2: Compared to other professions engineers are not good communicators

Compared with other professions engineers view themselves as poor communicators which they say is not good for the engineering profession. One engineer believes that "engineers lack the emotional intensity that they need to communicate to get a point across to people or to realise the impact of what they do on people's lives". He says that "others [non-engineers] seize that opportunity and that is why engineers are so often in the background". There is a view that mathematics work is "isolating" and that when an engineer tries to present mathematics to his work colleagues he notes that his audience is "nodding off". There is the difficulty of getting people to "grapple with an abstract concept" and there is a view that there is often a disconnection between the engineer who is "enthusiastic about the mathematical detail" and the decision maker and that it is not reasonable to expect the manager "to get up to the level of maths that the engineers are at". One engineer notes the challenge of converting mathematics into "ordinary English" and that while his documents might be as "clear as anything" to himself "other people" have difficulty reading them.

The engineers' view, that they are not good communicators, is somewhat supported in a longitudinal study of mathematically gifted adolescents, in section 2.5. This study found that “those with exceptional mathematical abilities relative to verbal abilities tend to gravitate toward mathematics, engineering and the physical sciences, while those with the inverse pattern are more attracted to the humanities, law and social sciences” (Benbow et al. 2000). Another study of graduates who didn’t come from the pool of mathematically gifted students found that male scientists have “exceptional quantitative reasoning abilities” compared to “ verbal reasoning ability” (Lubinski et al. 2001). In section 2.5, studies also show that engineering graduates lack communication skills required in engineering practice (Nair et al. 2009). A study investigating mathematics graduates’ transition to the workforce in terms of their communications skills, in section 2.6.1, found that prior to working the graduates had not considered the use of mathematics to communicate ideas. Their education did not teach them to use standard computer products such as Excel, Visual Basic or SAS. In the workplace, graduates are often the only ones who can speak the mathematical language and many graduates are unable to release the strength of their mathematics because they do not know how to communicate mathematically (Wood 2010). A study of the early work experiences of recent engineering graduates, in section 2.6.1, found that the social context of engineering in the workplace is a major driver of engineering work and that interpreting data was a new experience for many engineers. One engineer said he was “learning more about how to present my data to other people” (Korte et al., 2008).

A significant finding is that communicating mathematics is not only important in engineers’ work but that it is critically important for the engineering profession. One engineer asserts that “if engineers are to survive then they need to somehow harness communication skills”. There is a similar view in section 2.6.2 whereby it is presented that society values engineers who can apply their skills across disciplines and that it is important for engineers to communicate effectively with non-technical people. It is asserted that engineers should have the ability to explain technical problems (Grimson 2002). One engineer in this study maintains that if one doesn’t “bring the problem and the solution to people in their language” mathematics becomes “elitist”. Ernest reinforces this view, in section 2.2.3 where he states that the perception of mathematics “in which an elite cadre of mathematicians determine the unique and indubitably correct answers to mathematical problems and questions using arcane technical methods known only to them" puts "mathematics and mathematicians out of reach of common-sense and reason, and into a domain of experts and subject to their authority. Thus mathematics becomes an elitist subject of asserted authority, beyond the challenge of the common citizen" (Ernest 2009).

There is also a view in the literature, in section 2.6.2, that engineers "don't do a good job of explaining" engineering to people outside of engineering and consequently engineering is seen as a "bunch of technical things they can't grasp … and boring too". The perceived difficulty of technical aspects of engineering, especially mathematics and science, contributes to difficulties communicating what engineering is (National Academy of Engineering, 2008). The lack of public understanding of engineering is damaging the image of the profession (National Academy of Engineering 2005).

It is interesting to note at this point that engineers give importance to communicating mathematics in both the teaching of school mathematics and the use of mathematics in engineering practice. The ability to communicate mathematics is the predominant characteristic of the engineers' good mathematics teachers (F1.2). The importance of communication in learning mathematics is supported by Vygotsky's theory of social constructivism, in section 2.3.1, where learning is constructed in a social context and that classroom discussion, rather than teachers' transmission of knowledge is an essential part of mathematics learning (Vygotsky 1978). When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing and they also develop new levels of understanding mathematics. There is a view that communicating mathematics is neglected in mathematics education (National Council of Teachers of Mathematics 2000) and in Ireland there is little evidence of group work, and mathematics teachers generally rank lower-order abilities (e.g. remembering formulae and procedures) more highly, and higher-order abilities (e.g. providing reasons to support conclusions, thinking creatively and using mathematics in the real world) less highly than do teachers in many other countries (Lyons et al. 2003). Furthermore engineers in this study admit that they felt "alone" in their enjoyment of school mathematics and that there is an "isolation" associated with using mathematics in engineering practice. The engineer's feelings about school mathematics and mathematics in engineering practice supports the view, in section 2.5, that communication and team work contribute significantly to the gap between engineering education and engineering practice (Tang and Trevelyan 2009).

7.2.9 Theme 9: Engineering Practice, *Engaging* with Mathematics

The findings outlining the engineers' views about *engaging* with mathematics in engineering practice are presented in this section. Theme 9 is organised as follows:

Page number

7.2.9.1 Degree of necessity for a mathematical approach in engineers' work

A summary of engineers' need for a mathematical approach in their work is presented in Table A9-14, Appendix 9, Volume 2. A majority of engineers say that a specifically mathematical approach is not necessary in their work (A, B, C, D, E, F, G, H, K, L, M, N, and O). B says he doesn't see his job "in maths terms" even though "maths may be underlying some of it". E requires "just basic maths to do some things" in her work. F says he doesn't "have to actually use mathematics" in his job. For G mathematics "isn't usually necessary" in his current role. Both M and N rate the necessity of a mathematical approach in their work as "a little". O says that mathematics isn't specifically necessary in his work. C, who says his work "doesn't require a specifically mathematical approach", estimates that mathematics is less than "ten per cent" of his work and while "not a huge amount of what" he does "on a day to day requires a huge level of maths, invariably something will come along". K is also of the view that mathematics is necessary in only ten per cent of his work.

Only seven of the twenty engineers say that their job requires a mathematical approach. It is the top six *curriculum mathematics* users (P, Q, R, S, T, and U) and J who say that their work requires a mathematical approach. J, who works as a lecturer and a researcher, says that he couldn't do his work without mathematics. Q says that her career to date has involved *curriculum mathematics* and she rates the degree of necessity as between "quite a lot" and "a very great deal". R is of the view that mathematics is essential in her job. S says that mathematics is "necessary but not sufficient" in his work. When asked about the need for a specifically mathematically approach in her work, T says while she "could do ninety per cent" of her job without mathematics she "couldn't possibly do the other ten per cent without it". U says he needs mathematics "quite a lot" to get his job done.

Given that it is the top six *curriculum mathematics* users (P, Q, R, S, T, and U) and J who say their work requires a specifically mathematical approach and it also is the low *curriculum mathematics* users say that a specifically mathematical approach is not necessary in their work, this suggests that engineers view mathematics as *curriculum mathematics* usage and not *thinking* usage. This is consistent with the views of engineers A, B and D who do not consider estimation of engineering solutions to be mathematics (section 7.2.4) for example A is of the view that "having a feel for an answer or solution is more useful" than having an answer "correct to eight decimal places". Similarly B is of the view that doing "a reasonableness test to conceive a solution" to "within a good level of probability" is sufficient. D is "much more confident" in his work about "having the principles right and conclusions right from a good understanding of the problem with some checking by maths rather than doing a big long calculation, coming up with the answer and saying bang, there's the answer".

7.2.9.2 Value of a mathematical approach in engineering practice

The engineers give a variety of reasons for not requiring mathematical approach in their work. Of the lower *curriculum mathematics* users, there is little value in using mathematics in work that is not mathematical (A, B, C, and D). Mathematics is not a major element of A's job. B says that while "maths may be underlying some of" his work he doesn't see his job "in maths terms". C says that even though "not a huge amount of what" he does "on a day to day requires a huge level of maths, invariably something will come along". Any higher level mathematics required in C's company is "done by consultants". D's response of "very little, thank God" is due to the fact that he feared using mathematics and he would only use the mathematics he was "confident" about. D maintains that in engineering practice "the whole thrust is to reduce the figuring out to be done mathematically down to the minimum and … dumbing down the process all the time, so that you can shove it down to a less experienced or less qualified person".

Many engineers suggest that the value of mathematics in their work does not justify the time required to take a mathematical approach in their work (E, F, G, H, K, L, M, and N). E requires "just basic maths to do some things" in her work because of the problems she encounters in work require a solution that is "a number". She says that due to "time constraints" and finances, she has to re-use previous "setups" and "designs" in her work, often she does not get the opportunity "to come up with a better solution". F states that engineers don't set out to "develop a mathematical model for a problem because it would be nice thing to do". He says he just doesn't "have the time to do that". He says that he doesn't have "to actually use mathematics" in his job but that he does "need to understand" mathematics or get "someone to figure it out" for him to enable him to make the correct decisions. F adds that, in his company, there is a respect for "maths only to the extent that it is useful". He adds that one "wouldn't be thanked" for using mathematics if "it wasn't in answer to a specific problem". G states that as his experience increases, "there is a lot more judgement" than mathematics in his work which enables him to make decisions "quicker". H says that she is "at the stage where common sense applies more than the maths" and that she has to "look at the figures very quickly and make decisions". K asserts that his managers "don't care if you use mathematics" and for them "it's about getting the job done quickly". L says that it "is a lot easier to just input something into a program and get the result out the other end without having to do "the donkey work" in the middle". L notes that companies are "trying to make people's time more effective" and they "are trying to minimise the amount of work that's required in order to deliver an end product". He says it is "more cost effective" for engineering companies not to "revert back to the way things were done twenty or thirty years ago". M says that his team of engineers have a "tool box" of "problem solving analysis tools" and they decide "case by case" if they want to take a mathematical approach to a specific problem”. He says that some work “issues could be very complex, you might have a tooling mix, human interaction, raw materials and you need to take a mathematical approach or you could end up choosing the wrong solution”. M also says that “the problem may not be that complex and you could spend more time working on the solution through maths, than to just figure it out and move on”. M adds that “time to closure of the problem is a priority” for him and he doesn’t “have time for show boating … with maths”. He adds that his company are continuously looking for “cheaper ways of doing our engineering”. N says that as his career progresses he doesn’t “have the time to be going back into maths”. He adds that “if you’re sitting down doing all these calculations all the time”, “it’s not good for your career”. N says his priority is to “develop career wise” and that mathematics will not help him make the career transition from semi-management to total management. While these eight engineers are concerned about the time required to take a mathematical approach O is of the view that “if the maths works out … it’s a faster way of doing something”.

The engineers who say that their work requires a mathematical approach do so because mathematics is “essential” in their work (J, P, Q, R, S, T, and U). J says that mathematics is “essential” in his work. In his research work, he says that “the first step in a new area is to try and find the mathematical expression of some ideas and hypotheses”. P says that if the engineering business is “involved in mass production, you’ve got to take a statistical approach in order to identify what needs tweaking so that you reduce the failure rate … doing it be the seat of the pants doesn’t work”. R says “the work I am doing now I had to go back to my maths … I had to go back to my equations”. She says that “because we had a lot of floods lately … because we have had so much building … we had to go back to the design … of water pipes, water mains, attenuating and dispersion of rain water on sites, percolation and ground and soil conditions”. R notes that her “bosses demand the answer and once you can show them that the answer works” it doesn’t have to be mathematical. S says that mathematics is “absolutely necessary for the parts of the problem which I can frame mathematically but what I have realised is that the problem that I really need to be tackling has a lot of elements and some of these elements I am not able to model or frame mathematically". T says that her "work goes between front end design … to problem solving and that while she "could do ninety per cent" of her job without mathematics she "couldn't possibly do the other ten per cent without it". T adds that there are "times that you actually have to have the mathematical background to actually prove something or provide justification for something and … engineering is that extra ten per cent that you actually get paid for at the end of the day". U, who is of the view that he needs mathematics "quite a lot" to get his job done, says "I have to use numbers, in practically every action that I do … I don't think there is anything I can do in my work at the moment that I am not using numbers". He is of the view that the majority of "people who have qualified with engineering degrees do need a good understanding of basic mathematics".

7.2.9.3 Degree engineers seek a mathematical approach in their work

The majority of engineers say that they would like to take a mathematical approach in their work (E, G, H, J, K, L, O, P, Q, R, S, T, and U). It is mostly the low *curriculum mathematics* users who do not express a desire to take a mathematical approach in their work. Engineers A, B and C do not consider mathematics to be a significant part of their work. When using mathematics in work, D has "a nagging fear" that he has "got something wrong" and he says he would only use the maths that he was "confident" about. He also says that following college he focused on engineering and if he encountered a mathematics problem, he would "refer" to his colleagues. F is of the view that in his company he "wouldn't be thanked" for using mathematics. Similarly M says his company is continuously looking for "cheaper ways of doing our engineering and "you could spend more time working on the solution through maths, than to just figure it out and move on". N maintains that seeking a mathematical approach in his work will not help him to "develop career wise".

Of the engineers who would like to take a mathematical approach to their work, E is disappointed with how little mathematics there is in her job. She says she likes to do her work "the maths way, if there is a maths way". G says he "wouldn't bypass" the mathematics and he would "always like to understand how things work". H notes that “if there was a way of using maths to solve the problem” in her work, she would use mathematics. J says that while “other people would have opted for a solution which involved very little mathematics” he opts “for one that involved more maths”. He says that in his “mind-set”, he always assumes that “there is a mathematical solution” to his work. K says that while many of his colleagues have “no interest in presenting something through mathematics he “immediately” wants to solve a problem using “mathematical thinking” and he starts by “decomposing it, looking at it logically and from different viewpoints”. He says “it doesn’t always go down to *curriculum maths* but where necessary it does”. L says he “wouldn’t avoid maths” in his work and that working out the solution gives him “confidence” in his work. O says “if I could use it [mathematics] I would love it, yeah I mean if I could figure out all my problems using maths, I would absolutely yeah, yeah”. When asked if he would actively seek a mathematical approach in his work, P’s response is “very definitely”. He says that while he “would look to be able to represent things as far as possible mathematically”, he says that “for most engineering problems there is a myriad of strategies” and that there is a tendency to choose a strategy “that has worked for you in the past or one that you are comfortable with”. He adds that “it requires a hell of a lot less effort from you to repeat something than it does to reinvent the wheel”. Q says she likes to do something “the maths way” as she likes to “find out the answer”. R says “I will always go back to first principles on everything no matter what it is”. S says “I would like to formulate all my problems mathematically, I just can’t … I probably try and use it where I shouldn’t you know”. T always chooses the “maths way” of doing things because mathematics is “very easy to reference and verify, it is completely logical and nine times out of ten you are dealing with engineers who understand the maths”. U notes that he wouldn’t ever avoid using mathematics, he says “I wouldn’t be afraid to use the maths way, but I would do it the way that would be most productive at the time”.

7.2.9.4 Discussion of theme 9

There are two findings associated with theme 9, these are:

F9.1 The degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* in their organisation.

F9.2 Confidence in mathematical solutions motivates engineers to seek a mathematical approach in their work.

7.2.9.4-1 F9.1: The degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* in their organisation

Thirteen of the twenty engineers say that a specifically mathematical approach is not necessary in their work. The interview data shows that a mathematical approach is not a significant part of the low *curriculum mathematics* user's work and it is the high *curriculum mathematics* users and one other engineer who works in education and research who say their work requires a specifically mathematical approach. The technical nature of the engineers' [who say their work requires a mathematical approach] work includes: two engineers who use statistical process control in manufacturing environments; another engineer who designs water pipes and investigates the "attenuating and dispersion of rain water on sites" which is based on mathematical equations; two further engineers who work in education and research environments; one engineer who designs power transmission and distribution systems; and another engineer whose work includes the management and design of telecommunications networks and who says he has to use numbers "in practically every action that I do … I don't think there is anything I can do in my work at the moment that I am not using numbers".

The main reason engineers give for their work not requiring a mathematical approach is that it is "more cost effective" for their engineering companies not to use mathematics". The engineers say that companies are continuously looking for "cheaper ways of doing engineering" and in an attempt "to minimise the amount of work that's required in order to deliver an end product" engineers re-use previous "setups" and "designs". There is a view that engineers often have to make decisions "quickly" and they do not have "time" to "actually use mathematics" in their work.

One engineer says that "time to closure of the problem is a priority" for him and he doesn't "have time for show boating … with maths". He says "you could spend more time working on the solution through maths, than to just figure it out and move on". Another engineer is of the view that his company "don't care if you use mathematics" and that there is a respect for "maths only to the extent that it is useful". A further engineer goes so far as to present that mathematics is not good for his career and that mathematics will not help him make the career transition from semi-management to total management. The engineer who is the highest user of *curriculum mathematics* in the group of engineers interviewed says that "there is a certain respect for mathematics" in his company "but that seems to change as the management changes and I have seen that over the years where the CEO was an engineer there was a very large amount of respect for mathematics, whereas the current CEO is very much a marketing man and so definitely the emphasis is on sales and marketing and away from the maths right now".

While a specifically mathematical approach is not necessary in a majority of the engineers' work, the findings in section 7.6 show that a majority of engineers interviewed use aspects of either higher level Leaving Certificate mathematics or engineering level mathematics in their work (F6.1) and also a majority of engineers use *curriculum mathematics* in either *connecting* or *mathematising* ways (F6.3). Of the thirteen engineers who say their work does not require a mathematical approach, the bottom *curriculum mathematics* user is the only engineer whose *curriculum mathematics* usage is not greater than both ordinary level Leaving Certificate mathematics and type 1 (*reproducing*). The engineers' views that engineering practice is much more than mathematics in section 7.2.4 and that a mathematical approach is not necessary in their work suggest that engineering practice is multi-dimensional and that *curriculum mathematics* is a small proportion of their overall work. For example, two engineers who estimate that mathematics is ten per cent of their work each have different perspectives on the need for a mathematical approach in their work. One engineer, who is a low user of *curriculum mathematics* in his work, is of the view that a mathematical approach is not necessary in his work. He says that while "not a huge amount of what" he does "on a day to day requires a huge level of maths, invariably something will come along". However in his company, higher level mathematics is "done by consultants". On the other hand, another engineer, who is a high user of *curriculum mathematics* in her work, is of the view that a mathematical approach is necessary in her work. She says she "could do ninety per cent" of her job without mathematics, but that she "couldn't possibly do the other ten per cent without it" and she maintains that "engineering is that extra ten per cent that you actually get paid for at the end of the day". It is interpreted here that *curriculum mathematics* is a small proportion but necessary part of engineers' work. This interpretation is consistent with the engineers' view that while *curriculum mathematics* is "very useful for elements of problems particularly in engineering, it is not necessarily the full solution" and also with the finding that engineers' work is diverse and it comprises: degrees of mathematics, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions (F4.1).

The relationship between the engineers' *curriculum mathematics* usage and the necessity of a mathematical approach in engineering practice suggests that engineers view mathematics as *curriculum mathematics* usage and not their *thinking* usage. While *curriculum mathematics* usage is a small but a necessary part of engineering practice in general, *thinking* usage is another part of engineering practice which all the engineers in this study rate higher than their *curriculum mathematics* usage in their work (F7.3).

The observation that the degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* in engineering practice is similar to Wigfield and Eccles' social cognitive expectancy-value model of achievement motivation, presented in section 3.3.1 of this thesis, where students' perceptions of the importance, utility and interest in mathematics are strong predictors of their intentions to continue to take mathematics courses (Wigfield and Eccles 1992). According to the engineers the value (importance) of taking a mathematical approach in work relates to the technical aspects of the engineers' work and the costs (perceived negative aspects of engaging in mathematics) of taking a mathematical approach in work include: financial cost; time; effort; and the value or respect given to mathematics within engineering organisations.

While the value given to *curriculum mathematics* in engineering practice is determined by an appreciation or at least an awareness of engineers' *curriculum mathematics* usage within their organisation, the findings in section 7.8 show that engineers are not good communicators and that there is often a disconnection between the engineer who is "enthusiastic about the mathematical detail" and the decision maker.

7.2.9.4-2 F9.2: Confidence in mathematical solutions motivates engineers to seek a mathematical approach in their work

The majority of engineers are positively disposed to seeking a mathematical approach in their work. In fact two engineers are disappointed with how little mathematics is required in their work. One of these says she would "prefer to use maths more" in her work. The other engineer admits that he "was probably a little bit naive" going into electronic engineering education as he did not realise "it had to do so much with computers". He says that while he loves his work he would prefer a role where he does more mathematics.

Many engineers like to take a mathematical approach in their work because mathematics is their "mind-set" and using mathematics gives them "confidence" in their work. In her work, one engineer likes "a maths way to do something", she likes getting an "exact solution". She likes "to be able to prove that something is right with maths". Another engineer also says she likes to do something "the maths way" as she likes to "find out the answer". Another engineer always chooses the "maths way" of doing things and she says mathematics is "very easy to reference and verify, it is completely logical and nine times out of ten you are dealing with engineers who understand the maths". Another engineer says "I will always go back to first principles on everything no matter what it is". A further engineer says "I would like to formulate all my problems mathematically … I probably try and use it where I shouldn't you know".

Engineers' values (importance of *engaging* in mathematics) relate to the technical nature of the engineers' work and their confidence in mathematical solutions. Costs (perceived negative aspects of engaging in mathematics) include: "wouldn't be thanked" for using mathematics; cost and time requirements; and no benefit career wise. Engineers say that confidence in mathematical solutions and self-efficacy are factors in engineers' motivation to seek a mathematical approach to a work problem. Self-efficacy, as discussed in section 3.3.1, is "beliefs in one's capabilities to organise and execute the courses of actions required to produce given attainments" and it is influenced by mastery experiences, vicarious comparisons, social persuasions and physiological and affective responses (Bandura, 1997). Some engineers in this study show confidence in mathematical solutions and they prefer to take a mathematical approach in their work. For example, one engineer says he "wouldn't avoid maths" in his work and that working out the solution gives him "confidence" in his work and another engineer always chooses the "maths way" of doing things because mathematics is "very easy to reference and verify". On the other hand some engineers are fearful of using mathematics in work. For example, one engineer says he "was afraid of some of" the mathematics he encountered in engineering practice and another engineer says "if you were doing or using some maths for your solutions … where nobody has done it before and you can't copy a template … you are putting yourself up, putting your neck on the line … you don't want to be the guy that puts something in place that goes wrong or is fundamentally flawed".

7.2.10 Theme 10: Relevance of Engineering Education to Engineering Practice

The findings outlining the engineers' views on the relevance of engineering education to engineering practice are presented in this section. A summary of the value of mathematics education in engineering practice is included in Table A9-15, Appendix 9, Volume 2. Theme 10 is organised as follows:

Page number

7.2.10.1 Engineers support high level of mathematics in engineering education

While the engineers' views on the level of mathematics education required for engineering practice are mixed, the majority of engineers support the high level of mathematics taught in engineering education (A, B, C, D, F, G, H, J, K, L M, O, P, Q, R, S, T, and U). A is of the view that there is too much "specialist maths" in engineering education and that there isn't "a necessity for everyone to learn all the maths" nor "for most people to learn some of the stuff that is taught in college", however he estimates that engineers in general use just ten per cent of the mathematics learnt in university and the difficulty for engineering education is "figuring out which ten per cent for each individual" student. A is also of the view that because "engineering is very broad" and because there is no "sense" of the specific careers graduates take on, engineering education must adopt a "one size fits all" approach. Mathematics was not a factor in B's appointment to his job and while he didn't have the "opportunity to exploit" his mathematical "knowledge or skills" in work, his mathematics education did instil "great confidence" in him in terms of career progression. He says that "the grounding engineers get, prepares them for any manner of mathematical interpretation and understanding" in the real world. B supports the high level of mathematics in engineering education curricula; he says that courses have to be aimed at the graduates who take on "the highest consequence" of mathematics in their work and the "top five or ten per cent of engineers that are going to bear that design responsibility". He asserts that mathematics education is not wasted on engineers who pursue less numerate careers as these people benefit from the discipline and rigour of learning mathematics and they also infuse organisations with a "great deal of rigour and discipline". C notes that while his job does not "require a huge level of maths" that "invariably something will come along". He is of the view that the amount of mathematics in engineering education doesn't do "any harm to engineers". C recognises the security associated with a mathematical answer and he likens mathematics to "a safety valve". D is "not as convinced that the maths that you do in an engineering course needs to be as high as academia seems to think". He says that "an awful lot of the maths that you learn in engineering education, you never see it applied". He is of the view that "academics try to impress each other with the horse power of the maths" and that the "fundamental principles" of engineering are "bypassed" because students need a "very high standard of maths" to "grasp" subsequent engineering concepts. However D does say that the "whole four years in engineering education" are "very important" as graduates develop "logical, reasoning and analytical" skills. He is of the view that engineering education is "a good grounding" for engineering *thinking*. F is of the view that one "would need to have had higher level maths at some stage" to do his job. He says his higher level mathematics gives him confidence to use "models" and "black box solutions" and it gives him "an appreciation of the limitations" of the models. G says that engineering mathematics "is necessary" for his job and it makes his job "easier". He adds "if you have a fear of it or it turns you off it's just like not being able to use your driver in your golf bag, it's just going to handicap you". H says the need for higher level Leaving Certificate mathematics varies in her company. The engineers who do "modelling of drainage or water systems" need to know mathematics. J is of the view that while few engineers need "certain types of maths, applied maths and problem-solving techniques" in their work, there "are still quite a lot of engineers who couldn't do their jobs unless they can solve differential equations". He is also of the view that managers in engineering companies require an "appreciation of mathematics" and that if the managers never learnt the mathematics themselves they could not properly manage engineering work. J asserts that "doing maths is just very good training for the brain and teaches you concepts, like abstraction which you know make you a better thinker in general". J also says that the lecturer's job is to let the undergraduate engineering students out of university "with a level of maths that we think is appropriate for a professional engineer, a Chartered Engineer, which of course is a very high level of engineering". K says that mathematics is "valuable" in the ten per cent of his work where he uses mathematics. He says he sees "circumstances where others in the company would be better" if they had mathematics and that "when they don't have that level of mathematical ability it restricts them in terms of analysing situations". L states that "there is this belief that engineers should be good at maths and engineers generally are very good at maths and therefore maths should be part of any engineering curriculum no matter what the degree is or no matter what College is teaching it". L also considers the skill of "applying mathematics in a logical way" is necessary in engineering practice. M says that while "higher level Leaving Certificate maths isn't necessary" in his "day to day work", the "discipline that comes with it is a requirement". He is of the view that higher level Leaving Certificate mathematics usage "depends on engineering roles". He says that only "ten per cent of the engineers on site here would need some of the learning from higher level maths" and that "ninety per cent of us could do our roles without honours maths". O says that in his current role as a manager, he "would absolutely not need higher level Leaving Certificate maths". However he is of the view that while engineering managers generally wouldn't be using higher level Leaving Certificate mathematics "in their day to day jobs" that "they may need to understand certain parts of it". O says that he doesn't require his team to "to have an awful lot of mathematical knowledge coming in here". However he is "nervous" that the higher level Leaving Certificate mathematics exam will be made easier in an attempt to improve results and if it is made easier "kids will be utterly unable to cope when they eventually get to work". P is of the view that "if you don't have higher level maths, you're never going to appreciate the finer points of the topics that you need to master as you progress through an engineering course and if you don't master them you aren't ever going to be much of an engineer". He says "that a good grasp of maths is essential to being a good engineer" and that he wouldn't hire the "guys who have struggled through engineering school". P presents that "engineers start out with a very heavy emphasis on mathematics in the early part of their career, that very heavy emphasis continues and that tends to inform them how they approach the rest of their career." He believes that "mathematics is an extremely useful tool … early on one learns how useful it is and simply continues to use it in one way or another as one progresses through one's career". Q is of the view that while higher level mathematics may not be necessary for engineering practice in that "maybe you could get around it", she says "I do feel I am able to cope with things better because I have a grasp of the kind of maths and figures, particularly statistics required in my industry". She says that in addition to the people who "got into" engineering degree courses by doing a "certificate or diploma course first" and who "are still good engineers" there is "a need for mathematical engineers, because engineers need to be strong in maths to understand processes … or have a separate parallel function like a statistician". Q says that "some of the statistical analysis" she uses in her work is "more heavy weighted in the higher end of engineering and in theoretical maths than a graduate coming out of college would grasp". However Q says that there is "still a lot of maths" that she studied in college that she doesn't "know the application of". R is of the view that higher level Leaving Certificate mathematics is necessary for engineering practice because "in engineering you need to go into maths in a great depth … in the third degree … sometimes things are not in a straight line … everything you look at is in the third dimension … things like oscillations of springs". She says that mathematics is essential in her job and "the work I am doing now I had to go back to my maths … I had to go back to my equations". She says that "because we had a lot of floods lately … because we have had so much building … we had to go back to the design … of water pipes, water mains, attenuating and dispersion of rain water on sites, percolation and ground and soil conditions". S is of the view that while "there is a lot of engineering job specifications where maths is not necessary" he says that mathematics is "an advantage" in engineering practice. He notes that "given the broadness of the roles that engineers tend to end up in" engineers' use of mathematics “depends a lot exactly on what area of engineering you are in”. He adds that mathematics is “a really useful tool” in engineering in general. T is of the view that she couldn’t do her job without higher level Leaving Certificate mathematics. She says “I just think you can do certain aspects of it [her job], but I don’t think you understand the fundamental aspects unless you have a good grasp of maths”. U says that he “simply wouldn’t been able to do” his job without mathematics. In his current role he says he uses mathematics “in a financial sense”. He says that behind “the simple pie chart” in his financial reports there are calculations such as “turning things like man hours into megabits per second … there is a certain amount of estimation as well”. He adds that without mathematics he “wouldn’t be able to guess trends or calculate statistically”.

It is just two engineers who do not support the high level of mathematics taught in engineering education. E says she requires no higher than Leaving Certificate ordinary level mathematics to do her job and she is of the view that engineering mathematics “seems pointless” because she hasn’t “ever used it since”. While many of the students in E’s engineering class didn’t have higher level Leaving Certificate mathematics, E had previously covered most of the engineering mathematics in the higher level Leaving Certificate syllabus. E likes to do her work “the maths way if there is a maths way”, however she is disappointed with how little mathematics she requires in her work. She is a senior design engineer who designs water collection networks, water distribution systems and flood study measures and she says that all mathematical calculations in her work are done using “programs”. E is of the view that engineering mathematics is only necessary if “you wanted to go back to first principles and know the background behind how some programs work”. However while some engineers are of the view that one “just needs to have a basic understanding of how things work” when using computer tools, there is a stronger view that “the engineer should understand how the program is solving the equations and what it is doing, because it is always dangerous not to”. N is the other engineer who does not support the high level of mathematics in engineering education. He is of the view that while “everything should be covered in your education that would equip you to look at any engineering problem you have to solve at some stage of your career", "engineering training is too academically based". He maintains that graduate engineers are "so academic" and it is only after their education that they "pick up" the skills to tackle "the real world situation". N is of the view that "the amount of math that you use afterwards is never matched by the amount of maths you do in college". N suggests that by including so much mathematics in the engineering subjects, universities are making engineering "elitist".

7.2.10.2 Need to better match mathematics in engineering education with mathematics required in engineering practice

There is a view that *curriculum mathematics* is different to mathematics used in engineering practice and that there should be a greater focus in engineering education on the mathematics skills required to solve real world engineering problems. In engineering practice mathematics is used primarily as a tool to estimate and confirm solutions to real problems while in engineering education mathematics is about deriving exact solutions to theoretical problems from first principles. *Curriculum mathematics* differs from mathematics used in engineering practice in a number of ways.

In engineering education curricula there is an emphasis on high level academic mathematics while in engineering practice the focus is on the engineering problem solving (D, K, L, N, O, P, Q, S, and U). D sates that "an awful lot of the maths that you learn in engineering education, you never see it applied". L also notes this mismatch and he is critical of the mathematics in his own engineering education. He says that because mathematics was "taught by the mathematics department and most of the other subjects were done through the engineering school" his engineering mathematics education "didn't relate to the other elements of the degree". Q's engineering mathematics education was also theoretical; she says there is "still a lot of maths" she studied in college that she doesn't "know the application of". K is of the view that engineering education, where the focus is to design something "from scratch", does not match the requirements in modern engineering practice where much work is systems integration and connecting individual pieces of technology together. P is also of the view that “there is a big difference between the academic environment and engineering practice”. He says that engineering “practice very often doesn’t reflect the theory”, for example, practising engineers need “to look upon maths as a tool rather than as an end in itself” as it is taught in “school and indeed in college”. While S is of the view that mathematics in general is “a really useful tool” in engineering, he says that very often the level of mathematics needed to solve an engineering problem is “quite simple but nonetheless, the overall solution is now a real solution that real people really want and that can be very rewarding and challenging”. He adds that an engineer’s role is much more than mathematics. It is “to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker”. U also stresses the importance of learning how to communicate mathematics, he says that “sort of language” took him “a few years in the company to learn”. N is of the view that “projects are the way to go in engineering education” because “maths should follow the problems and not the reverse”. O suggests that engineering education, instead of doing the very complicated mathematics, should focus on the “stuff that might be more useful” in engineering practice and the “tools to do other things with maths”.

There is a view that many engineering problems do not require a precise solution and problem solving is often an iterative process of estimating, checking and refining a solution. There is a view that estimation is an important skill in engineering practice that is not taught in engineering education (A, B, D, F, K, N, and P). A is of the view that when an engineer is “trying to fix something” in engineering practice that “having a feel for an answer or solution is more useful” than having an answer “correct to eight decimal places”. B is also of the view that estimation skills are important in engineering practice. He says that “so much of the value an engineer brings to his job and brings to society is to be able to do a reasonableness test to conceive a solution and within a good level of probability to be able to say yeah, that will meet the need, but then not being afraid to modify that and evolve that in subsequent observations or in practice”. In his work, D says he is “much more confident” about “having the principles right and conclusions right from a good understanding of the problem with some checking by maths rather than doing a big long calculation, coming up with the answer and saying bang, there's the answer". P says that when learning mathematics students search for "a precise solution" and that engineers never find "a precise solution". He says that if in engineering "perfection is to be desired there is always an acceptable level of imperfection". An advantage of estimation is that it enables engineers to develop a working solution quickly. K notes that the "time you have in college you don't have in the real world" and that there is "pressure to get things done quicker in the real world". F is of the view that because of their education, engineering graduates are "more drawn to, black and white solutions" while other disciplines "are more comfortable with estimates". He presents that estimation should be taught in engineering education because it is an important skill in the real world. N is of the view that engineering students should get "a feel for the work" and they should be "given the freedom to say which tool to use". This he believes would make the students "intuitive as regards what would work and what wouldn't work" in engineering and their roles "end up confirming" their intuition.

In engineering practice, there is often more than one practical solution and the engineering challenge is to select the best solution that meets a number of different requirements unlike engineering educations where there is a unique mathematical answer (C, F, J, P and U). While engineering problems may comprise of many factors such as cost, time, resources and safety etc. in mathematics education, the challenge is to determine the single mathematical correct answer. According to F, problems are framed in engineering education so that there is just "one solution". He says that "this way of thinking is not applicable in all instances". Similarly P says "there is very seldom a unique right answer in engineering challenges". C contrasts mathematics education where "there must be one answer" with his current job whereby if he came "to one solution", he says "that would be a disaster". U says his mathematics education taught him "to give a factual answer unfortunately and there are times when not to give the factual answer" in engineering practice. J is of the view that modern engineering education is about "generic problem solving skills, like abstraction and choosing different solutions and designing experiments that would apply to any branch of engineering." He notes that "teamwork and communication skills" are important in problem solving. He says when he was in university there were "a lot of us putting our heads together trying to get solutions" and that with modern engineering students "there is not as much collaborating going on between as we might assume".

7.2.10.3 Graduate engineers lack real world engineering experience

Many engineers note that graduate engineers lack the practical experience required for engineering work (A, C, E, F, G, H, M, N, P, Q, R, S, and U). There is a view that graduating engineers are not ready to engineer and that work experience and practical application would help undergraduate engineering students develop the engineering skills that are required in engineering practice. H says when she "first came out of college" she recognised things but she didn't "know how to do anything". N says that graduate engineers are "so academic" and it is only after their education that they "pick up" the skills to tackle "the real world situation". R is of the view that graduate engineers are "green" in that they "don't know a lot" while U says that when he started off in his current company he "hadn't a notion" of engineering practice. M's company doesn't "expect much" from graduates in their first two years because they need time to "figure out their role in the company" and to "develop some of the softer skills". H is of the view that civil engineers need to be "about thirty years and have about 8 years' experience under their belt", before they know "how things actually work" on construction sites.

F claims that "experience is the most important thing" when hiring engineers. He states that if he had to choose between "two new graduates" he would opt for the person "with the higher qualification" because of the greater "potential to develop". However he "would have no problem giving a job to a technician who had a lot of experience over someone who had a first class honours degree but no experience". E has a similar view. She says that engineering students are educated "in the wrong things" and that many technicians know far more than the engineers". She believes that engineering education would benefit from “a reduction in some of the maths” in favour of “more practical solutions”.

A majority of engineers say that work experience prepares graduate engineers for engineering practice. Both A and C are of the view that they learned to do their work more from “experience” than from their engineering education. G is of the view that engineering experience is what makes a good engineer. N believes that “you are not expected to be a fully-fledged engineer until you have this practical experience” and he notes this should be “part of your engineering training”. P says when graduates “go out into the real world they move into a new phase of learning”. In the real world, graduates gain “insight into the more practical aspects of engineering”, “experience” and the confidence to make decisions. R says that prior to graduating she “never had any experience in an engineering environment”. This lack of experience caught her in her first job which “was to design a water treatment plant”. Having “studied waste water treatment in university”, she says she “thought they were all different treatment types” and she did not understand how to “put the whole system together”. R says that “life and experience teaches you more,” than college and that as she gained experience she developed an “automatic thinking because most of the problems and solutions … you have seen them before and it is not really rocket science. It’s wisdom and experience”. S is also of the view that engineering graduates need time to learn “real world engineering”. L, M, P, Q and U note the benefits of interacting with other engineers in work. M says he developed his “state of experience” from his “peers” and from “the templates throughout the company”. Q says that while her college education was “not very applied” she has learnt to do her job from experience and from consulting with her work colleagues. Q is the only engineer who says she did some work practice while in college. From her undergraduate work experience she developed an interest in bio-medical work and she says that having “read some validation procedures” while in work practice, she “got an idea where the maths comes in”. Q states that as a new engineer, she initially learned from people at work and with her four years’ experience, she has recently become “an independent thinker”. P suggests that “it would make a lot of sense if there was more communication between experienced engineers and student engineers". U feels that his ability to do his current job has come from his "experience of working in an engineering environment" in that he learned from people, senior managers, older engineers etc. how they estimated, how they worked out real problems" and how they looked at "the bigger picture". He is of the view that engineering education should incorporate that "experience" into its curricula.

R expresses an interesting view that engineering experience has a value whereby it prepares students for learning. She recalls doing an MBA where she says "if I did it fifteen years ago … I would have got better marks but I probably might not have learned as much because I wasn't ready". She argues that one may "not be ready" to learn when one has not experienced "enough life" and that that learning becomes "more relevant to you as you go along". R believes that incorporating work experience as part of engineering education would "make all the difference" to graduates and "one key benefit of engineering experience is that students learn quicker".

7.2.10.4 Engineering education should impart an importance of skills required in engineering practice

There is a view that while engineering education covers a "broad spectrum of stuff" and only a "very small percentage" of which is used in practice, that engineering education is not a waste as students cannot predict which aspect will be relevant to their future careers. According to H the consequence of a broad curriculum is that "it is very difficult to learn the theory and apply it at the same time". U notes that "because technology is moving at such a rapid rate that most knowledge that was available" when he was studying engineering" has been long swept away". He says he "had to go back and study statistics" after college because he needed a proper understanding" of statistics in his work. Given the breadth of engineering curricula, S is of the view that engineering education should teach students "how to learn for themselves and also give them an appreciation or an awareness of the importance of certain skills" rather than "packing more into the curriculum". P compares engineering curricula in Ireland which are very broad with those in the U.S. where engineering graduates "tend to be highly specialised". He believes that the "generalist approach" to engineering education "may not be quite as relevant today" but he says it was "enjoyable" for him.

J suggests that engineering education should be more about teaching "the skill of problem solving" than "imparting information" which is readily available on the internet. J's university department are implementing the CDIO[33] concept of engineering education. He delivers a final year course where he uses a "case based learning approach" without lectures; "it is just meetings and problem solving and the students get all their information from the internet". J says this system of education is "challenging because it is more difficult to quantify how you are getting on". He feels that the "teamwork and communication skills" developed are useful in engineering practice. Also on the subject of engineering education assessment, L is of the view that students "can predict" exam questions and they often "cut out the stuff" that "could be more beneficial to you at a future stage in engineering". T suggests that engineering education and assessment should be modified to include more practical and real life applications.

Q is of the view that engineering students do not take "soft skills" courses "seriously". She says she dropped "management" and "economics" subjects in favour of more technical subjects in college. Q is of the view that it is not possible to learn "the best problem solving road map" in college because "you wouldn't be applying it to anything in college". However some engineers say that their careers would have benefitted from engineering education containing subjects in the areas of finance and management. H says that her work "is probably a bit more contracting commercial based than maybe pure engineering" and that her job is "doing something that I never did in college". She says that many engineers go on "to do MBAs" rather than do "more engineering stuff" because economics and finance topics such as "commercial contracts" and "cash flow forecasting" while part of real world

[33]CDIO: Framework for educating engineers based on a premise that engineering graduates should be able to: Conceive – Design — Implement — Operate complex value-added engineering systems in a modern team-based engineering environment. It includes student projects and active group learning experiences, Crawley, E. F., Malmqvist, J., Östlund, S., and Brodeur, D. (2007). *Rethinking Engineering Education: The CDIO Approach*, New York: Springer Science+Business Media.

engineering are not sufficiently part of engineering education. B, who works in a commercial role also notes that finance and "costing" is not part of the general engineering curriculum. Similarly, U says that finance is a big part of his current work and he says that he got "no understanding of finance in school or in college and that was something that was lacking" in his education. U goes on to say "a course on politics, it would be very useful" for engineering practice and he also blames the universities for the dearth of engineers who are CEOs of engineering companies. He says that business graduates, many of whom know nothing about engineering, are more likely than engineers to become managers of engineering companies.

7.2.10.5 Discussion of theme 10

There are three findings associated with theme 10, these are:

F10.1 Engineers support the high level of mathematics in engineering education.

F 10.2 There is a need for a better match between the mathematics taught in engineering education and the mathematics required in engineering practice.

F 10.3 Graduate engineers lack the practical experience required for engineering work.

7.2.10.5-1 F10.1: Engineers support the high level of mathematics in engineering education

The majority of engineers support a high level of mathematics in engineering education. They say that mathematics in engineering education should be aimed at the graduates who take on "the highest consequence" of mathematics in their work and that engineers who pursue less numerate careers also reap advantages of mathematics learning particularly with regard to confidence in mathematical solutions and mathematical *thinking*. This view reflects engineers' own *curriculum mathematics* usage given that the majority of engineers' *curriculum mathematics* usage is at higher level Leaving Certificate mathematics or higher (F6.1) and a majority of engineers' *curriculum mathematics* usage is higher than type 1 (*reproducing*) (F6.3).

Engineers' support for high level mathematics is based on the view that "engineering is very broad" and because there is no "sense" of the specific careers graduates take on, engineering education must adopt a "one size fits all" approach. While the engineers estimate that they use ten per cent of their university mathematics in work, they say that students cannot predict which aspect of mathematics will be relevant to their future careers. There is a similar view in the research literature, in section 2.5, where "the use of mathematics within the job of engineer is not necessarily self-evident to an undergraduate student" (Wood, 2010, Wood et al., 2011). The research literature also indicates that the wide range of contexts in which engineering takes place lead to misconceptions, mystification and misunderstandings about what engineers do (Capobianco et al. 2011; Knight and Cunningham 2004; Oware et al. 2007a; Oware et al. 2007b; Prieto et al. 2009).

In addition to their *curriculum mathematics* usage, engineers note many advantages of mathematics learning in the context of engineers' work, these include: confidence in mathematical solutions; logical, reasoning and analytical skill development; a really useful tool; essential in job; and appropriate for professional engineers. This view is consistent with the findings regarding the engineers' *thinking* usage where the engineers say that mathematics education contributes to *thinking* skills (F7.2) and engineers' mathematics *thinking* usage is problem solving, big picture thinking, decision making, logical *thinking*, estimation and confirmation of solution (F7.1).

Given the concern in section 2.5 of the literature review about a "one-size-fits-all" approach to engineering mathematics and the view that engineering education delivers more mathematics education than is required by specific engineering disciplines (Manseur et al. 2010b), the engineers' support for high level of mathematics in engineering education is a significant contribution to the debate about mathematics in engineering practice. The engineers' view is substantiated by the diversity of engineers' work of which *curriculum mathematics* is only one part and by the transferability of engineers from one role to another within an organisation given that engineering graduates lose their discipline identity (F4.1). Furthermore for all engineers interviewed their mathematics *thinking* usage is greater than their *curriculum mathematics* usage (F7.3) and mathematics education contributes to engineers' *thinking* skills development (F7.2).

7.2.10.5-2 F10.2: There is a need for a better match between the mathematics taught in engineering education and the mathematics required in engineering practice

There is a view that *curriculum mathematics* is different to mathematics used in engineering practice and that there should be a greater focus in engineering education on the mathematics skills required to solve real world engineering problems. In engineering practice mathematics is used primarily as a tool to estimate and confirm multiple solutions to real problems while in engineering education mathematics is about deriving a unique and exact solution to theoretical problems from first principles. Following their education, engineering graduates are "drawn to, black and white solutions" while in engineering practice estimation is an important tool as there is "pressure to get things done quicker in the real world" compared to university. One engineer, who lectures in a university, is of the view that instead of "imparting information" which is readily available on the internet, modern engineering education is about "generic problem solving skills, like abstraction and choosing different solutions and designing experiments that would apply to any branch of engineering". He notes the importance of "teamwork and communication skills" in engineering practice. There is also a view that "because technology is moving at such a rapid rate" that engineering education should teach students "how to learn for themselves and also give them an appreciation or an awareness of the importance of certain skills" rather than "packing more into the curriculum". This view is supported in the research literature in section 2.5 where it is maintained that rather than "passing on a fixed body of mathematical knowledge by telling students what they must remember and do … society today needs individuals who can continue to learn, adapt to changing circumstances, and produce new knowledge" (Romberg, 1992). Also a study of undergraduate engineers, in section 2.7.1, found that while students did not remember all mathematical content knowledge in their engineering education, they did develop a foundation that prepared them to "relearn" the material if needed (Cardella, 2007).

Research literature supports the engineers' view that workplace mathematics differs from "textbook" mathematics (Chatterjee 2005; Ernest 2011; Schoenfeld 1992; Winkelman 2009) and that graduates enter the workforce ill-equipped for real-world engineered systems (Dym et al. 2005; Janowski et al. 2008 ; Korte et al. 2008; Nair et al. 2009; Wood 2010; Wulf and Fisher 2002).

The following interview findings give a picture of mathematics required in engineering practice:

- F4.1 Engineers' work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture *thinking*"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions.
- F4.2 Computer solutions are part of engineering practice.
- F6.1 Engineers use a high level of mathematics in their work.
- F6.2 *Statistics and probability* are important in engineering practice.
- F7.1 Engineers' mathematics *thinking* usage is problem solving, big picture thinking, decision making, logical thinking, estimation and confirmation of solution.
- F7.2 Mathematics education contributes to engineers' *thinking* skills development.
- F7.3 Engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage.
- F8.1 Communicating mathematics is an important part of engineers' work.

The main gaps, between engineering education and engineering practice, identified in this study relate to: tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience); communicating mathematics;

statistics and probability; and understanding the mathematics behind computer solutions. Consequences of this mismatch are that engineering education is disassociated from the "real world" and graduate engineers require a further "two to three years" to learn the way experts engage in mathematical practices in "the real world".

7.2.10.5-3 F 10.3: Graduate engineers lack the practical experience required for engineering work

There is a view that graduate engineers are not ready to engineer and that work experience and practical application would help engineering undergraduate students develop the engineering skills that are required in engineering practice. There is also a view that some employers prefer to hire technicians over engineers because of the practical nature of technician education. One engineer is of the view that "life and experience teaches you more," than college and another engineer feels that his ability to do his current job has come from his "experience of working in an engineering environment" in that he learned from people, senior managers, older engineers etc. how they estimated, how they worked out real problems" and how they looked at "the bigger picture". Only one of the twenty engineers interviewed engaged in work practice while in college; she says that from her undergraduate work experience she developed her interest in bio-medical work and she says that having "read some validation procedures", she "got an idea where the maths comes in". Another engineer believes that work experience as part of engineering education would "make all the difference" to graduates. She presents that one may "not be ready" to learn when one has not experienced "enough life" and that learning becomes "more relevant to you as you go along". She asserts that "one key benefit of engineering experience is that students learn quicker".

The concept of tacit knowledge (unwritten know-how carried in the minds of engineers developed through practice and experience), as discussed in the research literature (Ernest 2011; Schoenfeld 1992; Trevelyan 2010a), is similar to the engineers' views that "life and experience teaches you more" than college and an ability to do engineering work comes from the "experience of working in an engineering environment" watching other engineers estimate, work out real problems and how they view "the bigger picture". A study of new engineers in section 2.6.1 also found that graduate engineers "relied on their co-workers and managers to learn the subjective aspects of their work" (Korte et al. 2008). According to Trevelyan in section 2.6.1, the scarcity of systematic research on engineering practice makes it difficult for educators who wish to design learning experiences to enable students to manage the transition into commercial engineering contexts more easily (Trevelyan 2011).

7.3 SUMMARY OF INTERVIEW FINDINGS

The overall interview findings are:

F1.1 Mathematics is different compared to other school subjects.

F1.2 "Good" mathematics teachers communicate mathematics well; they are positive about mathematics and teaching; they know mathematics; they are able to teach a broad profile of students; they illustrate the relevance of mathematics; they are interesting; and they are organised and strict.

F2.1 Teachers, task value (why should I do mathematics?), feelings of success and family, peer and societal influences are key motivators to engage in mathematics learning.

F2.2 Mathematics education contributes positively to engineer's work and confidence in mathematical ability and in mathematical solutions are the main motivators for engineers to use mathematics in their work.

F3.1 Feelings about mathematics is the main influence on engineering career choice.

F3.2: Engineers say that the engineering profession currently has a poor image.

F3.3: Higher level Leaving Certificate mathematics is currently valued as a points earner and not as a stepping stone to engineering careers.

F4.1 Engineers' work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions.

F4.2 Computer solutions are part of engineering practice.

F 5.1 Graduate engineers are not ready to engineer.

F5.2 Majority of engineers become managers.

F6.1 Engineers use a high level of mathematics in their work.

F6.2 *Statistics and probability* are important in engineering practice.

F7.1 Engineers' mathematics *thinking* usage is problem solving, big picture thinking, decision making, logical thinking, estimation and confirmation of solution.

F7.2 Mathematics education contributes to engineers' *thinking* skills development.

F7.3 Engineers' mathematics *thinking* usage is greater than their *curriculum mathematics* usage.

F8.1 Communicating mathematics is an important part of engineers' work.

F8.2 Compared to other professions engineers are not good communicators.

F9.1 The degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* in their organisation.

F9.2 Confidence in mathematical solutions motivates engineers to seek a mathematical approach in their work.

F10.1 Engineers support the high level of mathematics in engineering education.

F 10.2 There is a need for a better match between the mathematics taught in engineering education and the mathematics required in engineering practice.

F 10.3 Graduate engineers lack the practical experience required for engineering work.

In this section the interview findings are discussed with respect to the main research questions and organised as follows:

Page number

7.3.1 What is the role of mathematics in engineering practice?

Engineers maintain that their work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions. Engineers' mathematics requirements range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths".

Engineers get "pleasure" when using mathematics, they are "comfortable with maths and using maths" and they show "confidence in mathematical solutions". However in engineering practice engineers' colleagues have a respect for "maths only to the extent that it is useful". The cost of doing mathematics in engineering practice includes the time required and the risk of being "wrong". For example, one engineer notes that "in engineering there is very seldom a unique solution, there is a balance between the amount of time you can spend on problem solving and the degree of certainty that you can have that the solution you've come up with is the ideal solution". Engineers say that "speed of response" is important in engineering practice and that engineers are required "to look at the figures very quickly and make decisions". There is also a view that using mathematics in engineering practice is an individual activity and engineers have difficulty communicating their mathematical solutions to their work colleagues.

Contrary to the view that engineers don't use higher level mathematics in their work, a majority of engineers in this study use both higher level Leaving Certificate mathematics and engineering level mathematics and much of this mathematics usage is either *connecting* or *mathematising* type usage. Consequently, engineers support the high level of mathematics in engineering education. However they say that "engineering is very broad" and engineering students cannot predict which aspect of mathematics will be relevant to their future careers. Engineers estimate that engineers in general use just ten per cent of the mathematics learnt in university. *Statistics and probability* stands out as one area of mathematics that is important in engineering practice particularly as engineers' decision making process is often based on data analysis and estimation of solutions. In addition to *curriculum mathematics*, all engineers in this study rate their *thinking* usage higher than their *curriculum mathematics* usage in their work. Engineers present that their *thinking* usage comprises of: problem solving; big picture thinking; decision making; logical thinking; estimation and confirmation of solution.

Computer solutions are part of engineering practice however the challenge for engineers is to correctly verify and interpret these solutions. Communicating mathematics is an important part of engineers' work. Engineers say there is "skill in communicating maths"; it is the "craft" of putting the mathematics "into a form that a non-engineer will understand". Engineers are poor communicators and consequences of poor mathematics communication skills are that calculations are "meaningless", the message can be "biased" or "abused" and engineers are left in the "background".

The degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* within their organisation. Engineers' difficulties communicating mathematics reduce the value of mathematics in engineering practice. For low *curriculum mathematics* users a specifically mathematical approach is not necessary and for high *curriculum mathematics* users a mathematical approach is necessary. The low *curriculum mathematics* users say it is "more cost effective" for their engineering companies not to use mathematics" and sometimes engineers who have to make decisions "quickly", do not have "time" to "actually use mathematics" in their work. The technical nature of the top *curriculum mathematics* engineers' work demands a mathematical approach. For example, statistical process control is required in manufacturing environments and "attenuating and dispersion of rain water on sites" is based on mathematical equations. Confidence in mathematical solutions and self-efficacy are factors in engineers' motivation to seek a mathematical approach to a work problem.

There is a view that early in the engineers' careers, *curriculum mathematics* usage is higher and mathematics *thinking* usage is lower compared to later in their careers and that *thinking* usage increases for technical, commercial and management roles over the course of engineering careers because the "higher up you're going in an organisation" the more "permutations" there are to consider and managers "apply the maths not just to engineering, but also to finance, to manpower and to people". There is also a view that the majority of engineers become managers and managers need to "understand the solutions other people are implementing".

There is a belief that graduate engineers are not ready to engineer and that they require "two or three years" of an "initialisation" period in engineering practice after which they are required "to make very important decisions". It is suggested that engineering education would benefit from "more communication between experienced engineers and student engineers". Furthermore there is a mismatch between the mathematics taught in engineering education and the mathematics required in engineering practice. In engineering practice mathematics is used primarily as a tool to estimate and confirm multiple solutions to real problems while in engineering education mathematics is about deriving a unique and exact solution to theoretical problems from first principles. There is also a "pressure to get things done quicker in the real world" compared to university. While engineering education mostly imparts knowledge, engineers' role is "to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker".

Engineers present a view that “life and experience teaches you more” than college and an ability to do engineering work comes from the “experience of working in an engineering environment” watching other engineers estimate, work out real problems and how they view “the bigger picture”. This view is similar to Ernest’s view that mathematics knowledge is either explicit (theorems, definitions) or tacit (personal know how) and that learning takes place in a social context (Ernest 2011). Vygotsky’s theory of learning is also based on the idea that learning is fundamentally a social process whereby knowledge exists in a social context and that learning environments should involve interaction with “ more capable peers” (Vygotsky 1978).

7.3.2 Is there a relationship between student’s experiences with school mathematics and their choice of engineering as a career?

The majority of the engineers say that their feelings about mathematics were the main influence in their decision to choose engineering. Engineers’ strong feelings about mathematics in the context of engineering career choice concern their “ability and enjoyment” of school mathematics. Family support with mathematics learning and positive school mathematics experiences all contributed to engineers’ good feelings about mathematics and consequently their decision to choose engineering. It is the engineers whose family supported their mathematics learning from a young age whose main reason for choosing engineering was their feelings about mathematics. The engineers, whose main reason for choosing engineering was for reasons other than their feelings about mathematics, didn’t get any family encouragement or home support with mathematics. For engineers who had particularly negative school mathematics experiences, their feelings about mathematics did not influence their choice of engineering.

Engineers present that school mathematics focuses on getting the “right answer” whilst other subjects lean towards “subjective analysis”. They contrast their ability to get the “right answer” and full marks in mathematics with other subjects whereby “no matter how much work” one puts into the “subjective” subjects one might not get “full marks”. Engineers enjoy the “feeling of success” provided by the “right answer". There is a sense that mathematics learning is more personal compared to other subjects. Each student learns mathematics "differently" and "every person takes responsibility" for their own understanding. In agreement with Vygotsky's theory of the zone of proximal development, engineers assert that an understanding of each topic is necessary prior to moving on to the next topic. The engineers believe that with rote learning mathematics, students do not experience success instead they "get stuck" and they "fall behind" very quickly. In addition to the knowledge base, the engineers maintain that mathematics is an "activity, it is a "process" of problem solving and/ or application of mathematics and for many students the problem solving nature of mathematics is time consuming.

In agreement with affective theory (Schunk et al. 2010), engineers hold that teachers, task value (why should I do mathematics?), feelings of success and family, peer and societal influences are key motivators to engage in mathematics learning. Engineers say that teaching is the "number one" factor in mathematics education and good mathematics teachers transform students' mathematics learning and their enjoyment of the subject. The ability to communicate mathematics is the predominant characteristic of good mathematics teachers. While one engineer's mathematics teacher was "excellent" because he "just connected with people through maths" the "plain ordinary bad" teacher "just could not explain the consequence" of any mathematics topic". Good mathematics teachers are also "positive" about mathematics and they are "enthusiastic to the point" where they "can foster interest and enthusiasm for the subject with a broad profile of students within the classroom". Teachers' own attitudes to mathematics contribute to students' affective engagement with the subject. Engineers believe that there are many "unqualified" mathematics teachers who are neither confident nor positive in their teaching of mathematics and who also fail to communicate the value of mathematics to students.

For some engineers the task value of mathematics (why should I do mathematics?) is evident where from a very young age when they enjoyed "mathematical type game playing" and engaged in authentic mathematical tasks in the home. In school, getting "the correct answer" is the key value of mathematics learning whereby engineers enjoy the recognition associated with success and consequently they were motivated to engage further with mathematics. The costs (perceived negative aspects of engaging in mathematics) include the time required to get “the correct answer” and the fear of getting the “wrong” answer. One engineer says he risked passing his Leaving Certificate exam because mathematics consumed more than half his allocated homework time period. A further cost of school mathematics is the lack of relevance of mathematics teaching to everyday life. Students view higher level Leaving Certificate mathematics in terms of both the value of CAO points and the cost of the effort required to take the higher level option. Engineers are also of the view that society does not value mathematics sufficiently and it is generally accepted by society that only a minority of students take higher level Leaving Certificate mathematics.

Engineers maintain that collaborative learning opportunities assisted their school mathematics learning. Advantages of peer mathematics learning include: the “comfort and positivity” of peers towards numerate subjects; compensation for poor teaching; playing “football together because nobody else would play football” with mathematics “geeks”; turning Leaving Certificate mathematics into this “fun thing”; and motivating students to “get an A in Leaving Certificate mathematics”. However engineers also present that there is a stigma associated with being good at mathematics and that being good at school mathematics causes social problems for students who consequently try to hide “the guilty pleasure of enjoying maths”.

Engineers say that the feeling of success is the main contributor to enjoyment of school mathematics and that confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. It is these feelings that influence students to choose engineering as a career. For example, one engineer’s career choice was influenced “a very great deal” by “love” of mathematics, he says engineering and mathematics “were hand in hand, I had very much an aptitude for mathematics in school, that’s the subject that I found easier, that’s the subject that I didn’t have to study and to me the engineering followed on from that”.

It is outside the scope of this study to determine if a mathematics-phobia exists that scares people away from engineering careers. However it is observed that two of the twenty engineers interviewed do not have higher level Leaving Certificate mathematics. Another three engineers took an engineering education route where higher level Leaving Certificate mathematics was not an admission requirement. Another engineer, who chose engineering because she "always wanted to build bridges", says she "had to do it [higher level Leaving Certificate mathematics] by hook or by crook in whatever way I could remember it to get a C in the honours exam". A further engineer, who had a negative Leaving Certificate mathematics experience, says his reasons to become an engineer had nothing "to do with love of maths" and he adopted a view that mathematics "is just one subject" and that one needs "other attributes to be a good engineer".

CHAPTER 8: CONCLUDING DISCUSSION

8.1 INTRODUCTION

This chapter discusses the survey findings and the interview findings in the context of the two main research questions. From a discussion of both the survey and interview findings the overall research findings are presented. This chapter identifies the contributions to knowledge and also explores the implications of this new knowledge. Limitations of the methodology employed are discussed. Suggestions for further work are included. This chapter is organised as follows:

Page number

8.2 USING THE INTERVIEW ANALYSIS TO BUILD ON THE SURVEY FINDINGS

In a sequential explanatory strategy mixed methods study, qualitative findings build on the survey findings. In this section the two main research questions are discussed with respect to both the five survey findings and the interview findings.

8.2.1 Is there a relationship between students' experiences with school mathematics and their choice of engineering as a career?

8.2.1.1 Survey finding # 1: Engineers' feelings about mathematics are a major influence on their choice of engineering as a career

Three quarters (75.9%) of the engineers who participated in the survey say that their feelings about mathematics impacted their choice of engineering as a career in the range “quite a lot” or “a very great deal”. A further 12.3% of engineers say that their feelings about mathematics impacted their choice of engineering career “a little”. It is just 4.1% of engineers whose feelings about mathematics impacted their choice of engineering as a career “very little” or “not at all”. Overall engineers’ feelings about mathematics impacted their choice of engineering as a career “quite a lot” (3.97 Likert units[34]).

The interview findings also confirm that feelings about mathematics are a major influence on engineering career choice. Engineers say that the feeling of success is the main contributor to enjoyment of school mathematics and that confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. Engineers’ confidence in their mathematics ability is the main influence on engineering career choice. For one engineer the key to mathematics learning is “finding that you are able to do it” and the sense of achievement another engineer experienced when he solved a difficult problem spurred him “to do more” mathematics. Engineers whose feelings about mathematics impacted their choice of engineering career were motivated to engage in more mathematics learning and they say that engineering education was “a logical progression” and “a very natural progression from one education phase into the next education phase”. The main finding in research literature concerning engineering career choice relates to women’s mathematical self-efficacy which is significantly lower than men’s perceptions of their capability to succeed in mathematics. This is the main reason why so few women compared to men choose engineering careers (Correll 2001; Løken et al. 2010; Zeldin and Pajares 2000). Betz & Hackett (1981) suggest that women’s lower self-efficacy expectations, with regard to occupations requiring competence in mathematics, may be due to “a lack of experiences of success and accomplishments, a lack of opportunities to observe women competent in math, and/or a lack of encouragement from teachers or parents” (Betz and Hackett 1981). In this study, it is found that the feeling of mathematics success motivates school leaving students to choose engineering careers.

[34] Likert units: Units on 5 point Likert scale, 1 = “not at all”, 2 = “a little”, 3 = “very little”, 4 = “quite a lot”, 5 = “a very great deal”.

While only twenty engineers were interviewed in this study, the interview data also shows that: (i) there is a high degree of correspondence between engineers whose family supported their mathematics learning from a young age and the engineers whose main reason for choosing engineering was their feelings about mathematics; (ii) engineers, whose main reason for choosing engineering was for reasons other than their feelings about mathematics, didn't get any family encouragement or home support with mathematics; and (iii) engineers who had negative school mathematics experiences say that their feelings about mathematics did not influence their career choice.

Two engineers in this study had bad school mathematics experiences and they were not scared away from engineering careers. For one of these engineers, whose Leaving Certificate mathematics teacher was "plain ordinary bad", higher level mathematics was a "career requirement" and his interest in engineering as a career motivated him to continue with higher level mathematics in school. However his "lack of maths caught" him all the way through college where he "endured the maths" and subsequently in engineering practice he only used mathematics that he "was confident about". The other engineer, whose Leaving Certificate mathematics teacher was a "manic depressive", says "I had to do it [higher level Leaving Certificate mathematics] by hook or by crook in whatever way I could remember it to get a C in the honours exam". She is currently an engineering manager and is a high user of both *curriculum mathematic*s and *mathematical thinking* in her work.

All twenty engineers interviewed are unanimous in the view that the "teacher is biggest influence" on students' relationships with mathematics. Concerns about mathematics teaching include: teachers' attitudes where mathematics is presented as a "hard" subject and where students "feel they can't do maths"; lack of relevance in mathematics teaching to real world applications; and "unqualified" mathematics teachers who are neither confident nor positive in their teaching of mathematics. Engineers' view is that teachers, task value (why should I do mathematics), feelings of success and peer and societal influences are key motivators to engage in mathematics learning.

8.2.1.2 Survey finding # 2: Teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics

80% of the engineers who participated in the survey enjoyed mathematics in school at the levels of "quite a lot" and "a very great deal". The overall mean value of engineers' enjoyment of school mathematics is "quite a lot" (4.11 Likert units).

Survey analysis shows that the teacher is the main factor that contributed to engineers' interest in and learning of mathematics from primary school through to Leaving Certificate. Affective factors such as success (self-efficacy), enjoyment (value), practical applications (value), interest (value), problem solving (metacognitive activity), relevance to science (value), required for engineering (value), careers (value) and points (value) also contribute to engineers' mathematics learning in school. Outside of school, family and parents (sociocultural influences) are a very big influence on engineers' mathematics learning.

When asked, how young people's affective engagement with mathematics could be improved, 92% of the engineers' views in the survey data relate to teacher or teaching. Engineers present that teachers should teach mathematics that illustrates the task value of mathematics. This includes: the usefulness of mathematics; the relevance of mathematics to modern living; mathematics that is used in various careers; and mathematics that has links with other school subjects. Engineers also maintain that "teachers must have the skills, enthusiasm and ability necessary to teach the subject" and engineers further maintain that teachers have a responsibility to correct the "stigma about the difficulty of higher level maths". Engineers say that "much of the problem sadly lies with" unqualified teachers. While the majority of engineers' views relate to affective variables, engineers also maintain that "a strong reason for students not enjoying maths is that they don't understand it" and they advocate that mathematics teaching should place "more emphasis on understanding".

A review of engineers' additional voluntary comments in the survey shows that more than half (52%) of the comments relate to task value. In their comments the engineers list benefits of mathematics education and how an awareness of these benefits would encourage students in their mathematics learning. Engineers are also of the view that sociocultural influences, both positive and negative, from families, teachers and peers significantly impact mathematics learning. In particular, the engineers express strong views about teachers' requirements for love and understanding of mathematics. Given that, in their voluntary comments in the survey, engineers associate mathematics and mathematics learning with values, attitudes, beliefs, self-efficacy, emotions and sociocultural influences, it is concluded that mathematics is a highly "affective subject".

Interview analysis reinforces the survey findings whereby teachers, task value (why should I do mathematics?), feelings of success and peer and societal influences are key motivators to engage in mathematics learning. In the interview data, engineers maintain that mathematics is different to most other subjects and that teachers are critical to successful mathematics learning and students' enjoyment of the subject. One difference between mathematics and many other subjects is that mathematics focuses on getting the "right answer" whilst other subjects lean towards "subjective analysis". A single "right answer" is regarded as an advantage of mathematics learning as students can objectively check their work which is a type of instant feedback. Compared to other subjects, mathematics learning is personal; one engineer asserts that the key to mathematics learning is "finding that you are able to do it" and this "unique skill doesn't come up much in any of the other subjects". Mathematics learning is "based on building on the fundamentals" and an understanding of each topic is necessary prior to moving on to the next topic. Engineers maintain that rote learning mathematics does not work and without an understanding of concepts and situations students "get stuck" and they "fall behind" very quickly. Compared to other subjects, mathematics is a diverse subject; in addition to its knowledge base, engineers say mathematics is an "activity, it is a "process" of problem solving and/ or application of mathematics. The problem solving nature of mathematics is time consuming because mathematics learning is "a lot about practice, it is about "trying to figure the stuff out" and students spend considerable amounts of their homework time "looking for a specific answer".

All twenty engineers interviewed are unanimous that “teacher is biggest influence” on students’ relationships with mathematics. The four engineers who don’t express any enjoyment of their school mathematics and who also had low confidence in their mathematics ability all had poor mathematics teachers. One engineer stands out in terms of his low confidence in his school mathematical ability. He says that due to “bad” teaching, he developed an “inferiority complex about maths” and a “blockage” to learning mathematics in secondary school that “caught” him all the way through college and work. In her Leaving Certificate year another engineer moved away from her “manic depressive” teacher to a grind school where her new mathematics teacher “totally revitalised her feelings of what maths was about”.

According to the engineers interviewed good mathematics teachers transform students’ mathematics learning and their enjoyment of the subject. Engineers say the ability to communicate mathematics is the predominant characteristic of good mathematics teachers. While one engineer’s mathematics teacher was “excellent” because he “just connected with people through maths” the “plain ordinary bad” teacher “just could not explain the consequence” of any mathematics topic”. A good mathematics teacher is “positive” about mathematics and he/ she is “enthusiastic to the point where he can foster interest and enthusiasm for the subject with a broad profile of students within the classroom”. Engineers are of the view that teachers need to be more positive about mathematics and “the idea that maths is actually something that a lot of people will enjoy” might get children started with mathematics and if they discover that they are “good at it” they might enjoy it more and “stick with it”. Good teachers “should encourage students to stay with it [mathematics]” and with good teaching students would “grasp the maths, understand it and feel good about it rather than just learn it off by heart”. On the other hand “bad” teachers” have poor attitudes and they often label specific parts of the course as “too hard” and they do not teach the entire syllabus. Engineers are of the view that teachers’ own attitudes to mathematics contribute to students’ affective engagement with the subject and that the many “unqualified” mathematics teachers in the early years of secondary school are neither confident nor positive in their teaching of mathematics. The engineers’ views are supported by Pape, Bell and Yetkin (2003), in the research literature in Chapter 2, who maintain that the teachers' role is to "establish the context for mathematical development" and to scaffold students' developing skills by presenting tasks that encourage students to value and enjoy mathematics and to articulate their thinking. By articulating their thinking over time, students learn to monitor their thinking and consequently they develop mathematical reasoning skills (Pape, Bell et al. 2003). Research literature also confirms that teachers' attitudes and beliefs about mathematics and teaching mathematics are a big influence on students' mathematics learning (Ernest 2011; Koehler and Grouws 1992; National Research Council 1989; Schoenfeld 1992; Schunk et al. 2010). Engineers believe that if students "feel they can't do maths they are just not going to do maths" and many students "going into secondary school have already decided to do ordinary level mathematics for their Junior Certificate exam". Thus, according to the engineers, many students are lost to engineering at a very young age. Task value (why should I do mathematics?) is a recurrent topic in this study and engineers say that mathematics teachers fail to communicate the value of mathematics and they also fail to demonstrate real world applications to students.

There is a strong view amongst the engineers that society is tolerant of "bad" mathematics teachers in Ireland in both primary and secondary schools. One engineer argues that "society needs to set certain expectations for kids coming out of school" and that mathematics teachers need to be accountable for achieving those expectations. Similarly Schoenfeld (1992), in Chapter 3, is of the view that teachers' beliefs are formed by their own schooling experience and the same beliefs are apparent in successive generation of teachers, which he calls a "vicious pedagogical/epistemological circle" (Schoenfeld 1992).

In the interview data engineers identify task value and feelings of success as affective factors contributing to engineers' interest in and learning of mathematics. In Chapter 3, Schunk, Pintrich and Meece (2010) say that goal setting is a key motivational process and learners with a goal and a sense of self-efficacy for attaining a goal engage in activities they believe will lead to attainment (Schunk et al. 2010). The engineers' "goal" was to get the "correct answer" in school mathematics. One engineer "persisted" until he "worked out the answer", another engineer says "I kept at it until I got the right answer" and a further engineer says she was "diligent", "methodical" and she would also "go back" over her work and she "filled in units" to verify that equations were "correct". Getting "the correct answer" was a key value of mathematics education for engineers as they enjoyed the recognition associated with success and consequently they were motivated to engage further with mathematics. Engineers say that confidence in school mathematics stems from recognition of success such as latest test grades, getting top marks or being the best in the class. From the "satisfaction" of getting the "right answer" one engineer says "I got confidence in the fact that I was getting good results in mathematics and then I realised this is something that I could be good at". Another engineer asserts that the key to mathematics learning is "finding that you are able to do it". The sense of achievement experienced by one engineer when he solved a difficult problem spurred him "to do more". Similarly Ernest, in Chapter 3, maintains that success at mathematical tasks leads to pleasure and confidence and a sense of self-efficacy and the resultant improved motivation leads to more effort and persistence (Ernest 2011).

In Chapter 3, it is claimed that sociocultural influences are a big influence on engineers' mathematics learning and subsequent motivation to use mathematics (Zeldin and Pajares 2000). From the interview data it is apparent that families, peers and society are all factors in students' motivation to engage in mathematics learning. Some engineers' families provided support and scaffolding for their mathematics learning where they "regularly discussed maths problems" and other related topics such as "methodology", "the right answer" and "negative views" about mathematics. Engineers present that engaging in social or group learning of mathematics with peers or role models has many advantages for students preparing for the Leaving Certificate mathematics exam. Advantages of having friends who are positively disposed to mathematics learning include: the "comfort and positivity" of peers towards numerate subjects; compensation for poor teaching; playing "football together because nobody else would play football" with "geeks"; turning Leaving Certificate mathematics into this "fun thing" and motivation to "get an A in Leaving Certificate mathematics". However engineers are also of the view that there is a general belief in society that mathematics is difficult and there is a stigma associated with being good at mathematics. One engineer is of the view that a "them and us culture" happens at quite an early age when "people decide that they can't do it [mathematics]" and "that the people who do it are somehow different from them". Being good at mathematics causes social problems for students, they feel "isolated", they hide "the guilty pleasure of enjoying maths" and they try to change their personality or appearance so as "not to be branded a geek".

8.2.2 What is the role of mathematics in engineering practice?

8.2.2.1 Survey finding # 3: While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, *mathematical thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*

In the survey engineers rate their mean mathematics usage for the 75 domain-level-usage combinations of *curriculum mathematics* as 2.73 Likert units. Survey analysis shows that almost two thirds of engineers (64.4%) use higher level Leaving Certificate mathematics in their work either "a little", "quite a lot" or "a very great deal". 57.3% of engineers use engineering mathematics and 41.4% of engineers use B.A./B.Sc. mathematics to the same degree.

Engineers rate their *mathematical thinking* usage as 4.02 Likert units which is considerably higher than their overall mean *curriculum mathematics* usage (2.73 Likert units), with a magnitude of the difference between 1.15 and 1.43 Likert units. *Thinking* usage is highest (4.19 Likert units) when engineers are within 2 years of graduating and reduces thereafter. The modes of *thinking* resulting from mathematics education, that influence engineers' work performance are: problem solving strategies (26.4%), logical thinking (26.2%); critical analysis (7.2%); modelling (7.2%); decision making (6.3%); accuracy/ confirmation of solution (4.8%); precision/ use of rigour (4.6%); organisational skills (4.6%); reasoning (3.6%); communication/ teamwork/ making arguments (3.2%); confidence/ motivation (3.1%); numeracy (2.2%); and use of mathematical tools (0.7%).

Interview data confirms that both higher level Leaving Certificate mathematics and engineering level mathematics are required in many engineers' work and that much of engineers' mathematics usage is at the higher types of connecting and mathematising. There is a view that engineers in general use just ten per cent of the mathematics learnt in university and the difficulty for engineering education is "figuring out which ten per cent for each individual".

The interview data shows that *statistics and probability* is often neglected in some engineers' education. One engineer says he was "never mad into statistics" and he prefers "concrete" problems that have an "exact answer". Another engineer didn't see the point of *statistics and probability*" and she omitted it from her Leaving Certificate preparation. A further engineer, whose teacher chose not to include statistics in the Leaving Certificate teaching, says that due to the nature of his engineering work he took up a statistics course after becoming an engineer. From the interview data it is apparent that *statistics and probability* is important in engineering practice. In particular estimation of solutions and an ability to understand data is required in all areas of engineering practice. Similarly in a study of the early work experiences of recent engineering graduates it was found that interpreting data was a new experience for many engineers (Korte et al. 2008).

All engineers interviewed rate their *mathematical thinking* usage higher than their *curriculum mathematics* usage in their work. For one engineer *thinking* usage is the "value" he brings to his job and another engineer says that *thinkin*g usage is "where it's all at … to me this is absolutely critical". Engineers present that their *thinking* usage comprises of: problem solving; "big picture thinking"; decision making; logical thinking; estimation and confirmation of solution. Problem solving is a major part of engineers' mathematics thinking usage. Engineers say that engineering problems have multiple answers and that their job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer. Big picture thinking is the term engineers use to describe mathematical thinking in "real world" engineering where engineers need to "have a real tangible understanding of the effect of one piece of work on another part of the system". It is defining a problem or identifying a question that meets the overall "objective" and “the overall concept of a situation”. According to the engineers, “engineering should be about trying to identify the right question, because a lot of the times, people are obsessing over the wrong question”. These findings are similar to the findings in a study of new engineers described in Chapter 2 where the new engineers describe their work as a “problem-solving process or way of thinking” where they try to “organise, define, and understand a problem; gather, analyse, and interpret data; document and present the results; and project-manage the overall problem-solving process” (Korte, Sheppard et al. 2008).

Engineers view the association between mathematics and thinking as “indirect” where thinking is how engineers use mathematics rather than the actual mathematics they use. They say that, when learning mathematics: doing “things in a particular order … teaches logical thinking”; the practice of working around a problem and getting “your brain going in different ways … transfers into other things that you do”; the emphasis on getting the right answer teaches one to “double check on everything”; and the discipline of “organising your study and the time it took to do your honours Leaving Certificate maths” is “something you bring through college and into to your working life”.

There is a view that early in the engineers’ careers, *curriculum mathematics* usage is higher and mathematics *thinking* usage is lower and that *thinking* usage increases for technical, commercial and management roles over the course of engineering careers. The engineer, whose *curriculum mathematics* is highest of all the engineers interviewed, says that his thinking usage is “probably higher than his *curriculum mathematics* usage because his role is management orientated and he has “to apply the maths not just to engineering, but also to finance, to manpower and to people”. Engineers maintain that graduate engineers with their “black and white solutions” are not ready to engineer. An ability to do engineering work comes from the “experience of working in an engineering environment” watching other engineers estimate, work out real problems and how they view “the bigger picture”. One engineer claims it took her four years to become an “independent thinker”. This is consistent with the views documented in Chapter 2 where newly graduated engineers are not ready to engineer (Korte et al. 2008; Trevelyan 2011).

The interview data also shows that computer solutions are widely used in modern engineering practice. Engineers say that computational tools have many advantages in engineering practice because they bypass the need to write down the fundamental engineering equations and solve them and they offer a standard methodology for developing solutions within organisations. Most engineers say they use Excel. Engineers note that results produced by computational tools can easily be misinterpreted. One engineer presents that using computational tools is "a different type of mathematics" and he is of the view that "the engineer should understand how the program is solving the equations and what it is doing, because it is always dangerous not to".

8.2.2.2 Survey finding # 4: Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and engineering role. Their mathematical *thinking* usage is independent of engineering discipline and engineering role

Survey analysis shows that the effect of engineering discipline and engineering role on engineers' overall mean *curriculum mathematics* usage depends on the other factor (role or discipline respectively). Neither engineering discipline, engineering role nor, the interaction of engineering discipline and role, has an effect on engineers' *mathematical thinking* usage.

One explanation for this as presented in the interview data is that engineers' work is diverse and that engineering roles are "so broad" that engineers to some extent lose their engineering discipline identity. Engineers say they are easily transferrable from one role to another within an organisation. Many engineers engage in the "social side" of engineering where they spend ninety per cent of their working day doing "project management and problem solving" tasks. Furthermore "engineers to a very large extent are influenced to move into management by the necessity to obtain financial reward". A second explanation is that there are tiers of *curriculum mathematics* requirements in engineering practice that range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths". Data analysis is one type of mathematics required in all engineering areas (engineering disciplines and roles) to inform engineering decisions. A third explanation is that many engineering problems cannot be formulated mathematically. In many situations "real world" applications involve "bigger picture thinking" (logical thinking about the complete project) and communicating the solution. A fourth explanation is that much of the mathematics required in engineering practice is done by software and the challenge for engineers is to correctly interpret computer solutions rather than do mathematics. It is noted in the research literature in Chapter 2 that the increasing availability of computerised tools and resources is contributing to the changing nature of engineering where IT (information technology) tools are dominating modern engineering practice (Anderson et al. 2010; Grimson 2002).

8.2.2.3 Survey finding # 5: Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation

Almost three quarters (74.0%) of the engineers who participated in the survey say that they enjoy using mathematics in their work either "quite a lot" or "a very great deal". Over 80% (80.6%) of the engineers surveyed feel confident dealing with mathematics in their work either "quite a lot" or "a very great deal. Engineers rate the degree they feel confident dealing with mathematics in their work: considerably greater (by 1.16 to 1.43 Likert units) than their overall *curriculum mathematics* usage; greater (by 0.31 to 0.52 Likert units) than the degree they actively seek a mathematical approach in their work; and also greater (by 0.07 to 0.23 Likert units) than the degree they enjoy using mathematics in work. The gap between engineers' confidence dealing with mathematics in their work and both their *curriculum mathematics* usage and the degree engineers seek a mathematical approach suggests that in addition to confidence, there are other factors that impact engineers' use of *curriculum mathematics* in work.

The survey data shows that engineers “love the challenge in solving problems mathematically”, they enjoy “the satisfaction of a result”, they find it easier to communicate using mathematics compared to words and they prefer “a 100% right answer rather than the ambiguity of non-mathematical solutions”. For the engineers who enjoy using mathematics in work, there is a sense that mathematics is “part of who” they are. Memories of school mathematics are the main reason engineers do not enjoy using mathematics in work. For example, one engineer has an “in built hatred of mathematics from secondary school”. Engineers’ “grounding” in mathematics and subsequent usage are two major confidence influencers. For many engineers high mathematical self-efficacy develops in school where engineers learn to check their answers and where they are “in the habit of getting 100% in maths and maths-based exams”. Engineers who have high confidence in using mathematics also show high confidence in mathematics solutions and in the “logical and objective nature of maths”. These engineers note the need to “check if a solution is correct” and they are also of the view that “there is no reason for ambiguity in maths; there is only a right or wrong answer”. Low confidence mathematics engineers avoid mathematics in their work while high confidence mathematics engineers readily “revise and brush up” on the required mathematics.

Interview analysis also shows that engineers’ confidence in their mathematical ability grew from recognition of success in school mathematics such as their latest test grades, getting top marks or being the best in the class. For one engineer the “sense” of getting “the answer right” and knowing that he had “the right answer” was “very direct gratification”. Another engineer asserts that the key to mathematics learning is “finding that you are able to do it” and this “unique skill doesn’t come up much in any of the other subjects. A further engineer says “I got confidence in the fact that I was getting good results in mathematics and then I realised this is something that I could be good at”. Due to “the very poor grounding” one engineer “got in maths” he says he “was afraid of some of” the mathematics he encountered in engineering practice and he has “a nagging fear that” he has “got something wrong” in his work. When he encounters a mathematics problem, he “refers” to his work colleagues.

Engineers' confidence in mathematical solutions in work is very evident in the interview data. Engineers like getting an "exact solution" and they tend to "double check" the mathematics before presenting a solution to co-workers. For one engineer mathematics is "a safety valve" in his work. Another engineer always chooses the "maths way" of doing things because mathematics is "very easy to reference and verify". Another engineer says that mathematics "is clean … it is completely logical, … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it".

Almost two thirds (64.6%) of the engineers who participated in the survey are of the view that a specifically mathematical approach is necessary in engineering practice either "quite a lot" or "a very great deal". Similarly, almost two thirds (63.3%) of engineers say that they actively seek a mathematical approach either "quite a lot" or "a very great deal". The overall mean rating for the degree engineers actively seek a mathematical approach in their work is in the range "a little" to "quite a lot" (3.62 Likert units). Engineers rate the degree they actively seek a mathematical approach in their work considerably greater (by 0.73 to 1.05 Likert units) than their *curriculum mathematics* usage and less than their *thinking* usage (by 0.26 to 0.51 Likert units). The value of engineers' engagement with mathematics in their work includes the usefulness of mathematics "for explaining results to others" and engineers' confidence in mathematics solutions. Costs of their mathematics engagement are the availability of sufficient ready-made solutions and "taking a mathematical approach may be risky and slow".

Only 3.9% of the engineers who participated in the survey say that they had a negative experience when using mathematics either "quite a lot" or "a very great deal". The majority of engineers, due to confidence in their mathematical ability and mathematical solutions, say that they did not have any negative experience using mathematics in the previous six months. However the "quirkiness of computational tools" and their "lack of understanding" and "over reliance of computer analysis" sometimes generate errors. For some engineers, mathematics consumes too much time, for example one engineer says "occasionally I have spent a long time trying to shoehorn something into mathematical language and failed, which was frustrating".

The greatest reason attributed by the engineers surveyed to negative experiences using mathematics relates to communicating mathematics and the negative feelings resulting from their colleagues' lack of understanding and consequently engineers' difficulty influencing business decisions. It is interpreted that when graduate engineers make the transition from an education environment where mathematics has high importance to engineering practice where many of their work colleagues do not understand mathematics and where there is less time to engage in mathematics that graduate engineers experience a reduction in motivational influences to use mathematics. While mathematics is "part of who" engineers are and while engineers prefer to communicate using mathematics, the task value of mathematics reduces when engineers move from engineering education into engineering practice where they encounter an affective hurdle.

Interview analysis also shows that confidence in mathematical ability and in mathematical solutions are the main motivators for engineers to use mathematics in their work. However engineers say that engineering is much more than mathematics. They say that there are tiers of mathematics requirements in engineering practice that range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths". Given the diversity of their work, engineers estimate that mathematics is "valuable" in only ten per cent of their work. While a majority of the engineers interviewed are of the view that a specifically mathematical approach is not necessary in their work, at the same time a majority of these engineers say they use aspects of either higher level Leaving Certificate mathematics or engineering level mathematics in their work and they also use *curriculum mathematics* in either *connecting* or *mathematising* ways. From the interview analysis it is interpreted that *curriculum* mathematics is a small proportion but necessary part of engineers' work and engineers view mathematics as *curriculum mathematics* usage and not mathematics *thinking* usage which is significantly greater than *curriculum mathematics* usage for all engineers interviewed.

One explanation for the gaps between engineers' confidence dealing with mathematics in their work and the degree they actively seek a mathematical approach in their work and their overall *curriculum mathematics* usage is given in the interview analysis. There, engineers suggest that the necessity of a specifically mathematical approach in engineers' work is related to the value given to *curriculum mathematics* in engineering practice. For example, in one engineer's company, it is "more cost effective" not to use mathematics and in another company engineers don't have "time" to "actually use mathematics". A further engineer claims that he "wouldn't be thanked" for using mathematics. Colleagues' respect for mathematics is a factor in the value of mathematics in engineering practice. For example, one engineer says that in his company there is a respect for "maths only to the extent that it is useful". Another engineer is of the view that the "respect for mathematics" in his company "seems to change as the management changes … the emphasis is on sales and marketing and away from the maths right now". Difficulty communicating mathematics reduces the value of mathematics in engineering practice. Engineers say there is "skill in communicating maths". It is the "craft" of putting the mathematics "into a form that a non-engineer will understand". Consequences of poor mathematics communication skills are that calculations are "meaningless" and the message can be "biased" or "abused". Compared to other professions, engineers say they are not good communicators and a consequence of poor mathematics communications is that engineers are left in the "background". One engineer asserts that "if engineers are to survive then they need to somehow harness communication skills". Another engineer asserts that if one doesn't "bring the problem and the solution to people in their language", mathematics becomes "elitist". Ernest has a similar view, he states that the perception of mathematics "in which an elite cadre of mathematicians determine the unique and indubitably correct answers to mathematical problems and questions using arcane technical methods known only to them" puts "mathematics and mathematicians out of reach of common-sense and reason, and into a domain of experts and subject to their authority. Thus mathematics becomes an elitist subject of asserted authority, beyond the challenge of the common citizen" (Ernest 2009).

8.3 DISCUSSION OF SURVEY AND INTERVIEW FINDINGS

8.3.1 Mathematics is a highly affective subject

Both the survey and interview findings confirm that feelings about mathematics are a strong influence on engineering career choice. In both sets of data engineers present mathematics as a highly "affective subject" where motivational beliefs such as affective memories (previous emotional experiences with mathematics), task value (why should I do mathematics?) and expectancy (am I able to do mathematics?) influence their engagement with mathematics. Throughout the engineers' education the task value of mathematics is mainly associated with engineers' feeling of success when they get the correct answer. Costs of learning mathematics include: the wrong answer; time requirements; lack of relevance/ usefulness; lack of respect for mathematics shown by peers and society and poor mathematics communication skills. Mathematics education that neglects the affective domain has consequences for both mathematics learning and for engineering career choice. When engineering graduates move from education to work environments they encounter an affective hurdle where they have difficulties communicating mathematics to non-mathematically competent people and mathematical solutions are consequently bypassed in decision making. While engineers say that the ability to communicate mathematics is an important skill for engineers themselves, they also maintain that it is the predominant characteristic of good mathematics teachers. Engineers hold mathematics teachers accountable for the lack of relevance in mathematics teaching to everyday life.

8.3.2 The focus on "objective" solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice

From both sets of data in this study it is apparent that *curriculum mathematics* is different to mathematics used in engineering practice. Solving real world engineering problems is more about how engineers use mathematics rather than the actual mathematics they use. According to one engineer, an engineers' role is "to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker". Engineers have a view that an ability to do engineering work comes from the "experience of working in an engineering environment", watching other engineers estimate, work out real problems and how they view "the bigger picture". Graduate engineers lack this tacit knowledge. This view is reinforced in the research literature in Chapter 2 (Korte et al. 2008; Trevelyan 2011).

It is concluded in this study that the focus on "objective" solutions in mathematics education at the expense of "subjective analysis" or tacit knowledge contributes to engineer's poor communication skills and reduces the value of mathematics in engineering practice thus creating an affective hurdle for graduate engineers to overcome when they begin working as engineers. It could be argued that engineer's confidence in mathematical solutions restricts their vision of engineering solutions. For example, one engineer presents that she enjoys using mathematics in work because "it is clean … it is completely logical … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it". However engineers maintain that "real life" engineering problems are "bigger" than mathematics, they have multiple answers and an engineer's job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer. This is supported in the research literature where it is maintained that "the unique charm of mathematics in engineering lies in the many levels and forms in which it is evoked, revoked, used, abused, developed, implemented, interpreted and ultimately put back in the box of tools, before the final engineering decision, made within the allotted resources of time, space and money, is given to the end user" (Chatterjee 2005). In both the survey and interview data analysis, a diversity of practising engineers highlight the importance of mathematics *thinking* usage in their work compared to *curriculum mathematics*. Mathematics *thinking* knowledge is a type of tacit knowledge, this is "unwritten know-how carried in the minds of engineers developed through practice and experience" (Trevelyan 2010a) and it differs from school mathematics (Ernest 2011; Schoenfeld 1992; Trevelyan 2010a; Trevelyan 2010b).

Engineers' task value of mathematics developed in school where the feelings of success associated with getting "the correct answer" made "quantitative" subjects more enjoyable than "qualitative" subjects. This is a major influence on engineering career choice. Furthermore engineers bring their confidence in mathematical solutions with them into the world of engineering practice where many engineers are also motivated to get the “exact solution” at the expense of engaging in mathematics *thinking* and effective mathematics communications. However in engineering practice mathematics is required to estimate and confirm multiple solutions to real problems unlike engineering education where mathematics is about deriving unique and exact solutions to theoretical problems from first principles. Engineers demonstrate an over-attachment to “objective” solutions at the expense of “real world” solutions. “Objective” solutions have limited value in engineering practice particularly when engineers have difficulty communicating mathematics. However while there is “seldom a unique right answer in engineering”, engineers prefer “a 100% right answer rather than the ambiguity of non-mathematical solutions”. This suggests a further finding that the focus on “objective” solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice. This finding has consequences for both mathematics education in secondary schools and in engineering education where tacit knowledge is neglected at the expense of “objective” knowledge. There is evidence in the research literature that learning mathematics in a social context enables students to enhance the tacit knowledge required in the workplace situations (Ernest 2011). It is concluded that the mathematics taught pre- and during engineering education could be better matched to the mathematics required in engineering practice.

8.4 CONTRIBUTIONS TO RESEARCH KNOWLEDGE

This research was inspired by the observation of the declining number of students entering professional engineering courses and the lacuna of information in the research literature concerning the research questions in this study:

1. What is the role of mathematics in engineering practice?
2. Is there a relationship between students’ experiences with school mathematics and their choice of engineering as a career?

The contributions to research knowledge arising from this study are centred around six findings, Figure 8-1 and these are:

1. Engineers' feelings about mathematics are a major influence on their choice of engineering as a career.

2. Teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics.

3. While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, *mathematical thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*.

4. Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and engineering role. Their *mathematical thinking* usage is independent of engineering discipline and engineering role.

5. Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation.

6. The focus on "objective" solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice.

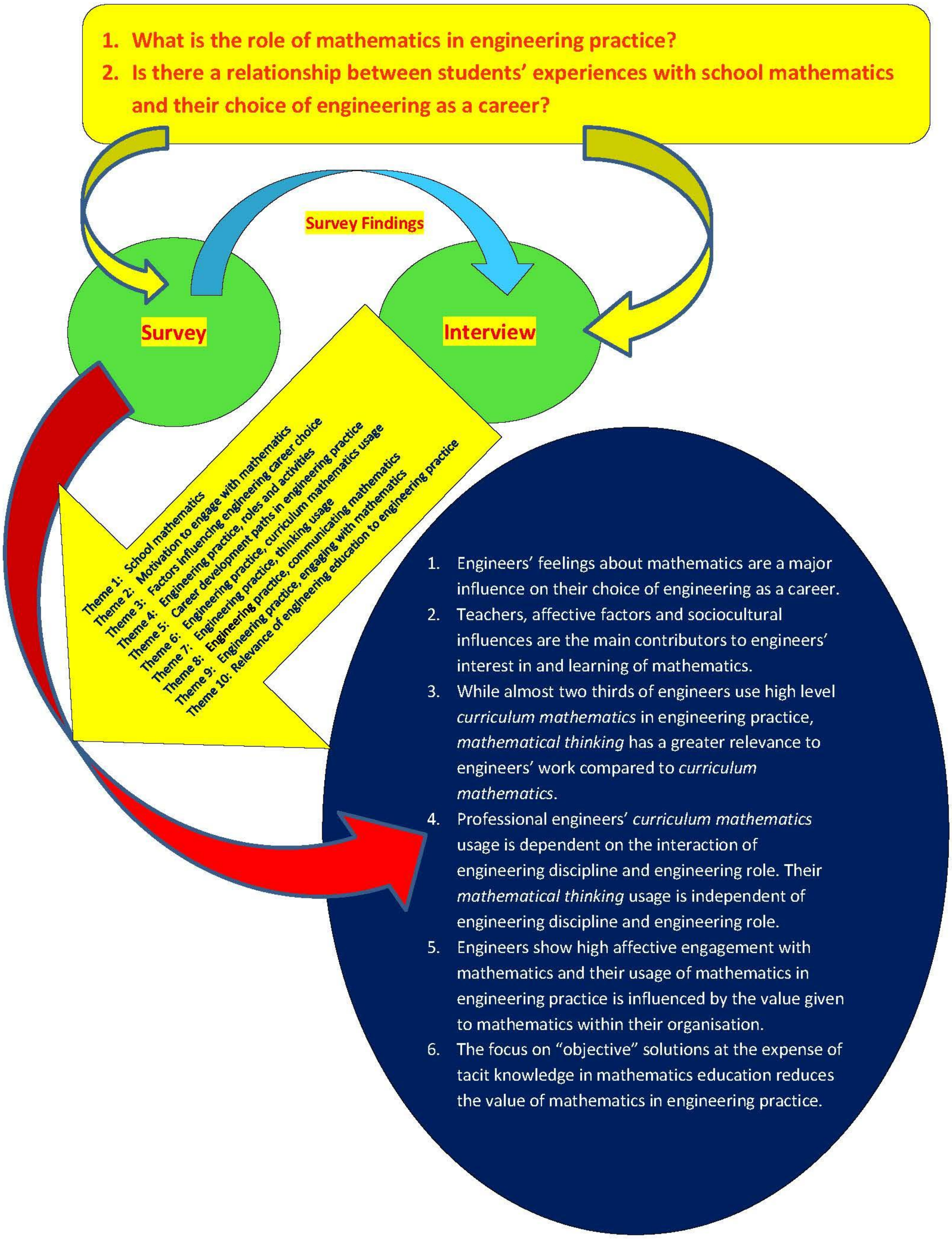

Figure 8-1: Contributions to research knowledge.

8.4.1 Engineers' feelings about mathematics are a major influence on their choice of engineering as a career

A major finding of this study is that feelings about mathematics are a major influence on engineering career choice. Three quarters (75.9%) of the engineers who participated in the survey say that their feelings about mathematics impacted their choice of engineering. Engineers' strong feelings about mathematics in the context of engineering career choice are presented in paragraph 7.2.3.1-1 and an example of these are: with "ability and enjoyment of mathematics" engineering "just made sense"; "I looked at my CAO application and said I would like to do more maths, so I just ticked all these boxes for engineering; "to me maths was everything, maths was where I wanted to be and to me it was the key to the career that I wanted, I wanted to be an engineer … I didn't want to do anything else"; interest in engineering came from "confidence from having done higher level maths"; engineering career choice was influenced "a very great deal" by "love" of mathematics; and engineering and mathematics "were hand in hand, I had very much an aptitude for mathematics in school, that's the subject that I found easier, that the subject that I didn't have to study and to me the engineering followed on from that".

The feeling of success is the main contributor to enjoyment of school mathematics and confidence in school mathematics stems from a recognition of success in school mathematics. Engineers' confidence in their mathematics ability is the main influence on engineering career choice. The study shows that teachers are the biggest influence on students' relationships with mathematics. In both the survey and the interview data engineers present that teachers, task value (why should I do mathematics?), feelings of success and peer and societal influences are key motivators to students' engagement in mathematics learning. Interview data shows that: (i) there is a high degree of correspondence between engineers whose family supported their mathematics learning from a young age and engineers whose main reason for choosing engineering was their feelings about mathematics; (ii) engineers, whose main reason for choosing engineering was for reasons other than their feelings about mathematics, didn't get any family encouragement or home support with mathematics; and (iii) engineers, who had negative school mathematics experiences, say that their feelings about mathematics did not influence their career choice.

It is noted that at the time of choosing their careers, engineers say that engineering was a prestigious career. For example, one engineer's "entry into the engineering profession" was "a due reward" for "excelling in maths" and when another engineer commenced engineering studies, she says the entry points for engineering were on par with medicine and there was an "ego" associated with engineering then and she felt she was "up there at the top".

8.4.2 Teachers, affective factors and sociocultural influences are the main contributors to engineers' interest in and learning of mathematics

From both the survey data and the interview data there is clear evidence that mathematics teachers have a powerful role in students' motivation to learn mathematics, section 5.6.2 and section 7.2.3.1-1. For example, one engineer's new mathematics teacher "transformed" him "from being someone who didn't like maths or didn't care about it to someone who loved it" and he "went from being this average student to being someone who was in the top five in the school". Mathematics is different to most other school subjects as shown in section 7.2.1.1 and consequently teaching has a greater influence in mathematics learning compared to other school subjects. Mathematics teaching focuses on getting the "right answer" whilst other subjects lean towards "subjective analysis". Students who get the "right answer" enjoy feelings of success and are motivated to engage in more mathematics learning. However many students who spend considerable amounts of their homework time "looking for a specific answer" may not experience the same success and consequently develop negative feelings and they can "fall behind" very quickly. The interview data illustrates that the key to mathematics learning is "finding that you are able to do it" and that confidence in school mathematics stems from recognition of success. For example, one engineer says that school mathematics was "instantly rewarding" and another engineer presents that from the "satisfaction" of getting the "right answer … I got confidence in the fact that I was getting good results in mathematics and then I realised this is something that I could be good at". Recognition of success is the main value of school mathematics for students. Teachers' role is to scaffold students and thus enable students to develop the necessary understanding and mastery to carry out mathematics tasks. Engineers say that with good teaching, students "feel good about it [mathematics] rather than just learn it off by heart" and if they discover that they are "good at it" they might enjoy it more and "stick with it".

The ability to communicate mathematics and its relevance is the predominant characteristic of good mathematics teachers. Good mathematics teachers are "positive" about mathematics and they are "enthusiastic to the point" where they "can foster interest and enthusiasm for the subject with a broad profile of students within the classroom". On the other hand "bad" mathematics teachers" have poor attitudes and they often label specific parts of course as "too hard" and they do not teach the entire syllabus. One engineer in this study stands out in terms of the consequences of "bad" mathematics teaching. He says that due to "bad" teaching, he developed an "inferiority complex about maths" and a "blockage" to learning mathematics in secondary school that "caught" him all the way through college and work. Engineers believe that if students "feel they can't do maths they are just not going to do maths" and there is a view that many students "going into secondary school have already decided to do ordinary level mathematics for their Junior Certificate exam". While mathematics teachers have the power to transform students' mathematics learning and their enjoyment of the subject from low to high levels, it is deemed unacceptable that unqualified mathematics teachers are given this power. Instead "teachers must have the skills, enthusiasm and ability necessary to teach the subject".

In this study engineers associate mathematics and mathematics learning with values, attitudes, beliefs, self-efficacy, emotions and sociocultural influences as shown in sections 5.8, 5.9 and 7.2, it is thus interpreted that mathematics is a highly "affective subject". In addition to feelings of success, engineers identify task value as a major factor that contributes to interest in and learning of mathematics. Engineers believe that mathematics teachers fail to communicate the value of mathematics and they also fail to demonstrate real world applications to students. Engineers say that teachers should teach mathematics that illustrates the task value of mathematics. This includes: the usefulness of mathematics; the relevance of mathematics to modern living; mathematics that is used in various careers; and mathematics that has links with other school subjects. Affective factors such as success (self-efficacy), enjoyment (value), practical applications (value), interest (value), problem solving (metacognitive activity), relevance to science (value), required for engineering (value), careers (value) and points (value) also contribute to mathematics learning in school.

Sociocultural influences, from families, peers and society are important factors in students' motivation to engage in mathematics learning. There is evidence that some engineers' families provided support and scaffolding for their mathematics learning. The correspondence between engineers whose family supported their mathematics learning from a young age and engineers whose main reason for choosing engineering was their feelings about mathematics and also the correspondence between the engineers whose main reason for choosing engineering was for reasons other than their feelings about mathematics and those who didn't get any family encouragement or home support with mathematics illustrate the value of family support in the formation of feelings about mathematics. There is also evidence that engaging in social or group learning of mathematics with peers or role models has many advantages for students preparing for the Leaving Certificate mathematics exam. However, according to the engineers, there is a general belief in society that mathematics is difficult and there is a stigma associated with being good at mathematics. A "them and us culture" happens at quite an early age when "people decide that they can't do it [mathematics]" and "the people who do it are somehow different" from those who can't. This culture causes social problems for students who are good at mathematics and consequently they feel "isolated" and hide "the guilty pleasure of enjoying maths". Society is deemed to be accepting of low numbers of students taking higher level Leaving Certificate mathematics in Ireland and also tolerant of "bad" mathematics teachers in Ireland in both primary and secondary schools. Engineers recommend that "society needs to set certain expectations for kids coming out of school" and mathematics teachers need to be accountable for achieving those expectations. Additionally teachers have the ultimate responsibility for correcting the "stigma about the difficulty of higher level maths".

8.4.3 While almost two thirds of engineers use high level *curriculum mathematics* in engineering practice, *mathematical thinking* has a greater relevance to engineers' work compared to *curriculum mathematics*

Engineers' mean *curriculum mathematics* usage score (for 75 domain-level-usage combinations of mathematics syllabi ranging from Junior Certificate ordinary to level 8 engineering and B.A./ B.Sc. mathematics) of 2.73 Likert units (out of a total score of 5 Likert units) illustrates the importance of *curriculum mathematics* in engineering practice. A major finding is that almost two thirds of engineers (64.4%) use higher level Leaving Certificate mathematics, 57.3% of engineers use engineering mathematics and 41.4% of engineers use B.A./ B.Sc. mathematics in their work. Much of engineers' mathematics usage is either *connecting* or *mathematising*. In the interview data, *statistics and probability*, particularly estimation of solutions and data analysis, stands out as one mathematics domain that is important in engineering practice. The interview data shows that *curriculum mathematics* has a diversity of uses in engineering practice and these are described in section 7.2.6.1.

A significant finding in this study is that engineers rate their mathematics *thinking* usage (4.02 Likert units) higher than their *curriculum mathematics* usage (2.73 Likert units) in their work. The modes of *thinking* resulting from mathematics education, that influence engineers' work performance are: problem solving strategies (26.4%), logical thinking (26.2%); critical analysis (7.2%); modelling (7.2%); decision making (6.3%); accuracy/ confirmation of solution (4.8%); precision/ use of rigour (4.6%); organisational skills (4.6%); reasoning (3.6%); communication/ teamwork/ making arguments (3.2%); confidence/ motivation (3.1%); numeracy (2.2%); and use of mathematical tools (0.7%). From the interview data, there is no overestimating the importance of *thinking* usage in engineering practice, for example, one engineer says that *thinking* usage is the "value" he brings to his job and another engineer says that *thinking* usage is "where it's all at … to me this is absolutely critical". There is a view that *thinking* usage increases for technical, commercial and management roles over the course of engineering careers because the "higher up you're going in an organisation" the more "permutations" there are to consider and managers "apply the maths not just to engineering, but also to finance, to manpower and to people". Engineers view the association between mathematics and *thinking* as "indirect" in that *thinking* is how engineers use mathematics rather than the actual mathematics they use. In section 7.2.7.1, the interview data shows that problem solving; big picture thinking; decision making; logical thinking; estimation and confirmation of solution are the main components of *thinking* usage. Problem solving is a major part of engineers' *thinking* usage; engineering problems have multiple answers and an engineer's job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer. Another aspect of *thinking* usage is "big picture thinking" which is taking the "the real world" into consideration where engineers need to "have a real tangible understanding of the effect of one piece of work on another part of the system" and "engineering should be about trying to identify the right question, because a lot of the times, people are obsessing over the wrong question". Engineers say that "speed of response" is important in engineering practice and that mathematics education contributes to an engineer's ability to think quickly. For example, one engineer says that what "the grounding in maths helps you do, is to look at the figures very quickly and make decisions".

This study shows that communicating mathematics is an important part of engineers' work**.** Engineers communicate mathematics when: expressing engineering concepts; expressing conclusions; writing reports; making arguments; explaining how "you have come to your conclusion"; justifying some decisions; rolling out IT solutions; reading reports; verifying consultants' work; communicating a concept to a decision-maker; asking the finance people to provide money; and selling products. Engineers say they communicate mathematics to a range of people including: other engineers; a variety of technical people on project sites; colleagues in Ireland and Singapore; clients;

managers; vendors; contractors; consultants; administrators; customers; decision makers; accountants; finance people; and human resources people.

This study gives an insight into engineering practice and the type of work engineers do. This is important knowledge given that many young people have a "blurred picture" of engineering in that they see an engineer as someone who is up to his or her "neck in equations for forty years" and not the "happy, successful engineer contributing to society". One key message about engineering practice that emerges from the study is summed up by one engineer who presents that in a "typical engineering company" only "a few people" do "maths at quite a high level", there are "people below them who need to understand and interpret what they are doing and then others who just need to know the big picture". The interview analysis in Chapter 7 gives a first-hand insight into engineering practice and a profile of 20 engineers' job descriptions is presented in Table 7-2 in Chapter7. The engineers' individual stories are included in Appendix 7 in Volume 2. The overall picture of engineering practice is that engineers' work is diverse and it comprises: degrees of *curriculum mathematics* usage, problem solving; "bigger picture thinking"; using computational tools; reusing solutions; analysing data; "real world" practicality; integrating units of technology; managing projects; and communicating solutions. The interview data shows that computer solutions are widely used in modern engineering practice. Engineers say that computational tools have many advantages in engineering practice in that the tools bypass the need to write down the fundamental engineering equations and solve them and they offer a standard methodology for developing solutions within organisations. However engineers note that results produced by computational tools can easily be misinterpreted. One engineer presents that using computational tools is "a different type of mathematics" and he is of the view that "the engineer should understand how the program is solving the equations and what it is doing, because it is always dangerous not to".

8.4.4 Professional engineers' *curriculum mathematics* usage is dependent on the interaction of engineering discipline and engineering role. Their mathematical *thinking* usage is independent of engineering discipline and engineering role

Survey analysis shows that the effect of engineering discipline and engineering role on engineers' overall mean *curriculum mathematics* usage depends on the other factor (role or discipline respectively) and neither engineering discipline, engineering role nor the interaction of engineering discipline and role have an effect on engineers' *thinking* usage. The absence of any clear profile of mathematics usage by engineering discipline or engineering role is explained in the interview data analysis where it is apparent that engineers' work is diverse and that engineering roles are "so broad" that engineers are easily transferrable from one role to another within an organisation. It is also apparent from the interview data that, with increasing experience, engineers to some extent lose their engineering discipline identity, for example, one engineer who manages a team of ten engineers says that none of his team of engineers is currently identifiable by their engineering qualification. There is also a view that "engineers to a very large extent are influenced to move into management by the necessity to obtain financial reward" and one engineer says that engineers who "graduate up through the management chain" don't use "maths on a daily basis" instead they manage people who use mathematics. Another engineer says that while "many engineers end up in management where they wouldn't necessarily be using maths regularly … they might have to talk to people who are using maths". There is also a view that Chartered Engineers are mostly managers who understand mathematics and who "are actually using more numbers that younger engineers" as they are managing budgets.

Across all engineering disciplines and engineering roles, there are tiers of mathematics requirements in engineering practice that range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths. This study shows that *curriculum mathematics* is only one part of engineering practice, engineers also engage in "project management and problem solving" tasks, "real world" applications and "bigger picture thinking" (logical thinking about the complete project). Engineers estimate that they use ten per cent of their university mathematics and because there is no “sense” of the specific careers graduates take on, engineering education must adopt a “one size fits all” approach. One engineer says that this size should be aimed at the graduates who take on “the highest consequence” of mathematics in their work and that the engineers who pursue less numerate careers reap the benefit of “rigour and discipline” from learning higher level mathematics.

One mathematics domain that stands out in the interview analysis is *statistics and probability* where engineers say that data analysis is required in all engineering areas to inform engineering decisions. For example, one engineer’s mathematics usage is “more about interpreting stuff” and being “able to understand data” than doing “calculations” and another engineer has to “look at data, make decisions and give directions”. Much of the mathematics required in modern engineering practice is done by software where the challenge for engineers is to correctly interpret computer solutions rather than do mathematics.

8.4.5 Engineers show high affective engagement with mathematics and their usage of mathematics in engineering practice is influenced by the value given to mathematics within their organisation

Almost three quarters of the engineers who participated in the survey say that they enjoy using mathematics in their work and over 80% of the engineers who participated in the survey feel confident dealing with mathematics in their work. Engineers “love the challenge in solving problems mathematically”, they enjoy “the satisfaction of a result”, they find it easier to communicate using mathematics compared to words and they prefer “a 100% right answer rather than the ambiguity of non-mathematical solutions”. For the engineers who enjoy using mathematics in work, there is a sense that mathematics is “part of who” they are. Engineers’ confidence in mathematical solutions in work is very evident in both the survey and interview data. For example, one engineer always chooses the “maths way” of doing things because mathematics is “very easy to reference and verify” and another engineer says that mathematics “is clean … it is completely logical, … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it".

The study shows that low confidence mathematics engineers avoid mathematics in their work and high confidence mathematics engineers readily "revise and brush up" on the required mathematics. Engineers' confidence in their mathematical ability grew from recognition of success in school mathematics such as their latest test grades, getting top marks or being the best in the class. Memories of school mathematics are the main reason engineers have low confidence using mathematics in work, for example due to "the very poor grounding" one engineer "got in maths" he says he "was afraid of some of" the mathematics he encountered in engineering practice and he has "a nagging fear that" he has "got something wrong" in his work. When he encounters a mathematics problem, he "refers" to his work colleagues.

Almost two thirds (64.6%) of engineers who participated in the survey are of the view that a specifically mathematical approach is necessary in their work and similarly almost two thirds (63.3%) of engineers who participated in the survey say that they actively seek a mathematical approach in their work. Engineers say there are tiers of mathematics requirements in engineering practice that range from a majority of engineers who "need to understand" mathematics to a minority of engineers who "require a very high standard of maths". Engineers estimate that mathematics is "valuable" in only ten per cent of their work, for example one engineer, who is a high user of *curriculum mathematics*, says she "could do ninety per cent" of her job without mathematics, but that she "couldn't possibly do the other ten per cent without it" and she maintains that "engineering is that extra ten per cent that you actually get paid for at the end of the day".

From the interview analysis it is apparent that engineers view mathematics as *curriculum mathematics* usage and not thinking usage or using computer solutions. Interview analysis shows that the degree a specifically mathematical approach is necessary in engineers' work is related to the value given to *curriculum mathematics* in engineering practice, for example, in one engineer's company, it is "more cost effective" not to use mathematics and in another company engineers don't have “time” to “actually use mathematics”. Colleagues’ respect for mathematics is also a factor in the value of mathematics in engineering practice, for example one engineer presents that in his company there is a respect for “maths only to the extent that it is useful”. Difficulty communicating mathematics reduces the value of mathematics in engineering practice. Engineers say there is “skill in communicating maths”; it is the “craft” of putting the mathematics “into a form that a non-engineer will understand”. Compared to other professions, engineers say they are not good communicators and a consequence of poor mathematics communications is that engineers are left in the “background”. One engineer asserts that “if engineers are to survive then they need to somehow harness communication skills”. It is interpreted that when graduate engineers make the transition from an education environment where mathematics is highly valued to engineering practice where mathematics is perceived to have a lesser value and where there is less time to engage in mathematics that there are associated changes in sociocultural influences and in motivational influences. While mathematics is “part of who” engineers are and while engineers prefer to communicate using mathematics, the task value of mathematics reduces when engineers move from engineering education where mathematics is a requirement into engineering practice where mathematics is often bypassed and consequently graduate engineers encounter an “affective hurdle”.

8.4.6 The focus on “objective” solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice

Engineers’ value of mathematics stems from getting the “right answer” in school mathematics. The resultant feeling of success when students get the correct answer is the main contributor to enjoyment and confidence in school mathematics. In this study engineers show a preference for mathematics where they have an ability to get the “right answer” and full marks compared to school subjects that lean towards “subjective analysis” whereby “no matter how much work” one puts into the “subjective” subjects one might not get “full marks”. It is the engineers’ strong feelings about mathematics and particularly their ability and enjoyment of school mathematics that influenced their decision to choose engineering.

Graduate engineers bring their confidence in mathematical solutions with them into the world of engineering practice where many engineers are also motivated to get the "exact solution" at the expense of engaging in "subjective analysis". For example, one young engineer presents that she enjoys using mathematics in work because "it is clean … it is completely logical, … it is totally transparent and basically once you are happy with it yourself, no one else can really question the validity of it". Another engineer never liked statistics which he describes as "vague"; he prefers "concrete" problems that have an "exact answer". However statistics is required in engineering practice and "objective" solutions have limited value in engineering practice particularly when engineers have difficulty communicating mathematics. There is strong evidence in this study that engineering problems have multiple answers, an engineer's job is to determine "what the answer means", which is "the best answer for all participants" and what "is the knock on effect" of the answer and "real world" engineers need to "have a real tangible understanding of the effect of one piece of work on another part of the system". It is concluded in this study that engineers' over-attachment to "objective" solutions restricts their vision of engineering solutions and "the bigger picture" of engineering practice particularly where "real world" practicality is often constrained by cost and safety factors and "a background of incomplete information". This is further supported by engineers who say that in engineering practice mathematics is used primarily as a tool to estimate and confirm multiple solutions to real problems while in engineering education mathematics is about deriving unique and exact solutions to theoretical problems from first principles. Engineers have a corresponding belief that graduate engineers, who are "drawn to black and white solutions", are not ready to engineer and that an ability to do engineering work comes from the "experience of working in an engineering environment", watching other engineers estimate, working out real problems and how they view "the bigger picture". Engineers claim that graduate engineers lack this tacit knowledge. The focus on "objective" solutions in mathematics education at the expense of tacit knowledge contributes to engineer's poor communication skills and consequently reduces the value of mathematics in engineering practice thus creating an "affective hurdle" for graduate engineers to overcome when they begin working as engineers. Engineers are adamant that engineering education would benefit from "real world practicality" and that experiencing how mathematics is used in the real world would benefit students' learning. One engineer believes she "wasn't ready" for some aspects of engineering education and that these aspects become "more relevant" with "experience". Another engineer, who experienced engineering practice while in college, says that having "read some validation procedures" during this work experience practice, she "got an idea where the maths comes in". This young engineer, with just four years' experience as a practising engineer, maintains that that she learned how to do engineering from her work colleagues and she has recently become "an independent thinker". There is a strong view among the twenty engineers interviewed that the mismatch between engineering education and engineering practice could be reduced by incorporating "real life" engineering experiences in engineering education.

It is also interpreted that the focus on "objective" solutions and the "right answer" in school mathematics at the expense of tacit knowledge has implications for engineering career choice. Students' school mathematics ability is categorised by a "hierarchy" of mathematics grades and according to one engineer, in mathematics "you get your answer right or you get it wrong and you either get an A or a D grade". Another engineer maintains that, in mathematics learning, a "them and us culture" happens at quite an early age when "people decide that they can't do it [mathematics]" and "that the people who do it are somehow different from them". Consequently students experience feelings of either success or failure, there is no gradation. However engineers maintain that modern students who get an "A1 in maths" are unlikely to opt for an engineering course "that is only 350 points", this is also supported by (Devitt and Goold 2010) and reinforced by one engineer who points out that there are engineering education paths where students can get "the same level 8 degree without higher level maths". It is concluded in this thesis that the declining interest in engineering careers is compounded by "elitism" at the top of the mathematics hierarchy and also by a perceived inability to do mathematics at the bottom of the hierarchy. While students at the top of the hierarchy are likely to opt for high points courses and thus do not choose level 8 engineering courses, students at the bottom may not have the required grade C3 in higher level Leaving Certificate mathematics for entry into level 8 engineering courses or they may have a fear of mathematics. In section 7.2.1.1-3 engineers maintain that mathematics is a “special” subject because it is “unique, it’s precise, there is a right answer”. Engineers contrast mathematics where “you either get an A or a D grade” with English that “is so subjective”; “no matter how much work” one engineer put into it, her best grade ever was a “C1”. It is therefore suggested that incorporating more “subjective analysis” (tacit knowledge) into the school mathematic syllabus would give a better distribution of mathematics results. It is also anticipated that a better distribution of students’ success in the subject would improve many students’ feelings about mathematics which, according to the findings in this study, would ultimately lead to greater interest in engineering careers.

8.5 IMPLICATIONS OF MAIN FINDINGS

This study informs mathematics teachers, engineering educators, practising engineers, students, parents and society. For each of these groups this study gives an insight into engineering practice and how mathematics is used in the workplace. This study also illustrates that feelings about mathematics are an important factor in mathematics learning and usage. One implication for mathematics curricula development and assessment is that mathematics learning generally focuses on objective analysis while *thinking* usage, subjective analysis and communicating mathematics are also required in engineering practice and possibly in other numerate professions such as economics.

Another implication for educators, parents and society, arising from this study, is that mathematics is a highly affective subject where student feelings about mathematics are a major influence on their engagement with the subject.

The findings from this study have particular implications for teaching mathematics and for engineering education.

8.5.1 School Mathematics Teachers

The key messages for mathematics teachers arising from this study is that the teacher is the "biggest influence" on students' relationships with mathematics and mathematics is a highly affective subject where motivational beliefs such as affective memories (previous emotional experiences with mathematics), goals, task value (why should I do mathematics?) and expectancy (am I able to do mathematics?) are major influences on students' engagement with mathematics. Students develop mathematical self-efficacy in school when they discover that they are able to do mathematics and they bring this confidence with them to university, work and into society.

Concerns about mathematics teaching include: teachers' own attitudes where mathematics is presented as a "hard" subject; lack of recognition of student success; lack of encouragement where students "feel they can't do maths"; failure to communicate the value of mathematics; emphasis on rote learning rather than on understanding; difficulty communicating mathematics, focus on objective solutions at the expense of tacit knowledge; lack of relevance in mathematics teaching to real world applications; and "unqualified" mathematics teachers who are neither confident nor positive in their teaching of mathematics.

According to the findings in this study, teachers should present tasks that encourage students to value and enjoy mathematics and "teachers must have the skills, enthusiasm and ability necessary to teach the subject". Engineers in this study maintain that teachers should "emphasise more the applications of maths … say that this is why we are doing it, the place of maths in the world and make that part of the taught and examined subject". The provision of career guidance at an early stage of secondary school, conveying the career value of higher level mathematics, would assist students' task value and take-up of higher level Junior Certificate mathematics where more than 50% of the student population are lost to higher level mathematics and consequently do not meet the entry requirements to level 8 engineering education. According to engineers, it is teachers' responsibility to correct the "stigma about the difficulty of higher level maths".

There is strong evidence that mathematics learning requires a social environment whereby students benefit from group discussion and peer learning. The ability to communicate mathematics and its relevance is the predominant characteristic of good mathematics teachers. Teachers need to help students acquire a task value of mathematics and they need to engage with students in mathematics discussions and subjective analysis. According to Vygotsky's social constructivist mathematics learning theory, teachers' role is to provide scaffolding on which students construct their learning. Scaffolding is a means whereby a more skilled person imparts knowledge to a less skilled person and discussion between teacher and students and amongst students themselves enhance students' mathematical thinking and communication (Vygotsky 1978). A social mathematics learning environment enables students to enhance their tacit knowledge and this type of knowledge is required in workplace situations (Ernest 2011). These findings have implications for mathematics teacher training.

8.5.2 Engineering Education

This study provides evidence for a requirement to better match the mathematics taught in engineering education with the mathematics required in engineering practice. Engineers maintain that engineering education mostly imparts knowledge while the role of practising engineers is "to frame the problem correctly and maybe express it in maths, then they have to solve it and then they have to interpret the solution and communicate that to the decision maker".

The key message for engineering education is that building a mathematics curriculum that more closely represents the way mathematics is used in engineering practice will strengthen it. This study provides evidence that while a majority of engineers use both higher level Leaving Certificate mathematics and engineering level mathematics in their work, *curriculum* mathematics is different to much of the mathematics used in engineering practice. In engineering practice, mathematics is used primarily as a tool to estimate and confirm multiple solutions to real problems while in engineering education mathematics is about deriving a unique and exact solution to theoretical problems from first principles. Data analysis, which is often neglected in engineering education, is required in all engineering areas to inform engineering decisions. Similarly computer analysis is widely used in modern engineering practice where engineers do not know how the computer is solving the problem.

A significant difference between engineering practice and engineering education is practising engineers' reliance on tacit knowledge while engineering education is based on explicit knowledge. Workplace problems often lack data and are more ambiguous compared to problems encountered in engineering education and an engineer's job is to determine "what the answer means", "which is the best answer for all participants" and "what is the knock on effect" of the answer. Engineers have particular difficulty interpreting computer solutions which have become a significant part of modern engineering practice. They say that the "quirkiness of computational tools" and their "lack of understanding" and "over reliance of computer analysis" sometimes generate errors.

Another difference between engineering education and practice is the social aspect of work compared to education. Tackling workplace problems is usually a team effort while in engineering education problem solving is mostly an individual effort. Graduate engineers' difficulty communicating mathematics is a significant weakness of engineering education and consequently when engineers move from engineering education into engineering practice where mathematics is given a lower value compared to in education environments, they do not realise their mathematical ability.

To better prepare engineering students for engineering practice, they need to engage in "real world" practicality where "speed of response" and cost factors are important factors, subjective analysis and group work. Many engineers have an opinion that an ability to do engineering work comes from the "experience of working in an engineering environment", watching other engineers estimate, work out real problems and how they view "the bigger picture". Big picture thinking is taking the "the real world" into consideration where engineers need to "have a real tangible understanding of the effect of one piece of work on another part of the system" and “engineering should be about trying to identify the right question, because a lot of the times, people are obsessing over the wrong question”.

The findings in this study suggest that engaging in active or social learning environments that emulate engineering practice would benefit engineering education. This type of learning environment would provide a greater focus on: engineering practice; real world applications of mathematics; working with tacit knowledge; teamwork; communicating mathematics; data analysis and decision making; and interpreting computer solutions. Students would be required to present and defend their mathematical solutions to both their peers and their lecturers. Based on the findings in this study, it is anticipated that this type of learning environment would develop students’ mathematics communications skills and would also enhance their mathematics thinking and confidence.

8.6 LIMITATIONS

An advantage of the mixed methods research approach taken in this study is that two different methodologies are used to collect and analyse data relating to the same research questions while also allowing new knowledge to emerge. The survey methodology produced data from a large sample of professional engineers that was objectively analysed while in the interview stage the researcher had direct access to a small number of engineers and the opportunity to explore the phenomenon in depth. The findings, contained herein, are a combination of statistical findings which are generalised to the professional engineering population and insights from the personal stories and perspectives of twenty engineers working in engineering practice.

Survey analysis has potential to produce objective knowledge that is almost free from research bias. However generalisation of statistical findings is dependent on minimum sample size requirements and the randomness of the sample. The sample size in this study is satisfactory for precision to within 0.15 units (on a Likert scale with five outcomes) and 95% confidence i.e. 95% probability that the findings from the survey questionnaire represent the population of Chartered Engineers in Ireland. While the response rate was noted to be broadly representative across disciplines, gender and geography, it cannot be verified that the respondents do in fact constitute a random sample. Any sampling bias that may exist in this study is due to non-responsive sampling; all Chartered Engineers were invited to participate in the survey and those who did respond are more likely to be those that have stronger interest in the research topic and consequently the survey data may not be representative of the entire population (Panzeri et al. 2008). However if Chartered Engineers were randomly chosen to participate in this study, they may not agree to participate and furthermore there is no guarantee that such engineers' views are representative of the entire population of Chartered Engineers practising in Ireland. In economics studies sample selection models are used to test if individuals who do not participate in studies are systematically different from those who do; these models are rarely used in social work research (Cuddeback et al. 2004). Furthermore these models are of no use when data for non-participants are unavailable (Hill et al. 2008). Given the dearth of research concerning practising engineers' mathematics usage, their feelings about mathematics in the context of engineering career choice and the diversity of engineering disciplines and roles, it is not possible to test if the survey data represents a random sample of Chartered Engineers practising in Ireland. The survey findings are limited by the assumption that the survey participants comprise a random sample of Chartered Engineers in Ireland. To compensate for any limitation, engineers who participated in the interview stage comprised: a diversity of mean *curriculum mathematics* users; a diversity of engineering disciplines and roles, a diversity of employers, a diversity of urban and rural backgrounds; a diversity of Leaving Certificate mathematics levels; a diversity of engineering education routes and a diversity of ages.

While qualitative interviews, where the researcher has direct access to the participants and the opportunity to explore the research phenomenon in depth, enhance the validity of mixed methods studies, there is also a concern that the researcher's subjectivity influences the research. The researcher has taken every effort to minimise subjective influences; the data analysis is based on the interviewees' stories and what they say. However, the validity and reliability of any self-report study must be given consideration. The mixed-methods research approach cross-referenced the survey and interview data and both sets of data are consistent. There is however the possibility that engineers' recollection of their school mathematics experiences or indeed their work experiences have become distorted with time, either consciously or sub-consciously. There is a view in the research literature that the accuracy of self-report data is an unresolved research topic in itself. Research literature shows that students with lower actual test scores tend to recall their scores with less accuracy more than students with high test scores. Social desirability bias where the student wishes to preserve self-esteem and reconstructed memory process are the main causes of such bias. Literature reports relatively high correlation between students' self-reported and actual test scores generally and particularly so for cumulative academic experiences (Herzog and Bowman 2011; Kuncel et al. 2005; Mayer et al. 2007). Given that the engineers in this study have all successfully completed engineering education; they have experienced engineering practice first hand; their views and personal stories are based on their experiences rather than on their accomplishments; and much of the interview data is based on affective factors, the researcher is confident that the data has high validity.

Another possible limitation of this study is that the engineers' views about school mathematics and mathematics in engineering education are constructed only by their own education experiences and these may be somewhat out of date. However many engineers and especially Chartered Engineers, through their contact with the profession, with young engineers who come into the workforce and with local schools, are aware of developments in mathematics and engineering education.

8.7 SUGGESTIONS FOR FURTHER WORK

One finding in this study is that engineers' feelings about mathematics are a major influence on their decision to choose engineering careers. This study also identifies factors that contribute to students' interest and learning of mathematics. Coincidental with this study is the introduction of a revised mathematics syllabus, called Project Maths, into both Junior Certificate and Leaving Certificate mathematics in Ireland. Project Maths aims to provide for an enhanced student learning experience with increased use of contexts and applications that will enable students to relate mathematics to everyday experience (National Council for Curriculum and Assessment 2010b). Given the significance of students' feelings about mathematics particularly in the context of engineering career choice, as discovered in this study, it is likely that a study of students' feelings about mathematics arising from the new mathematics syllabus would provide curriculum developers with important new knowledge.

Another finding in this study is that the focus on "objective" solutions at the expense of tacit knowledge in mathematics education reduces the value of mathematics in engineering practice. The completion of this study coincides with the first complete State exam in Project Maths. The 2012 Project Maths examination sought to place greater emphasis on student understanding of mathematical concepts, with increased usage of contexts and applications compared to the previous syllabus. However there were some criticisms of the subjective nature of the 2012 examination paper with one question described as "verbose" and "confusing" and that "candidates sitting the economics paper next week would have found the subject matter more familiar". The "language" used in another question was considered "slightly unfair on students". On the positive side, the treatment of statistics "relied in part on students' common sense and general knowledge" (Donnelly 2012). A study, investigating both the "objective" and tacit knowledge learning dimensions within the Project Maths curriculum, would be interesting. Similarly a study investigating engineering practice competencies e.g. communications and tacit knowledge in mathematics in engineering education would be worthwhile.

While this study investigated mathematics as a factor in the formation of engineers, the researcher is also curious to explore other factors influencing engineering career choice. During this study it became apparent that the early introduction to engineering on family farms steered some students towards engineering careers. It is speculated that outside of farming modern young people have little opportunity for tinkering with gadgets especially with increasing miniaturisation and modularisation of modern technology. An analysis of rural versus urban backgrounds in the formation of engineers would, in the researcher's eyes, be interesting.

8.8 CONCLUDING REMARKS

This data presented in this study is based on practising engineers' experiences of school mathematics, engineering education and engineering practice. It includes new knowledge about the type of mathematics required by engineers in their work and their feelings about mathematics. Recommendations for mathematics and engineering education are also included. The survey findings have been published (Devitt and Goold 2011) and the complete findings have been accepted for presentation at the 2012 European Society for Engineering Education annual conference (Goold and Devitt upcoming September 2012). It is anticipated that, the findings of this study, if addressed particularly by providers of mathematics education in both second and third level education, could revitalise engineering career choice.

GLOSSARY OF IRISH EDUCATION TERMINOLOGY

Junior Certificate: Examination at mid secondary school in Ireland.

Leaving Certificate: Examination at completion of secondary school in Ireland.

Foundation, Ordinary and Higher [honours] Level: For both Junior Certificate and Leaving Certificate mathematics is provided at three syllabus levels: foundation, ordinary and higher with corresponding levels of examination papers. The higher level is sometimes called honours level.

Grade A: ≥85%

Grade A1: ≥ 90%

Grade B: ≥70%, <85%

Grade C: ≥55%, <70%

Grade C1: ≥65%, <70%

Grade C3: ≥55%, <60%

Grade D: ≥40%, <55%

Project Maths: Major revision of the second level school mathematics curriculum in Ireland.

Transition year: Optional, one-year, standalone, full-time programme taken in the year after the Junior Certificate in Ireland with a strong focus on personal and social development and on education for active citizenship.

Grind school: In Ireland, grinds are private tuition; grind schools are private secondary schools that provide students with intensive coaching in preparation for Junior Certificate and Leaving Certificate exams.

CAO: Central Applications Office, Ireland's central administration for management of the competitive points system for entry to third level education.

Points [CAO Points]: Points are awarded to students based on their achievements in the Leaving Certificate examination. Points are calculated from students' top 6 subjects and the maximum number of points is 600 (up to 2011). Students who score A1 grades in 6 higher level Leaving Certificate subjects are awarded 600 points. Students applying for third level education courses apply to the Central Applications Office (CAO) and those who meet the minimum points required for a course for which they have applied are offered places. When the demand for a particular course exceeds the number of available places, places are offered to those students with the highest score in the CAO points system.

Level 6 qualification: Certificate (e.g. technician); typically 2 year undergraduate course.

Level 7 qualification: Ordinary Bachelor Degree (e.g. technologist); typically 3 year undergraduate course.

Level 8 qualification: Honours Bachelor Degree (e.g. professional engineer); typically 4 year undergraduate course.

Level 9 qualification: Masters Degree.

Entry to level 8 engineering courses: In addition to the points required, students entering level 8 engineering courses are also required to have a grade of C3 (55-59.9%) or higher in higher level Leaving Certificate mathematics.

Third Level Education: Third level education in Ireland mostly comprises universities and institutes of technologies. There are seven universities and fourteen institutes of technology in Ireland. The institute of technology system of engineering education allows students to progress from two year programmes (level 6) to three year programmes (level 7) or to honours degree programmes (level 8). Unlike direct entry to level 8 engineering education, students entering level 6 engineering courses are not required to have a grade of C3 (55-59.9%) or higher in higher level Leaving Certificate mathematics.

REFERENCES

ABET Engineering Accreditation Commission. (2010). "ABET Criteria for Accrediting Engineering Programs, Effective for Evaluations During the 2008-2009 Cycle." City: Baltimore, MD.

Ahlfors, L. V., Bacon, H. M., Bell, C. V., Bellman, R. E., Bers, L., Birkhoff, G., Boas, R. P., Brauer, A. T., Britton, J. R., Buck, R. C., Carrier, G. F., Cohen, H., Courant, R., Coxeter, H. S. M., Dawson, D. T., Douglis, A., Erdelyi, A., Freiberger, F., Friedrichs, K. O., Garabedian, P. R., Gilbarg, D., Goldstein, S., Goldstine, H., Greenberg, H., Hancock, J. D., Hutchinson, C. A., Kac, M., Kaplan, W., Kempner, A., Kinney, L. B., Kline, M., Kolodner, I. I., Langer, R. E., Larsen, C. M., Lax, P. D., Leighton, W., Levison, N., Mann, H. L., Mann, W. R., Montgomery, D., Morse, M., Nehari, Z., Neyman, J., Pohle, F. V., Pollak, H. O., Pôlya, G., Poritsky, H., Prager, W., Protter, M. H., Rado, T., Sawyer, W. W., Schiffer, M. M., Serrin, J. B., Smith, L. T., Sokolnikoff, I. S., Sternberg, E., Stoker, J. J., Taub, A. H., Truesdell, C. E., Walker, R. J., Wasow, W., Weil, A., and Wittenberg, A. (1962). "On The Mathematics Curriculum Of The High School." *The Mathematics Teacher* 55(3), 191-195.

Alpers, B. (2010a). "The Mathematical Expertise of Mechanical Engineers –The Case of Mechanism Design", in R. Lesh, P. L. Galbraith, C. R. Haines, and A. Hurford, (eds.), *Modeling Students' Mathematical Modeling Competencies*. New York, London: Springer pp. 99-110.

Alpers, B. (2010b). "Methodological Reflections on Capturing the Mathematical Expertise of Engineers", in A. Araújo, A. Fernandes, A. Azevedo, and J. F. Rodrigues, (eds.), *Educational Interfaces Between Mathematics and Industry*. City: Lisbon, Portugal, pp. 41-51.

Alpers, B. (2010c). "Studies on the Mathematical Expertise of Mechanical Engineers." *Journal of Mathematical Modelling and Application*, 1(3), 2-17.

Anderson, K. J. B., Courter, S. S., McGlamery, T., Nathans-Kelly, T. M., and Nicometo, C. G. (2010). "Understanding Engineering Work and Identity: A Cross-Case Analysis of Engineers Within Six Firms." *Engineering Studies*, 2(3), 153-174.

Bandura, A. (1986). *Social Foundations of Thought and Action: A Social Cognitive Theory*, Englewood Cliffs, NJ: Prentice-Hall.

Bandura, A. (1997). *Self-Efficacy: The Exercise of Control*, New York: W. H. Freeman and Company.

Baranowski, M., and Delorey, J. (2007). *Because Dreams Need Doing: New Messages for Enhancing Public Understanding of Engineering*. National Academy of Engineering, Washington, DC.

Barrett, C. (2008). *Tapping America's Potential, Gaining Momentum, Loosing Ground*. The TAP coalition, Washington, DC.

Baytiyeh, H., and Naja, M. K. (2010). "Impact of College Learning on Engineering Career Practice." *40th American Society for Engineering Education (ASEE) / Institute of Electrical and Electronics Engineers (IEEE) Frontiers in Education Conference* City: Washington, DC.

Becker, F. S. (2010). "Why Don't Young People Want to Become Engineers? Rational Reasons for Disappointing Decisions." *European Journal of Engineering Education*, 35(4), 349-366.

Benbow, C. P., Lubinski, D., Shea, D. L., and Efterkhari-Sanjani, H. (2000). "Sex Differences in Mathematical Reasoning Ability at Age 13: Their Status 20 Years Later." *Psychological Science*, 11(6), 474-480.

Betz, N., and Hackett, G. (1981). "The Relationship of Career-Related Self-Efficacy Expectations to Perceived Career Options in College Women and Men." *Journal of Counseling Psychology* 28(5), 399-410.

Betz, N. E., and Hackett, G. (1983). "The Relationship of Mathematics Self-Efficacy Expectations to the Selection of Science-Based Majors." *Journal of Vocational Behaviour*, 23(3), 329-345.

Bissell, C., and Dillon, C. (2000). "Telling Tales: Models, Stories and Meanings." *For the Learning of Mathematics*, 20(3), 3-11.

Boaler, J. (2006). "Opening Our Ideas: How a Detracked Mathematics Approach Prompted Respect, Responsibility and High Achievement." *Theory into Practice*, 45(1), 72-81.

Bogdan, R. C., and Biklen, S. K. (1997). *Qualitative Research for Education: An Introduction to Theory and Methods*, Boston, MA: Allyn & Bacon.

Bordogna, J. (1992). "Engineering - The Integrative Profession." *NSF Directions*, 5(2), 1.

Borrego, M., Douglas, E. P., and Amelink, C. T. (2009). "Quantitative, Qualitative, and Mixed Research Methods in Engineering Education." *Journal of Engineering Education Education* 98(January), 53–66.

Borrus, M., and Stowsky, J. (1997). "Technology Policy and Economic Growth", in L. Branscomb, (ed.), *Investing in Innovation*. Cambridge, MA: MIT Press.

Boskin, M. J., and Lau, L. J. (1992). "Capital, Technology, and Economic Growth", in N. Rosenberg, R. Landau, and D. C. Mowery, (eds.), *Technology and the Wealth of Nations*. Stanford, CA: Stanford University Press.

Boskin, M. J., and Lau, L. J. (1996). "The Contribution of R&D to Economic Growth: Some Issues and Observations", in B. L. R. Smith and C. E. Barfield, (eds.), *Technology, R & D, and the Economy*. Washington, DC: The Brookings Institution and American Enterprise Institute, pp. 75-107.

Bowen, E., Prior, J., Lloyd, S., Thomas, S., and Newman-Ford, L. (2007). "Engineering More Engineers - Bridging the Mathematics and Careers Advice Gap." *Engineering Education: Journal of the Higher Academy*, 2(1), 23-32.

Breen, S., and O'Shea, A. (2010). "Mathematical Thinking and Task Design." *Irish Mathematical Society Bulletin*, 66, 39-49.

Brickhouse, N. W., Lowery, P., and Schultz, K. (2000). "What Kind of a Girl Does Science? The Construction of School Science Identities." *Journal of Research in Science Teaching*, 37(5), 441-458.

Brown, M., Brown, P., and Bibby, T. (2008). "“I Would Rather Die”: Attitudes of 16 year-olds towards their future participation in mathematics." *Research in Mathematics Education*, 10(1), 3-18.

Brown, R., and Porter, T. (1995). "The Methodology of Mathematics." *Math Gazette*, 79 (July), 321-334.

Brown, S., and Burnham, J. (2012). "Engineering Student's Mathematics Self-Efficacy Development in a Freshman Engineering Mathematics Course." *International Journal of Engineering Education*, 28(1), 113-129.

Bucciarelli, L. L. (2002). *Designing Engineers*, Cambridge MA and London: MIT Press.

Buechler, D. N. (2004). "Mathematical Background Versus Success in Electrical Engineering." *American Society for Engineering Education Annual Conference & Exposition* City: Salt Lake City, UT.

Burton, L. (1984). *Thinking Things Through: Problem Solving in Mathematics*, Oxford: Basil Blackwell.

Burton, L. (2004). *Mathematicians as Enquirers: Learning about Learning Mathematics,* Berlin: Springer.

Capobianco, B., Diefes-Dux, H. A., Mena, I., and Weller, J. (2011). "What is an Engineer? Implications of Elementary School Student Conceptions for Engineering Education." *Journal of Engineering Education*, 100(2), 304-328.

Cardella, M. (2007). "What Your Engineering Students Might Be Learning From Their Mathematics Pre-Reqs (Beyond Integrals and Derivatives)." *37th American*

Society for Engineering Education (ASEE) / Institute of Electrical and Electronics Engineers (IEEE) Frontiers in Education Conference. City: Milwaukee, WI, pp. S4F1-S4F6.

Cardella, M., and Atman, C. J. (2007). "Engineering Students' Mathematical Thinking: In the Wild and with a Lab-Based Task." *Annual American Society of Engineering Education Conference*. City: Honolulu, HI.

Cardella, M. E. (2008). "Which Mathematics Should We Teach Engineering Students? An Empirically Grounded Case for a Broad Notion of Mathematical Thinking." *Teaching Mathematics and its Applications*, 27(3), 150-159.

Cardella, M. E. (2010). "Mathematical Modeling in Engineering Design Projects", in R. Lesh, P. L. Galbraith, C. R. Haines, and A. Hurford, (eds.), *Modeling Students' Mathematical Modeling Competencies.* New York, London: Springer, pp. 87-98.

Cardella, M. E., and Atman, C. C. J. (2004). "A Qualitative Study of the Role of Mathematics in Engineering Capstone Design Projects." *International Conference on Engineering Education*. City: Gainesville, FL.

Cardella, M. E., and Atman, C. J. (2005). "Engineering Students' Mathematical Problem Solving Strategies in Capstone Projects." *American Society for Engineering Education Annual Conference & Exposition*. City: Washington, DC, pp. 5537-5550.

Carmichael, C. S., and Taylor, J. A. (2005). "Analysis of Student Beliefs in a Tertiary Preparatory Mathematics Course." *International Journal of Mathematical Education in Science and Technology*, 36(7), 713-719.

Central Applications Office. (2008). *CAO Board of Directors' Reports*. Dublin, Ireland.

Chamberlin, S. A. (2010). "A Review of Instruments Created to Assess Affect in Mathematics." *Journal of Mathematics Education*, 3(1), 167-182.

Chambers, P. (2008). *Teaching Mathematics (Developing as a Reflective Secondary Teacher)*, London: Sage.

Chatterjee, A. (2005). "Mathematics in Engineering." *Current Science*, 88(3), 405-414.

Cockcroft, W. H. (1982). *The Cockcroft Report: Mathematics Counts,* Department of Education and Science*,* London: Her Majesty's Stationery Office.

Cohen, L., Manion, L., and Morrison, K. (2008). *Research Methods in Education*, Oxon, NY: Rutledge.

Collis, J., and Hussey, R. (2009). *Business Research A Practical Guide For Undergraduate & Postgraduate Students*, London: Palgrave Macmillan.

Correll, S. J. (2001). "Gender and the Career Choice process: The Role of Biased Self-Assessments." *American Journal of Sociology*, 106(6), 1691-1730.

Costello, J. (1991). *Teaching and Learning Mathematics 11-16*, New York: Routledge.

Coupland, M., and Gardner, A. (2008). "Mathematics for Engineering Education: What Students Say", M. Goos, R. Brown, and K. Makar, (eds.), *The 31st Annual Conference of the Mathematics Education Research Group of Australasia*. City: Brisbane, Australia.

Courter, S. S., and Anderson, K. J. B. (2009). "First-Year Students as Interviewers: Uncovering What It Means to be an Engineer." *39th American Society for Engineering Education (ASEE) / Institute of Electrical and Electronics Engineers (IEEE) Frontiers in Education Conference* City: San Antonio, TX.

Crawley, E. F., Malmqvist, J., Östlund, S., and Brodeur, D. (2007). *Rethinking Engineering Education: The CDIO Approach*, New York: Springer Science+Business Media.

Creswell, J. W. (2005). *Research Design Qualitative, Quantitative, and Mixed Methods Approaches*, Thousand Oaks, New Delhi, London, Singapore: Sage.

Croft, T., and Grove, M. (2006). "Mathematics Support: Support for the Specialist Mathematician and the More Able Student." *MSOR Connections*, 6(2), 1-5.

Crotty, M. (1998). *The Foundations of Social Research: Meaning and Perspective in the Research Process*, London, California, New Delhi: Sage Publications Ltd.

Csíkszentmihályi, M. (1992). "The Flow Experience and its Significance for Human Psychology", in M. Csíkszentmihályi and I. S. Csikszentmihalyi, (eds.), *Optimal Experience: Psychological Studies of Flow in Consciousness*. New York: Cambridge University Press pp. 15-35.

Cuddeback, G., Wilson, E., Orme, J. G., and Combs-Orme, T. (2004). "Detecting and Statistically Correcting Sample Selection Bias." *Journal of Social Service Research* 30(3), 19-34.

Cunningham, C. M., Lachapelle, C., and Lindgren-Streicher. (2005). "Assessing Elementary School Students' Conceptions of Engineering and Technology." *American Society for Engineering Education Annual Conference & Exposition*. City: Portland, OR.

De Corte, E., Verschaffel, L., and Op ’t Eynde, P. (2000). "Self-Regulation: A Characteristic and Goal of Mathematics Education", in M. Boekaerts, P. R. Pintrich, and M. Zeidner, (eds.), *Handbook of Self-Regulation*. San Diego, CA: Academic Press, pp. 687-726.

De Lange, J. (1994). "Assessment: No Change Without Problems", in T. A. Romberg, (ed.), *Reform in School Mathematics and Authentic Assessment*. Albany, NY: Suny Press.

De Lange, J. (1999). *Framework for Classroom Assessment in Mathematics*. Freudenthal Institute, Utrecht, The Netherlands.

De Lange, J. (2001). *Mathematics for Literacy*. National Council on Education and the Disciplines, Princeton, NJ.

De Lange, J., and Romberg, T. A. (2004). "Monitoring Student Progress", in T. A. Romberg, (ed.), *Standards-Based Mathematics Assessment in Middle School: Rethinking Classroom Practice*. New York: Teachers College Press, pp. 5-24.

Devitt, F., and Goold, E. (2010). "Schools Evaluate the Appliance of Science."*The Sunday Business Post*. City: Dublin, pp. 14.

Devitt, F., and Goold, E. (2011). "The Role of Mathematics in Engineering Practice and in the Formation of Engineers." *Presented at 18th International Conference on Adults Learning Mathematics*, Dublin.

Donnelly, K. (2012). "Leaving Cert: New Style Maths Papers Tough But Mostly Manageable." *Irish Independent*. City: Dublin, pp. 5.

Doppelt, Y., Mehalik, M. M., Schunn, C. D., Silk, E., and Krysinski, D. (2008). "Engagement and Achievements: A Case Study of Design-Based Learning in a Science Context." *Journal of Technology Education*, 19(2), 22-39.

Du, X., and Kolmos, A. (2009). "Increasing the Diversity of Engineering Education - a Gender Analysis in a PBL Condext." *European Journal of Engineering Education*, 34(5), 425-437.

Duderstadt, J. J. (2008). "Engineering for a Changing World: A Roadmap to the Future of American Engineering Practice, Research and Education", in D. Grasso and M. Brown Burkins, (eds.), *Holistic Engineering Education*. New York: Springer Science+Business Media.

Dym, C., Agogino, A. M., Eris, O., Frey, D. D., and Leifer, L. J. (2005). "Engineering Design Thinking, Teaching and Learning." *Journal of Engineering Education*, 94(1), 103-120.

Eckert, C., Blackwell, A., Bucciarelli, L. L., Clarkson, P. J., Earl, C. F., Knight, T. W., McMillan, S., K., S. M., and D, W. (2004). "What Designers Think We Need to Know about their Processes: Early Results from a Comparative Study", D.

Marjanovic, (ed.) *International Design Conference* City: Dubrovnik, Croatia, pp. 995-1002.

Eisner, E. W. (1991). *The Enlightened Eye: Qualitative Inquiry and the Enhancement of Educational Practice*, New York, NY: Macmillan.

Elliott, L. (2009). "UK 'must find 600,000 new engineers in seven years'", *The Observer*. City: London.

Ellis, M., Williams, B., Sadid, H., Bosworth, K., and Stout, L. (2004). "Math Usage by Practicing Engineers: What does it mean to Curriculum Planners?" *American Society for Engineering Education (ASEE) Annual Conference & Exposition*. City: Salt Lake City, UT.

Engineers Ireland. (2011). *Engineers Ireland Annual Report 2010-2011*. Engineers Ireland Dublin.

Engineers Ireland. (2012). "Engineers Ireland Web Page". City: Dublin.

English, L. D. (2007). "Cognitive Psychology and Mathematics Education: Reflections on the Past and the Future." *The Montana Mathematics Enthusiast*, 2, 119-126.

Ernest, P. (2004a). "Images of Mathematics, Value and Gender: A Philosophical Perspective", in B. Allen and S. Johnston-Wilder, (eds.), *Mathematics Education: Exploring the Culture of Learning*. New York: Routledge Falmer, pp. 12-25.

Ernest, P. (2004b). "What is the Philosophy of Mathematics Education?" *Philosophy of Mathematics Education Journal* (18), 1-16.

Ernest, P. (2009). "Values and the Social Responsibility of Mathematics", in P. Ernest, B. Greer, and B. Sriraman, (eds.), *Critical Issues in Mathematics Education*. Charlotte, NC: Information Age Publishing pp. 207-216.

Ernest, P. (2010). "Add It Up: Why Teach Mathematics?" *Professional Educator* 9(2), 44-47.

Ernest, P. (2011). *The Psychology of Learning Mathematics: The Cognitive, Affective and Contextual Domains of Mathematics Education*, Saarbrücken, Germany: Lambert Academic Publishing.

Evans, J. (2000). *Adults' Mathematical Thinking and Emotions: A Study of Numerate Practice*, London: Routledge Falmer.

Fennema, E. (1989). "The Study of Affect and Mathematics: A Proposed Generic Model for Research", in D. B. McLeod and E. Adams, (eds.), *Affect and Mathematical Problem Solving: A New perspective*. New York, : Springer-Verlag, pp. 205-219.

Fennema, E., and Sherman, J. A. (1976). "Fennema-Sherman Mathematics Attitude Scales: Instruments Designed to Measure Attitudes Toward the Learning of Mathematics by Females and Males." *Journal for Research in Mathematics Education*, 7(5), 324-328.

Fennema, E., and Sherman, J. A. (1977). "Sex-Related Differences in Mathematics Achievement, Spatial Visualisation and Affective Factors." *American Educational Research Journal* 14(1), 51-71.

Fennema, E., and Sherman, J. A. (1978). "Sex-Related Differences in Mathematics Achievement and Related Factors: A further Study." *Journal for Research in Mathematics Education* 9(3), 189-203.

Ferla, J., Valcke, J., and Cai, Y. (2009). "Academic Self-Efficacy and Academic Self-Concept: Reconsidering Structural Relationships." *Learning and Individual Differences*, 19, 499-505.

Flegg, J., Mallet, D., and Lupton, M. (2011). "Students' Perceptions of the Relevance of Mathematics in Engineering." *International Journal of Mathematical Education in Science and Technology*, 0(0), 1-16.

Forfás. (2008). *The Expert Group on Future Skills Needs Statement of Activity 2007*. Department of Enterprise, Trade and Employment, Dublin.

Forgasz, H. J., Leder, G. C., and Taylor, C. (2007). "Research Versus the Media: Mixed or Single-Gender Settings?" *Australasian Association for Engineering Education Conference*. City: Canberra.

Fraenkel, J., and Wallen, N. (2008). *How to Design and Evaluate Research in Education*, New York, NY: McGraw-Hill.

Fuller, M. (2002). "The Role of Mathematics Learning Centres in Engineering Education." *European Journal of Engineering Education*, 27(3), 241-247.

Gainsburg, J. (2005). "School Mathematics in Work and Life: what we know and how we can learn more." *Technology in Society*, 27, 1-22.

Gainsburg, J. (2006). "The Mathematical Modeling of Structural Engineers." *Mathematical Thinking and Learning* 8(1), 3-36.

Ginzberg, E., Ginsburg, S. W., Axelrad, S., and Herma, J. L. (1951). *Occupational Choice: An Aproach To General Theory*, New York: Columbia University Press.

Gleason, J., Boykin, K., Johnson, P., Bowen, L., Whitaker, K., Micu, C., Raju, D., and Slappey, C. (2010). "Integrated Engineering Math-Based Summer Bridge Program for Student Retention." *Advances in Engineering Education*, 2(2), 1-17.

Goldin, G. A. (2002). "Affect, Meta-Affect, and Mathematical Belief Structures", in Gilah C. Leder, E. Pehkonen, and G. Törner, (eds.), *Beliefs: A Hidden Variable in Mathematics Education?* Dordrecht, The Netherlands Kluwer Academic Publishers.

Goold, E. (1999). *Survey of Young People's Attitudes to Science and Technology*. Institute of Technology Tallaght, Dublin.

Goold, E. (2000). "A Taste of Science and Technology in Transition Year", I. o. T. Tallaght, (ed.). City: Dublin.

Goold, E., and Devitt, F. (2012). "The Role of Mathematics in Engineering Practice and in the Formation of Engineers." To be p*resented at 40th SEFI Annual conference: Engineering Education 2020: Meet the Future*, Thessaloniki, Greece.

Graves, E. (2005). "The Usefulness of Mathematics as Seen by Engineering Seniors", *American Society for Engineering Education Annual Conference & Exposition*. City: Portland, OR.

Greer, B., and Mukhopadhyay, S. (2003). "What is Mathematics Education For? Guest Editorial." *The Mathematics Educator* 13(2), 2–6.

Grimson, J. (2002). "Re-Engineering the Curriculum for the 21st Century." *European Journal of Engineering Education* 27(1), 31-37.

Grübler, A. (1998). *Technology and Global Change,* Cambridge (England) and New York, NY: Cambridge University Press.

Hackett, G., and Betz, N. (1989). "An Exploration of the Mathematics Self-Efficacy/ Mathematics Performance/ Correspondence." *Research in Mathematics Education*, 20(3), 261-273.

Hannula, M. S. (2002). "Attitude Towards Mathematics: Emotions, Expectations and Values." *Educational Studies in Mathematics*, 49(1), 25-46.

Hannula, M. S. (2006). "Motivation in Mathematics: Goals Reflected in Emotions." *Educational Studies in Mathematics*, 63(2), 165-178.

Hannula, M. S., Maijala, H., and Pehkonen, E. (2004). "Development of Understanding and Self-Confidence in Mathematics; Grades 5-8." *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. City: Bergen, Norway, pp. 17-24.

Hardré, P. L., Sullivan, D. W., and Crowson, H. M. (2009). "Student Characteristics and Motivation in Rural High Schools." *Journal of Research in Rural Education*, 24(16), 1-19.

Henderson, S., and Broadbridge, P. (2007). "Mathematics for 21st Century Engineering Students" *Australasian Association for Engineering Education Conference*. City: Melbourne.

Henderson, S., and Broadbridge, P. (2008). *Mathematics Education for 21st Century Engineering Students. Final Report*. Carrick Institute for Learning and Teaching in Higher Education Ltd., Melbourne.

Herzog, S., and Bowman, N. A. (2011). *Validity and Limitations of College Student Self-Report Data: New Directions for Institutional Research*, San Francisco: Jossey-Bass.

Heywood, J. (2005). *Engineering Education: Research and Development in Curriculum and Instruction*, Hoboken, NJ: John Wiley and Sons, Inc.

Higher Education Authority. (2010). *09/10 Higher Education Key Facts and Figures*. Higher Education Authority, Dublin, Ireland.

Higher Education Authority. (2011). *Summary Enrolment Reports 2010/11*. Higher Education Authority, Dublin, Ireland.

Hill, L. G., Goates, S., and Rosenman, R. (2008). "Detecting Selection Bias in Community Disseminations of Universal Family-Based Prevention Programs "*School of Economic Sciences: Working Paper Series*. Washington State University.

Hodgen, J., Küchemann, D., Brown, M., and Coe, R. (2009). "Lower Secondary School Students’ Attitudes to Mathematics: Evidence from a large-scale survey in England", in M. Joubert, (ed.) *Proceedings of British Society for Research into Learning Mathematics* City: Loughborough, UK, pp. 49-54.

Hodgen, J., Pepper, D., Linda, S., and Ruddock, G. (2010). *Is the UK an outlier? An international comparison of upper secondary mathematics education*. Nuffield Foundation, London.

Hoyles, C., Noss, R., Kent, P., and Bakker, A. (2010). *Improving Mathematics at Work: The Need for Techno-Mathematical Literacies*, Oxon and New York: Routledge.

Hoyles, C., Wolf, A., Molyneux-Hodgson, S., and Kent, P. (2002). *Mathematical Skills in the Workplace*. Institute of Education, University of London, London.

International Association for the Evaluation of Educational Achievement. (2011). *TIMSS 2011 Mathematics Framework*. International Association for the Evaluation of Educational Achievement, Amsterdam, Hamburg.

Irish Academy of Engineering. (2004). *The Role of Mathematics in Engineering Education*. Irish Academy of Engineering, Dublin.

Jacobs, J. E., Lanza, S., Osgood, W. D., Eccles, J. S., and Wigfield, A. (2002). "Changes in Children's Self-Competence and Values: Gender and Domain Differences across Grades One through Twelve." *Child Development*, 73(2), 509-527.

Jagacinski, C. M., LeBold, W. K., Linden, K. W., and Shell, K. D. (1985). "Factors Influencing the Choice of an Engineering Career." *Institute of Electrical and Electronics Engineers (IEEE) Transactions on Education*, E-28(1), 7.

James, W., and High, K. (2008). "Freshman-Level Mathematics in Engineering: A Review of the Literature in Engineering Education", *American Society for Engineering Education (ASEE) Annual Conference & Exposition*. City: Pittsburgh, PA.

Janowski, G. M., Lalor, M., and Moore, H. (2008). "A New Look at Upper-Level Mathematics Needs in Engineering Courses at UAB." *American Society for Engineering Education (ASEE) Annual Conference and Exposition* City: Pittsburgh, PA.

Jaworski, B. (2002). "Social Constructivism in Mathematics Learning and Teaching", in L. Haggarty, (ed.), *Teaching Mathematics in Secondary Schools*. London and New York: Rutledge Falmer.

Jeffers, G. (2011). "The Transition Year Programme in Ireland. Embracing and Resisting a Curriculum Innovation." *The Curriculum Journal*, 22(1), 61-76.

Johnson, R. B., Onwuegbuzie, A. J., and Turner, L. A. (2007). "Toward a Definition of Mixed Methods Research." *Journal of Mixed Methods Research* 1(2), 112 - 133.

Katehi, L. (2005). "The Global Engineer", in N. A. o. Engineering, (ed.), *Educating The Engineer of 2020*. Washington, DC: The National Academies Press, pp. 151-155.

Kent, P., and Noss, R. (2002). "The Mathematical Components of Engineering Expertise: The Relationship Between Doing and Understanding Mathematics. "*Institution of Electrical Engineers (I.E.E.) 2nd Annual Symposium on Engineering Education* City: London, England.

Kent, P., and Noss, R. (2003). *Mathematics in the University Education of Engineers*. The Ove Arup Foundation, London.

King, N., and Horrocks, C. (2010). *Interviews in Qualitative Research*, London, California, New Delhi, Singapore: Sage.

King, R. (2008). *Addressing the Supply and Quality of Engineering Graduates for the New Century*. University of Sydney, Sydney.

Klingbeil, N., Rattan, K., Raymer, M., Reynolds, D., and Mercer, R. (2004). "Rethinking Engineering Mathematics Education: A Model for Increased Retention, Motivation and Success in Engineering." *American Society for Engineering Education Annual Conference & Exposition* City: Salt Lake City, UT.

Knight, D. W., Carlson, L. E., and Sullivan, J. F. (2007). "Improving Engineering Student Retention through Hands-On, Team Based, First-Year Design Projects." *31st*

International Conference on Research in Engineering Education. City: Honolulu, HI.

Knight, M., and Cunningham, C. (2004). "Draw an Engineer Test (DAET): Development of a Tool to Investigate Students' Ideas about Engineers and Engineering." *American Society for Engineering Education Annual Conference & Exposition*. City: Salt Lake City, UT.

Koehler, M. S., and Grouws, D. A. (1992). *Mathematics Teaching Practices and their Effects*, New York: Macmillan.

Korte, R., Sheppard, S., and Jordan, W. (2008). "A Qualitative Study of the Early Work Experiences of Recent Graduates in Engineering." *American Society for Engineering Education (ASEE) Annual Conference & Exposition*. City: Pittsburgh, PA.

Kuncel, N. R., Credé, M. R., and Thomas, L. L. (2005). "The Validity of Self-Reported Grade Point Averages, Class Ranks, and Test Scores: A Meta-Analysis and Review of the Literature." *Review of Educational Research*, 75(1), 63-82.

Lampert, M. (1990). "When the Problem is Not the Question and the Solution is Not the Answer: Knowing and Teaching Mathematics." *American Educational Research Journal*, 27(1), 29-63.

Leder, G. (1984). "Sex Differences and Attributions of Success and Failure." *Psychological Reports*, 54(1), 57-58.

Leder, G. (2008). "High Achievers in Mathematics: What Can We Learn From and About Them?", in M. Goos, R. Brown, and K. Makar, (eds.), *Mathematics Education Research Group of Australasia 31*. City: Brisbane.

Lent, R., Brown, S., and Hackett, G. (2002). "Social Cognitive Theory", in D. Brown, (ed.), *Career Choice and Development*. San Francisco: John Wiley & Sons.

Lent, R. W., Brown, S. D., and Hackett, G. (1994). "Towards a Unifying Social Cognitive Theory of Career and Academic Interest, Choice, and Performance." *Journal of Vocational Behavior* 45(1), 79-122.

Lent, R. W., Brown, S. D., and Larkin, K. C. (1986). "Self-Efficacy in the Prediction of Academic Performance and Perceived Career Options." *Journal of Counseling Psychology* 33(3), 265-269.

Lesh, R., and English, L. D. (2005). "Trends in the Evolution of Models and Modeling Perspectives on Mathematical Learning and Problem Solving." *International Reviews on Mathematical Education (Zentralblatt fürDidaktik der Mathematik)* 37(6), 487-489.

Lohmann, J., Rollins, H. A., and Hoey, J. (2006). "Defining, Developing and Assessing Global Competence in Engineers." *European Journal of Engineering Education*, 31(1), 119-131.

Løken, M., Sjøberg, S., and Schreiner, C. (2010). "Who's That Girl? Why Girls Choose Science - in Their Own Words ", in S. Sjøberg, (ed.) *Socio-Cultural and Human Values in Science and Technology Education*. City: Bled, Slovenia.

Lubinski, D., and Benbow, C. P. (2006). "Study of Mathematically Precocious Youth After 35 Years: Uncovering Antecedents for the Development of Math-Science Expertise." *Psychological Science*, 1(4), 316-345.

Lubinski, D., Benbow, C. P., Shea, D. L., Efterkhari-Sanjani, H., and Halvorson, M. B. J. (2001). "Men and Women at Promise For Scientific Excellence: Similarity Not Dissimilarity." *Psychological Science*, 12(2), 309-315.

Lynch, R., and Walsh, M. (2010). "Are We Educating Our Students Out of Engineering?" *The Engineers Journal*, 64(10), 373-376.

Lyons, M., Close, S., Boland, P., Lynch, K., and Sheerin, E. (2003). *Inside Classrooms: the Teaching and Learning of Mathematics in Social Context*, Dublin: Institute of Public Administration.

Magajna, Z., and Monaghan, J. (2003). "Advanced Mathematical Thinking in a Technological Workplace." *Education Studies in Mathematics* 52(2), 101- 122.

Male, S. A., Bush, M. B., and Chapman, E. S. (2010). "Perceptions of Competency Defeciencies in Engineering Graduates." *Australasian Journal of Engineering Education*, 16(1), 55-67.

Male, S. A., Bush, M. B., and S., C. E. (2009). "Identification of Competencies Required by Engineers Graduating in Australia." *Australasian Association for Engineering Education Conference*. City: Adelaide, Australia.

Malmivuori, M.-L. (2006). "Affect and Self-Regulation." *Educational Studies in Mathematics*, 63(2), 149-164.

Maltese, A. V., and Tai, R. H. (2010). "Eyeballs in te Fridge: Sources of Early Interest in Science." *International Journal of Science Education*, 32(5), 669-685.

Maltese, A. V., and Tai, R. H. (2011). "Pipeline Persistence: Examining the Association of Educational Experiences With Earned Degrees in STEM Among U.S. Students." *Science Education*, 95(5), 877-907.

Manseur, Z. Z., Ieta, A., and Manseur, R. (2009). "Panel - Reforming Mathematics Requirements for a Modern Engineering Education." *39th American Society for Engineering Education (ASEE) / Institute of Electrical and Electronics Engineers (IEEE) Frontiers in Education Conference* City: San Antonio, TX.

Manseur, Z. Z., Ieta, A., and Manseur, R. (2010a). "Modern Mathematics Requirements in a Developing Engineering Program." *American Society for Engineering Education (ASEE) Annual Conference & Exposition*. City: Louisville, KY.

Manseur, Z. Z., Ieta, A., and Manseur, R. (2010b). "Work in Progress - Mathematics Preparation for a Modern Engineering Program." *40th ASEE/ IEEE Frontiers in Education Conference*. City: Washington, DC.

Mason, J., and Johnston-Wilder, S. (2004). *Designing and Using Mathematical Tasks*, Milton Keynes: Tarquin Publications.

Masouros, S. D., and Alpay, E. (2010). "Mathematics and Online Learning Experiences: a Gateway Site for Engineering Students." *European Journal of Engineering Education*, 35(1), 59-78.

Matthews, A., and Pepper, D. (2007). *Evaluation of Participation in GCE Mathematics: Final Report. QCA/07/3388*. Qualifications and Curriculum Authority, London.

Matusovich, H., Streveler, R., Miller, R., and Olds, B. (2009). "I'm Graduating This Year! So What is an Engineer Anyway?" *American Society for Engineering Education* City: Austin, TX.

Mayer, R. E., Stull, A. T., Campbell, J., Almeroth, K., Bimber, B., Chun, D., and Knight, A. (2007). "Overestimation Bias in Self-Reported SAT Scores." *Educational Psychological Review* 19(4), 443-454.

McKinsey. (2011). *Growth and Renewal in the United States: Retooling America's Economic Engine*. The McKinsey Global Institute, Washington, DC.

McLeod, D. B. (1989). "The Role of Affect in Mathematical Problem Solving", in D. B. McLeod and V. M. Adams, (eds.), *Affect and Mathematical Problem Solving A New perspective*. New York: Springer-Verlag.

McLeod, D. B. (1992). "Research on Affect in Mathematics Education: A Reconceptualization", in D. A. Grows, (ed.), *Handbook of Research on Mathematics Teaching and Learning* New York: Macmillan, pp. 575-596.

McLeod, D. B., and Adams, V. M. (1989). *Affect and Mathematical Problem Solving*, New York: Springer-Verlag.

McMasters, J. H. (2006). "Influencing Student Learning: An Industry Perspective." *International Journal of Engineering Education* 22(3), 447-459.

McPhan, G., Morony, W., Pegg, J., Cooksey, R., and Lynch, T. (2008). *Maths? Why Not?* Australian Department of Education, Employment and Workplace Relations (DEEWR), Canberra.

McWilliam, E., Poronnik, P., and Taylor, P. G. (2008). "Re-designing Science Pedagogy: Reversing the Flight from Science." *Journal of Science Education and Technology* 17(3), 226-235.

Middleton, J. A., and Spanias, P. A. (1999). "Motivation for Achievement in Mathematics: Findings, Generalizations and Criticisms of the Research." *Journal for Research in Mathematics Education*, 30(1), 65-68.

Miles, M. B., and Huberman, A. M. (1994). *Qualitative Data Analysis: A Expanded Sourcebook*, Thousand Oaks, London, New Delhi: Sage.

Mooney, O., Patterson, V., O'Connor, M., and Chantler, A. (2010). *A Study of Progression in Irish Higher Education*. Higher Education Authority, Dublin.

Morgan, C., Isaac, J. D., and Sansone, C. (2001). "The Role of Interest in Understanding the Career Choices of Female and Male College Students." *Sex Roles*, 44(5&6), 295-320.

Nair, C. S., Patil, A., and Mertova, P. (2009). "Re-engineering Graduate Skills." *European Journal of Engineering Education*, 34(2), 131-139.

Nardi, E., and Steward, S. (2003). "Is Mathematics T.I.R.E.D? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom." *British Educational Research Journal*, 29(3), 345-367.

National Academy of Engineering. (2005). *Educating the Engineer of 2020*, Washington, DC: The National Academies Press.

National Academy of Engineering. (2008). *Changing the Conversation: Messages for Improving Public Understanding of Engineering*, Washington, DC: The National Academy Press.

National Academy of Sciences, National Academy of Engineering, and Institute of Medicine. (2010). *Rising Above the Gathering Storm, Revisited: Rapidly Approaching Category 5*, Washington, D.C.: The National Academies Press.

National Council for Curriculum and Assessment. (2007). *ESRI Research into the Experiences of Students in the Third Year of Junior Cycle and in Transition to Senior Cycle*. National Council for Curriculum and Assessment, Dublin.

National Council for Curriculum and Assessment. (2010a). *Leaving Certificate Mathematics. Draft Syllabus: Strands 1-5. Initial 24 Schools Only. For examination in June 2012*. National Council for Curriculum and Assessment, Dublin.

National Council for Curriculum and Assessment. (2010b). *Overview of Project Maths*. National Council for Curriculum and Assessment, Dublin.

National Council of Educational Research and Training. (2006). *Teaching of Mathematics*. National Council of Educational Research and Training, New Delhi.

National Council of Teachers of Mathematics. (1988). *Curriculum and Evaluation Standards for School Mathematics*. National Council of Teachers of Mathematics, Reston, VA.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*, Reston, VA: NCTM.

National Research Council. (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* National Academy Press, Washington, DC.

National Science Foundation. (2010). *Science and Engineering Indicators 2010*. National Science Foundation, Arlington, VA.

Niss, M. A. (2003). "Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project", in A. Gagatsis and S. Papastavridis, (eds.), *3rd*

Mediterranean Conference on Mathematical Education City: Athens, The Netherlands.

Op ’t Eynde, P., De Corte, E., and Verschaffel, L. (2006). ""Accepting Emotional Complexity”: A Socio-Constructivist Perspective on the Role of Emotions in the Mathematics Classroom." *Educational Studies in Mathematics*, 63(2), 193-207.

Op ’t Eynde, P., and Hannula, M. S. (2006). "The Case Study of Frank." *Educational Studies in Mathematics*, 63(2), 123-129.

Organisation for Economic Co-Operation and Development. (2009). *PISA 2009 Assessment Framework. Key Competencies in Reading, Mathematics and Science*. OECD Publishing, Paris.

Organisation for Economic Co-Operation and Development. (2010). *The High Cost of Low Educational Performance. The Long-Run Economic Impact of Improving PISA Outcomes*. OECD, Paris.

Orton, A., and Wain, G. (1994). "The Aims of Teaching Mathematics", in A. Orton, (ed.), *Issues in Teaching Mathematics*. London: Cassell.

Otung, I. (2002). *Putting Engineering First and Mathematics Second in Engineering Education*. UK Higher Education Academy, London.

Oware, E., Capobianco, B., and Diefes-Dux, H. A. (2007a). "Gifted Students' Perceptions of Engineers? A Study of Students in a Summer Outreach Program." *American Society for Engineering Education (ASEE) Annual Conference and Exposition* City: Honolulu, HI.

Oware, E., Capobianco, B., and Diefes-Dux, H. A. (2007b). "Young Children’s Perceptions of Engineers Before and After a Summer Engineering Outreach Course." *37th American Society for Engineering Education (ASEE) / Institute of Electrical and Electronics Engineers (IEEE) Frontiers in Education Conference* City: Milwaukee, WI.

Oxford English Dictionary. (1989). "Oxford English Dictionary"*Oxford English Dictionary*. City: Oxford: Oxford University Press.

Pajares, F., and Miller, D. M. (1994). "Role of Self-Efficacy and Self-Concept Beliefs in Mathematical Problem Solving: A Path Analysis." *Journal of Educational Psychology*, 86(2), 193-203.

Panitz, B. (1998). "Opening New Doors." *PRISM*, 13(5), 24-29.

Panzeri, S., Magri, C., and Carraro, L. (2008). "Sampling Bias." *Scholarpedia*, 3(9), 4258.

Pape, S. J., Bell, C. V., and Yetkin, I. E. (2003). "Developing Mathematical Thinking and Self-Regulated Learning: A Teaching Experiment in a Seventh-Grade Mathematics Classrroom." *Educational Studies in Mathematics*, 53(3), 179-202.

Park, H., Behrman, J. R., and Choi, J. (2011). "Causal Effects of Single-Sex Schools on Students' STEM Outcomes by Gender and Parental SES." *Meeting of Population Association of America*. City: Washington, DC.

Patton, M. Q. (2002). *Qualitative Research and Evaluation Methods.*, Thousand Oaks, London, New Delhi: Sage Publications.

Pearson, R. W. (1991). "Why Don't Most Engineers Use Undergraduate Mathematics in Their Professional Work?" *Undergraduate Mathematics Education Trends*, 3(4), 8.

Perkins, R., Moran, G., Cosgrove, J., and Shiel, G. (2010). *PISA 2009: The Performance and Progress of 15 Year-Olds in Ireland*. Educational Research Centre, Dublin.

Petocz, P., and Reid, A. (2006). "The Contribution of Mathematics to Graduates' Professional Working Life", in P. L. Jeffery, (ed.) *Australasian Association for Engineering Education (AARE) International Engineering Education Research Conference* City: Melbourne, Australia.

Petocz, P., Reid, A., Wood, L. N., Smith, G. H., Mather, G., Harding, A., Engelbrecht, J., Houston, K., Joel, H., and Perrett, G. (2007). "Undergraduate Students' Conceptions of Mathematics: An International Study." *International Journal of Science and Mathematics Education*, 5(3), 439-459.

Pietsch, J. (2009). *Teaching and Learning Mathematics Together: Bringing Collaboration to the Centre of the Mathematics Classroom*, Newcastle upon Tyne, UK: Cambridge Scholars Publishing.

Pintrich, P. R. (1999). "The Role of Motivation in Promoting and Sustaining Self-Regulated Learning." *International Journal of Educational Research* 31(6), 459-470.

Pólya, G. (1945). *How to Solve it*, New Jersey: Princeton University Press.

Prieto, E., Holbrook, A., Bourke, S., O'Connor, J., Page, A., and Kira, H. (2009). "Influences on Engineering Enrolments. A Synthesis of the Findings of Recent Reports." *European Journal of Engineering Education*, 34(2), 183-203.

Punch, K. F. (2005). *Introduction to Social Research: Quantitative and Qualitative Approaches*, London: Sage.

Radzi, N. M., Abu, M. S., and Mohamad, S. (2009). "Math-Oriented Critical Thinking Skills in Engineering." *International Conference on Engineering Education*. City: Kuala Lumpur.

Reed, E. (2003). "A Review of Mathematics Strategies in Engineering Education." *PROGRESS 3 Conference, Higher Education Funding Council for England*. City.

Reid, A., Petocz, P., Smith, G. H., Wood, L. N., and Dortins, E. (2003). "Mathematics Students' Conception of Mathematics." *New Zealand Journal of Mathematics*, 32(supplement), 163-172.

Reilly, J. (2006). *Using Statistics*, Dublin: Gill & Macmillan.

Ridgway, J. (2002). "The Mathematical Needs of Engineering Apprentices", in A. Bessot and J. Ridgway, (eds.), *Education for Mathematics in the Workplace*. Dordrecht, The Netherlands: Kluwer Academic Press.

Roberts, G. (2002). *SET for Success: the Report of Sir Gareth Roberts' Review. The Supply of People with Science, Technology, Engineering and Mathematical Skills*. Research Councils UK, London.

Robinson, J. C. (2010). "Engineering Education: The Future is Sharpening Up, Not Dumbing Down." *The Engineers Journal*, 64(1), 59-62.

Romberg, T. A. (1992). "Assessing Mathematics Competence and Achievement", in H. Berlak, F. M. Newman, E. Adams, D. A. Archbald, T. Burgess, J. Raven, and T. A. Romberg, (eds.), *Towards A New Science of Educational Testing and Assessment*. Albany, NY: State University of New York Press.

Saldaña, J. (2011). *The Coding Manual for Qualitative Researchers*, London, Thousand Oaks, New Delhi and Singapore: Sage.

Schoenfeld, A. H. (1988). "When Good Teaching Leads to Bad Results: The Disasters of Well Taught Mathematics Classes." *Educational Psychologist*, 23(2), 145-166.

Schoenfeld, A. H. (1992). "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense-Making in Mathematics", in D. A. Grouws, (ed.), *Handbook for Research on Mathematics Teaching and Learning, New York, NY:* Macmillan.

Schoenfeld, A. H. (1994). *Reflections on Doing and Teaching Mathematics*, Hillsdale, NJ: Lawrence Erlbaum Associates.

Schunk, D. H., Pintrich, P. R., and Meece, J. L. (2010). *Motivation in Education: Theory, Research, and Applications*, Upper Saddle River, NJ: Pearson Educational International.

Sedig, K. (2007). "Toward Operationalization of Flow in Mathematics Learnware." *Computers in Human Behavior* 23(4), 2064-2092.

Sheppard, S., Colby, A., Macatangay, K., and Sullivan, W. (2006). "What is Engineering Practice?" *International Journal of Engineering Education*, 22(3), 429-438.

Sheppard, S., Macatangay, K., Colby, A., and Sullivan, W. M. (2009). *Educating Engineers: Designing for the Future of the Field*, Stanford, CA: Jossey-Bass.

Shivy, V. A., and Sullivan, T. N. (2003). "Engineering Students' Perceptions of Engineering Specialties." *Journal of Vocational Behavior*, 67(1), 87-101.

Silverman, D. (2010). *Interpreting Qualitative Data*, London, California, New Delhi, Singapore: Sage.

Sjöberg, S., and Schreiner, C. (2011). "The Next Generation of Citizens: Attitudes to Science Among Youngsters", in M. W. Bauer, R. Shukla, and N. Allum, (eds.), *The Culture of Science - How Does the Public Relate to Science Across the Globe?* New York: Routledge.

Skemp, R. R. (1987). *The Psychology of Learning Mathematics* , Hillsdale, NJ: Lawrence Erlbaum Associates.

Smith, A. (2004). *Making Mathematics Count: The Report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education*. Department for Education and Skills (DfES), London.

Smith, R., Hollebrands, K., Parry, E., Bottomly, L., Smith, A., and Lynn, A. (2009). "The Ways in which K-8 Students' Participation in a GK-12 Program Affects Achievement in and Beliefs about Mathematics." *American Society for Engineering Education*. City: Austin, TX.

Solow, R. M. (1957). "Technical Change and the Aggregate Production Function." *The Review of Economics and Statistics*, 39(3), 312-320.

State Examinations Commission. (2011a). *State Examination Statistics*. Dublin, Ireland.

State Examinations Commission. (2011b). *State Examinations Statistics 2011*. State Examinations Commission, Dublin.

Tang, S., and Trevelyan, J. (2009). "Engineering Learning and Practice - a Brunei Practice?" *Australasian Association for Engineering Education Conference*. City: Adelaide, Australia.

Tapping America's Potential Coalition. (2008). *Tapping America's Potential: Gaining Momentum, Loosing Ground*. The TAP coalition, Washington, DC.

Tilli, S., and Trevelyan, J. (2008). "Longitudinal Study of Australasian Engineering Graduates: Preliminary Results." *American Society for Engineering Education (ASEE) Annual Conference*. City: Pittsburgh, PA.

Trevelyan, J. (2009). "Steps Toward a Better Model of Engineering Practice." *Research in Engineering Education Symposium* City: Palm Cove, QLD.

Trevelyan, J. (2010a). "Mind the Gaps: Engineering Education and Practice." *Australasian Association for Engineering Education (AAEE) Conference*. City: Sydney.

Trevelyan, J. (2010b). "Reconstructing Engineering from Practice." *Engineering Studies*, 2(3), 175-195.

Trevelyan, J. (2011). "Are We Accidently Misleading Students about Engineering Practice?" *Research in Engineering Education Symposium*. City: Madrid.

Triantafillou, C., and Potari, D. (2006). "Mathematical Activity in a Technological Workplace: Results from an Ethnographic Study", in J. Novotná, H. Moraová, M. Krátká, and S. N., (eds.), *30th Conference of the International Group for the Psychology of Mathematics Education*. City: Prague, pp. 297-304.

Tully, D., and Jacobs, B. (2010). "Effects of Single-Gender Mathematics Classrooms on Self-Perception of Mathematical Ability and Post Secondary Engineering Paths: an Australian Case Study." *European Journal of Engineering Education*, 35(4), 455-467.

U.S. Department of Labor. (2007). *Occupational Outlook Handbook: Engineers*, Washington, DC: U.S. Department of Labor and Bureau of Labor Statistics.

U.S. Department of Labor website. (2010-11). "Occupational Outlook Handbook". City: Washington, DC.

Underwood, D. (1997). "Is Mathematics Necessary?" *The College Mathematics Journal*, 28(5), 360-364.

United Nations Educational Scientific and Cultural Organisation. (2007). *Single-Sex School for Girls and Gender Equality in Education*. UNESCO, Bangkok.

Verhage, H., and De Lange, J. (1997). "Mathematics Education and Assessment." *Pythagoras*, 42, 14-20.

Vygotsky, L. S. (1978). "Mind in Society: The Development of Higher Psychological Processes", in M. Cole, V. John-Steiner, S. Scribner, and E. Souberman, (eds.) Mind in Society: The Development of Higher Psychological Processes". Cambridge, MA Harvard University Press.

Watson, A., and Mason, J. (2008). *Mathematics as a Constructive Activity*, New York: Routledge.

Weiner, B. (1994). "Integrating Social and Personal Theories of Achievement Striving." *Review of Educational Research*, 64(4), 557-573.

Wigfield, A. (1994). "Expectancy-Value Theory of Achievement Motivation: A Developmental Perspective." *Educational Psychology Review*, 6, 49-78.

Wigfield, A., and Eccles, J. S. (1992). "The Development of Achievement Task Values: A Theoretical Analysis." *Developmental Review*, 12, 265-310.

Wigfield, A., and Eccles, J. S. (2000). "Expectancy-Value Theory of Achievement Motivation." *Contemporary Educational Psychology*, 26, 68-71.

Wigfield, A., and Eccles, J. S. (2002). "The Development of Competence Beliefs, Expectations for Success and Achievement Values from Childhood through

Adolescents", in A. Wigfield and J. S. Eccles, (eds.), *Development of Achievement Motivation*. San Diego: Academic Press.

Wigfield, A., Eccles, J. S., and Pintrich, P. R. (1996). "Development between the Ages of 11 and 25", in D. Berlinger and R. Calfee, (eds.), *Handbook of Educational Psychology*. New York: Macmillan.

Williams, R. (2003). "Education for the Profession Formerly Known as Engineering." *The Chronicle of Higher Education*, 49(20), B12.

Winkelman, P. (2009). "Perceptions of Mathematics in Engineering." *European Journal of Engineering Education*, 34(4), 305-316.

Wood, L. N. (2008). "Engineering Mathematics - What do Students Think?" *ANZIAM Journal*, 49(4), C513-525.

Wood, L. N. (2010). "Graduate Capabilities in Mathematics: Putting High Level Technical Skills into Context." *International Journal of Mathematical Education in Science and Technology*, 41(2), 189-198.

Wood, L. N., Mather, G., Petocz, P., Reid, A., Engelbrecht, J., Harding, A., Houston, K., Smith, G. H., and Perrett, G. (2011). "University Students' Views of the Role of Mathematics in Their Future." *International Journal of Science and Mathematics Education* 10(1), 99-119.

Wulf, W. A., and Fisher, G. M. C. (2002). "A Makeover for Engineering Education" *Issues in Science and Technology*, 18(3), 35-39.

Yara, P. O. (2009). "Relationship between Teachers' Attitude and Students'Academic Achievement in Mathematics in Some Selected Senior Secondary Schools in Southwstern Nigeria." *European Journal of Social Sciences*, 11(3), 364-369.

Zan, R., Brown, L., Evans, J., and Hannula, M. S. (2006). "Affect in Mathematics Education: An introduction." *Educational Studies in Mathematics*, 63(2), 113-121.

Zeldin, A. L., and Pajares, F. (2000). "Against the Odds: Self-Efficacy Beliefs of Women in Mathematical, Scientific, and Technological Careers." *American Educational Research Journal* 37(1), 215-246.

Zevenbergen, R. (2000). "Research Methods for Mathematics at Work", in A. Bessot and J. Ridgway, (eds.), *Education for Mathematics in the Workplace*. Dordrecht, The Netherlands: Kluwer Academic.

Zimmerman, B. J. (2000). "Attaining Self-Regulation: A Social cognitive Perspective ", in M. Boekaerts, P. R. Pintrich, and M. Zeidner, (eds.), *Handbook of Self-Regulation*. San Diego, CA: Academic Press, pp. 13-39.

Zimmerman, B. J., and Schunk, D. H. (2003). "Educational Psychology: A Century of Contributions". Mahwah, NJ: Lawrence Erlbraum Associates.